12.

nejde VOAL · [illegible] → [illegible] Bläk

$$\lim_{x\to 0^+}\left(\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x}}}}-\sqrt{\frac{1}{x}-\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x}}}}\right)\cdot\frac{\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x}+\frac{1}{x}}}+\sqrt{\frac{1}{x}-\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x}}}}}{\text{dtto}}$$

$$=\lim_{x\to 0^+}\frac{\frac{1}{x}+\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x}}}-\left(\frac{1}{x}+\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x}}}\right)}{\text{dtto}}=$$

$$=\lim_{x\to 0^+}\frac{2\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x}}}}{\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x}}}}+\sqrt{\frac{1}{x}-\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x}}}}}=\lim_{x\to 0^+}\frac{2\sqrt{\frac{\frac{1}{x^2}-\frac{1}{x}}{\frac{1}{x}-\sqrt{\frac{1}{x}}}}}{\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x}}}}+\sqrt{\frac{1}{x}-\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x}}}}}$$

$$=\lim_{x\to 0^+}\frac{2\sqrt{\frac{1-x}{x^2}}}{\left(\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x}}}}+\sqrt{\frac{1}{x}-\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x}}}}\right)\cdot\sqrt{\frac{1}{x}-\sqrt{\frac{1}{x}}}}=$$

$$=\lim_{x\to 0^+}\frac{2\sqrt{1-x}}{x\left(\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x}}}}+\sqrt{\frac{1}{x}-\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x}}}}\right)\cdot\sqrt{\frac{1}{x}-\sqrt{\frac{1}{x}}}}=$$

$$=\lim_{x\to 0^+}\frac{2\sqrt{1-x}}{\sqrt{x}\left(\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x}}}}+\sqrt{\frac{1}{x}-\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x}}}}\right)\cdot\sqrt{x}\cdot\sqrt{\frac{1}{x}-\sqrt{\frac{1}{x}}}}$$

$$=\lim_{x\to 0^+}\frac{2\sqrt{1-x}}{\left(\sqrt{\frac{x}{x}+\sqrt{\frac{x^2}{x}+\sqrt{\frac{x^4}{x}}}}+\sqrt{\frac{x}{x}-\sqrt{\frac{x^2}{x}+\sqrt{\frac{x^4}{x}}}}\right)\cdot\sqrt{\frac{x}{x}-\sqrt{\frac{x^2}{x}}}}$$

$$=\lim_{x\to 0^+}\frac{2\sqrt{1-x}}{\left(\sqrt{1+\sqrt{x+\sqrt{x^3}}}+\sqrt{1-\sqrt{x+\sqrt{x^3}}}\right)\left(\sqrt{1-\sqrt{x}}\right)}=$$

nyní můžeme použít VOAL, ve jm. není nula, odmocnina je spojitá na svém def. oboru => má limitu

$$\frac{2\sqrt{1-0}}{\left(\sqrt{1+\sqrt{0+\sqrt{0^3}}}+\sqrt{1-\sqrt{0+\sqrt{0^3}}}\right)\sqrt{1-0}} =$$

$$= \lim_{x\to 0^+} \frac{2}{2\cdot 1} = \underline{\underline{1}}$$

Plíšk

1. Dokažte z definice, že

a) $\lim_{x\to 1} \left(\frac{x}{2}\right)^3 = \frac{1}{8}$

musí dokázat $\forall \varepsilon > 0 \ \exists \delta > 0 \ \forall x$

$\left|\frac{x^3}{8} - \frac{1}{8}\right| < \varepsilon$

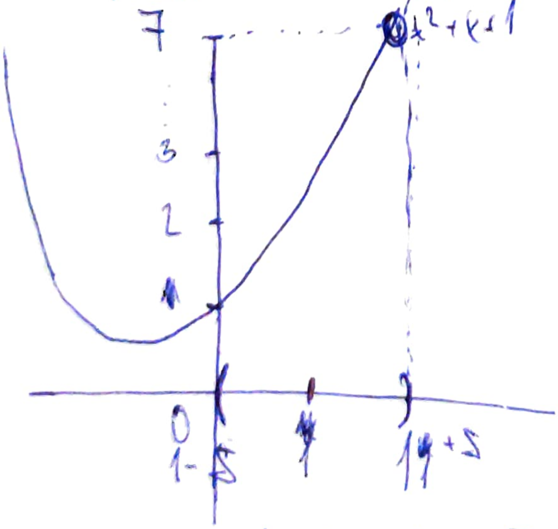
$8|x^3 - 1| < \varepsilon$

$|x^3 - 1| < \frac{\varepsilon}{8}$

$|x-1|\cdot|x^2+x+1| < \frac{\varepsilon}{8}$

$0 < |x-1| < \delta$

$1-\delta < x < 1+\delta$



pokud $\delta \le 1$ pak

$|x^2+x+1| < 7$

$\Rightarrow |x-1|\,|x^2+x+1| < |x-1|\cdot 7 < \frac{\varepsilon}{8}$

$|x-1| < \frac{\varepsilon}{35}$

$\Rightarrow \delta = \frac{\varepsilon}{35}$

$\Rightarrow \forall \varepsilon > 0 \ \exists \delta = \min\left\{1, \frac{\varepsilon}{35}\right\}$

$0 < |x-1| < \delta$ $\qquad\qquad$ $\delta = \min\left\{1, \frac{\varepsilon}{35}\right\}$

$$|x^3 - 1| = |x-1|\,|x^2+x+1|$$

$$|x-1|\cdot|x^2+x+1| < |x-1|\cdot 7$$

— tato nerovnost určitě platí pro $\delta \leq 1$, což je vždy protože δ je min. z 1 a $\frac{\varepsilon}{35}$

$$< \frac{\varepsilon}{35}\cdot 7 = \frac{\varepsilon}{5}$$

$$|x^2+x+1| < 7$$

- když $\frac{\varepsilon}{35}$ je menší než 1 můžeme položit
  $|x-2| < \delta = \frac{\varepsilon}{35}$ a nerovnost platí
- když minimum je 1 můžeme
  $$\underbrace{|x-2| < 1 < \frac{\varepsilon}{35}}_{}$$
  plyne z toho, že 1 je min z $\left\{1, \frac{\varepsilon}{35}\right\}$ $\rightarrow$
  $1 < \frac{\varepsilon}{35}$
- pro úplnost bychom mohli ještě vynásobit celou nerovnost δ, abychom měli ε na jedné straně nerovnice.