Limita posloupnosti - komplexní úloha III

Určete limitu posloupnosti

$$\lim_{n\to\infty} (\cancel{n^4}\, n^2 + \sin(n+1))(\sqrt{n^4+2} - \sqrt{n^4+1}).$$

Řešení

$$= \lim_{n\to\infty} \frac{(n^2+\sin(n+1))(\sqrt{n^4+2}-\sqrt{n^4+1})(\sqrt{n^4+2}+\sqrt{n^4+1})}{\sqrt{n^4+2}+\sqrt{n^4+1}} = \lim_{n\to\infty} \frac{(n^2+\sin(n+1))(n^4+2-n^4-1)}{\sqrt{n^4+2}+\sqrt{n^4+1}} =$$

$$= \frac{n^2\left(1+\frac{\sin(n+1)}{n^2}\right)}{\sqrt{n^4\left(1+\frac{2}{n^4}\right)}+\sqrt{n^4\left(1+\frac{1}{n^4}\right)}} = \lim_{n\to\infty} \frac{\cancel{n^2}\left(1+\frac{\sin(n+1)}{n^2}\right)}{\cancel{n^2}\left(\sqrt{1+\frac{2}{n^4}}+\sqrt{1+\frac{1}{n^4}}\right)} \overset{\text{VOAL}}{=} \frac{1+0}{1+1} = \underline{\underline{\frac{1}{2}}}$$

Věta o limitě součinu omezené a mizející posl.

$\sin(n+1)$ je omezená ; $\frac{1}{n^2} \to 0 \Rightarrow \frac{\sin(n+1)}{n^2} \to 0$

$\sqrt{1+\frac{2}{n^4}}$; neboť $\frac{2}{n^4} \to 0$; dále platí $\lim\limits_{n\to\infty} a_n^q = \left(\lim\limits_{n\to\infty} a_n\right)^q$, $q \in \mathbb{Q}$, $a_n > 0$

$1+\frac{2}{n^4} > 0$

tedy $\lim\limits_{n\to\infty} \sqrt{1+\frac{2}{n^4}} = \sqrt{\lim 1+\frac{2}{n^4}} \overset{\text{VOAL}}{=} \sqrt{1+0} = 1$

Podobně i pro $\sqrt{1+\frac{1}{n^4}}$