9. HODINA

Integrování parciálních zlomků

— jen předběžně, ale složitější případy —

(1.) $\alpha \in \mathbb{R}$: $\int \frac{1}{x-\alpha}\,dx = \ln|x-\alpha| + C$ | | DŮLEŽITÉ

$x \neq \alpha$

(2.) $\alpha \notin \mathbb{R}$:

K $\frac{C}{x-\alpha}$ najít $\frac{\bar{C}}{x-\bar{\alpha}}$ a sečíst: $\frac{Cx+\bar{C}x - C\bar{\alpha} - \bar{C}\alpha}{x^2-2\mathrm{Re}(\alpha)x+|\alpha|^2}$

$C+\bar{C}$ je reálné

Tj. $\frac{Ax+B}{x^2-2\mathrm{Re}(\alpha)x+|\alpha|^2}$.

$C\bar{\alpha}+\bar{C}\alpha$ je také reálné

!! Ponaučení: $\alpha \notin \mathbb{R} \wedge n=1 \Rightarrow$ vidět jako kvadratické členy ve jmenovateli.

Toto budeme integrovat na ln a arctg rozdělením:

TJ ABSTRAKTNÍ SCHÉMA:

$$\frac{Ax}{x^2-2\mathrm{Re}(\alpha)x+|\alpha|^2} + \frac{B}{x^2-2\mathrm{Re}(\alpha)x+|\alpha|^2} =$$

pr

$$= \frac{A}{2}\frac{(x^2-2\mathrm{Re}(\alpha)x+|\alpha|^2)'}{x^2-2\mathrm{Re}(\alpha)x+|\alpha|^2} + \underbrace{\frac{\cancel{2}\mathrm{Re}(\alpha)A}{x^2-2\mathrm{Re}(\alpha)x+|\alpha|^2} + \frac{B}{\cdots}}$$

$\Downarrow \int$

$\Downarrow$ TOTO NA arctg

$$A\ln|x^2-2\mathrm{Re}(\alpha)x+|\alpha|^2| + c$$

$$\int \frac{f'(x)}{f(x)}\,dx = \int [\ln(f(x))]'\,dx = $$

$$\left[\uparrow \int (\ln f(x))' dx = \int f'(x)\frac{1}{f(x)}\,dx \right]$$

$$= \ln|f(x)| + c$$

$$\int \frac{1}{x^2+1} = \mathrm{arctg}\, x + C$$

Ve (2) jde tedy o log. derivaci a pak ... o doplnění na čtverec

3. $\alpha \in \mathbb{C}$ avšak $\int (x-\alpha)^{-n} dx \quad n>1$

SNAŽŠÍ $\mathbb{R}$

$\alpha \in \mathbb{R}$ zjevně: $\int (x-\alpha)^{-n} dx = \frac{-1}{-n+1} (x-\alpha)^{-n+1} + c$

$$\int (x-3)^{-5} dx = \frac{-1}{-4} (x-3)^{-4} = -\frac{1}{4} \frac{1}{(x-3)^4}$$

Pro $\alpha \in \mathbb{C}$ mohu udělat totéž:

$$\int \left[\frac{c}{(x-\alpha)^n} + \frac{\bar{c}}{(x-\bar{\alpha})^n}\right] dx = \int \frac{c\, dx}{(x-\alpha)^n} + \int \frac{\bar{c}\, dx}{(x-\bar{\alpha})^n} =$$

stále $n>1$ $\quad \int y^n = \frac{y^{n+1}}{n+1} + C$

$$= \frac{c(x-\alpha)^{-n+1}}{-n+1} + \frac{\bar{c}(x-\bar{\alpha})^{-n+1}}{-n+1} = \frac{1}{1-n}\left[\frac{\overbrace{c(x-\alpha)^{n-1}}^{A} + \overbrace{\bar{c}(x-\bar{\alpha})^{n-1}}^{\bar{A}}}{[(x-\alpha)(x-\bar{\alpha})]^{n+1}}\right]$$

$A + \bar{A} \in \mathbb{R}$!

$$\boxed{\text{SUBSTITUCE}}$$

Dvě věty: $\varphi: I \to J$, $\varphi' \exists$

1. $F(x) := \int f(x)dx \ \exists \Rightarrow \exists \int f(\varphi(x))\varphi'(x)dx = F(\varphi(x))$

2. φ prosté a $\varphi' \neq 0$ a $\exists G(t) := \int f(\underbrace{\varphi(t)}_{(=x)})\varphi'(t)dt$.
   (neboť $\varphi' > 0$ popř. $\varphi' < 0$)

Pak $\exists \int f(x)dx = G \circ \varphi^{-1}(x)$.

Př.: 1. a) $\int x e^{-x^2} dx = -\frac{1}{2}\int \underline{(-2x)} e^{\underline{-x^2}} dx = -\frac{1}{2}\int (e^{-x^2})' dx =$

$= -\frac{1}{2} e^{-x^2} + C$

Umím $\int \underbrace{e^y}_{f(y)} dy$ a spočtu $\int f'(\varphi(x))\, g'(x)\, dx$ popř. $\underline{\underline{\text{1.VOS}}}$

$-\frac{1}{2}$ krát $\int f(\varphi(x))\, \varphi'(x)\, dx$

b) Jest tedy $f(y) = e^y$, $\varphi(x) = -x^2$

$-\frac{1}{2}\int f(\varphi(x))\varphi'(x)\, dx = -\frac{1}{2}\int e^{-x^2}(-2x)\, dx = \int x e^{-x^2} dx$

Věta říká, že jde o $-\frac{1}{2}(F\circ\varphi)(x) = -\frac{1}{2} e^{\varphi(x)} = -\frac{1}{2} e^{-x^2}$,

což nám vyšlo.

2. a) $\int \sin^3 x \cos x\, dx = \frac{1}{4}\int (\sin^4 x)'\, dx = \frac{1}{4}\sin^4 x + C$

NOVÁ FCE PŮVODNÍ

b) $f(y) = y^3$, $y = \varphi(x) = \sin x$ $\underline{\underline{\text{1. VOS}}}$

$F(y) = \frac{1}{4} y^4$ $\int f(\varphi(x))\varphi'(x)\, dx = \int \sin^3 x \cos x\, dx$

!! 1. $\int$-VOS: $F(\varphi(x)) = \frac{1}{4}\sin^4 x + C$

c) formálně $y = \sin x$ $\int y^3 dy = \frac{1}{4} y^4 + C$

$dy = \cos x\, dx$

STARÁ NOVÁ STARÁ JE FCE NOVÉ

3. $\int \sqrt{a^2 + x^2}\, dx = \left| \begin{array}{l} x = a \sinh t \\ dx = a\cosh t\, dt \\ \varphi: t \mapsto x \end{array} \right| = \int \underbrace{\sqrt{a^2 + a^2 \sinh^2 t}}_{f(\varphi(t))}\ \underbrace{a \cosh t\, dt}_{\varphi'(t)}$

$a > 0$

$= \int a^2 \cosh^2 t\, dt$ umím! Umím tedy $\int f(g(t)) g'(t)\, dt$

a pomocí něj $\int f(x)\, dx$. Jde o $\underline{\text{2-VOS}}$

$$\int \cosh^2 t\,dt = \left| \begin{matrix} f = \cosh t & f' = \sinh t \\ g' = \cosh t & g = \sinh t \end{matrix} \right| = \cosh t \sinh t - \int \sinh^2 t\,dt =$$

$$= \cosh t \sinh t - \int \underline{(\cosh^2 t - 1)}\,dt = \cosh t \sinh t - \int \cosh^2 t + \underline{t} \Rightarrow$$

$$\Rightarrow \int \cosh^2 t\,dt = \underline{\frac{1}{2}(t + \sinh t \cdot \cosh t)}$$

Celkově $\displaystyle\int \sqrt{a^2+x^2}\,dx = \frac{a^2}{2}\left(\operatorname{arcsinh}\frac{x}{a} + \frac{x}{a}\cosh\left(\operatorname{arcsinh}\frac{x}{a}\right)\right) + C$

Výsledek má odpovídat $G(t) := \int f(\varphi(t))\varphi'(t)\,dt$

$G \circ \varphi^{-1}(x)$, tj. do G dosadím za $\underline{t}$ inverzní fci.

To jsme přesně učinili $\operatorname{arcsinh}\frac{x}{a} = t$

H na přednášce. My jsme psali G.

(3')

Př. $\int \frac{1}{\sqrt{1-x^2}(\arcsin x)^2}\,dx \quad \left| \begin{matrix} x=\sin t \\ t=\arcsin x \\ dt=\frac{1}{\sqrt{1-x^2}}dx \end{matrix} \right| \quad \int \frac{dt}{t^2} = -\frac{1}{t} = -\frac{1}{\arcsin x}$.

$x\in(-1,1)$

$$\int \sin^7 x\,dx = \int \sin x\,(1-\cos^2 x)^3\,dx = \left| \begin{matrix} t=\cos x \\ dt=-\sin x\,dx \end{matrix} \right|$$

$$= -\int (1-t^2)^3\,dt = -\int (1-3t^2+3t^4-t^6)\,dt = -t+t^3-\frac{3t^5}{5}+\frac{t^7}{7} =$$

$$= -\cos x+\cos^3 x-\frac{3\cos^5 x}{5}+\frac{\cos^7 x}{7}+C$$

Př.:

$$\int 1\arccos x\,dx = \left| \begin{matrix} f=\arccos x & f'=\frac{-1}{\sqrt{1-x^2}} \\ g'=1 & g=x \end{matrix} \right| = x\arccos x+\int \frac{x\,dx}{\sqrt{1-x^2}}$$

1.VOS-$\int$

$$= x\arccos x + \int -\frac{1}{2}\frac{-2x}{\sqrt{1-x^2}}\,dx = x\arccos x - \int \left(\sqrt{1-x^2}\right)'dx$$

$\left(\sqrt{1-x^2}\right)'$

$$= x\arccos x - \sqrt{1-x^2} + C.$$

Pozn.: Obdobně $\int \operatorname{arcsinh} x\,dx$, $\int \operatorname{arctanh} x\,dx$.

Př.: $\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx = \left| \begin{matrix} t=\cos x \\ dt=-\sin x\,dx \end{matrix} \right| = -\int \frac{dt}{t} =$

$$= -\ln|t|+C = -\ln|\cos x|+C$$

0. Integrace parc. zlomků – podrobněji !!

Př.: Předp. příklad vede na $\int$ rac. lom. fce. Nechť dojdeme k tomuto rozkladu

$$\frac{1}{x-3} + \frac{i}{(x-i)^2} - \frac{i}{(x+i)^2} + \frac{2+i}{x-1-2i} + \frac{2-i}{x-1+2i}$$

- $\int \frac{1}{x-3}\,dx = \ln|x-3| + c$

$\int$ zvlášť

- $$\int\left[\frac{i}{(x-i)^2} - \frac{i}{(x+i)^2}\right]dx = \frac{i}{x-i}\cdot\frac{1}{-1} - \frac{i}{x+i}\cdot\frac{1}{-1} + c =$$

$$= \frac{-i}{x-i} + \frac{i}{x+i} = \frac{-ix+1+ix+1}{x^2+1} = \frac{2}{x^2+1}$$

Lze i nejdřív sečíst, pak $\int$, ale zdlouhavé a zde zbytečné.

- Pro $\alpha \in \mathbb{C}\setminus\mathbb{R} \wedge n=1$ je jediné, co neumíme zintegrovat.

Nejdřív sečíst.

$$\frac{4x-8}{x^2-2x+4} = 2\,\frac{\boxed{2x-2}-2}{x^2-2x+4} =$$

$$= 2\,\frac{2x-2}{x^2-2x+4} - 4\,\frac{1}{x^2-2x+4} \Rightarrow$$

$$2\int\frac{2x-2}{x^2-2x+4}\,dx = 2\ln|\underbrace{x^2-2x+4}_{(>0)}| + C$$

$$\int\frac{1}{x^2-2x+4}\,dx = \int\frac{1}{(x-1)^2-1+4}\,dx = \int\frac{1}{(x-1)^2+3}\,dx =$$

3'''

$$\left|\begin{array}{l} y = x-1 \\ dy = dx \end{array}\right| \int \frac{1}{y^2+3}\,dy = \left|\begin{array}{l} \varphi(z) = \sqrt{3}z \\ y = \sqrt{3}\,z \\ dy = \sqrt{3}\,dz \end{array}\right| = \int \frac{\sqrt{3}\,dz}{\underline{3z^2+3}\ \cancel{\sqrt{3}}} =$$

$$= \frac{1}{\cancel{3}\sqrt{3}} \int \frac{1}{z^2+1}\,dz = \frac{1}{\cancel{3}\sqrt{3}} \operatorname{arctg}(\sqrt{3}\,y) + C = \frac{1}{\cancel{3}\sqrt{3}} \operatorname{arctg}\left[\sqrt{3}\,(x-1)\right] + C.$$

Př.: Cvičení se již jen v $\dfrac{px+q}{ax^2+bx+c}$.

Př.: Spočtěte: $\displaystyle\int \frac{x+1}{x^2+3x+\cancel{7}4}\,dx = \frac{1}{2}\int \frac{2x+3}{x^2+3x+4}\,dx - \frac{1}{2}\int \frac{dx}{x^2+3x+4}$

$$= \frac{1}{2}\ln|x^2+3x+\cancel{7}4| - \frac{1}{2}\int \frac{dx}{x^2+3x+4}$$

Na whiteboardu misto 4 priklad se 7.

- $D = 9-16 = -5 < 0$, tj. $\displaystyle\int \frac{1}{\underbrace{x^2}_{a} + 2\cdot\underbrace{\tfrac{3}{2}}_{b}\underbrace{x}_{a} + \underbrace{\tfrac{9}{4}}_{b^2} + 4 - \tfrac{9}{4}}\,dx =$

$$= \int \frac{dx}{\left(x+\frac{3}{2}\right)^2 + \frac{5}{4}} = \int \frac{dx}{\frac{5}{4}\left[\frac{4}{5}\left(x+\frac{3}{2}\right)^2 + 1\right]} =$$

$$= \frac{4}{5}\int \frac{dx}{\left[\frac{2}{\sqrt{5}}\left(x+\frac{3}{2}\right)\right]^2+1}\,dx \quad \left| \ y = \frac{2}{\sqrt{5}}\left(x+\frac{3}{2}\right) \ \right| =$$

$$= \frac{4}{5}\int \frac{\frac{\sqrt{5}}{2}\,dy}{y^2+1}\,dy = \frac{2}{\sqrt{5}}\operatorname{arctg} y + C =$$

$$= \frac{2}{\sqrt{5}}\operatorname{arctg}\frac{2}{\sqrt{5}}\left(x+\frac{3}{2}\right) + C$$

Celkem $\displaystyle\frac{1}{2}\ln|x^2+3x+7| - \frac{1}{\sqrt{5}}\operatorname{arctg}\frac{2}{\sqrt{5}}\left(x+\frac{3}{2}\right) + C$

(Jde$\hat{}$ o 2. VOS)
predevsim

4

1. $R(e^{ax})$

$y = e^{ax}$, více exponenc., pak volím nejmenší a...

Př.: $\int \frac{1}{e^{2x}+e^x-2}\,dx = \left| \begin{array}{l} y = e^x \\ dy = e^x dx \\ dx = e^{-x}dy = \frac{dy}{y} \end{array} \right| = \int \frac{1}{y}\frac{dy}{(y^2+y-2)} =$

$$\frac{1}{y}\,\frac{1}{y^2+y-2} = \frac{1}{y(y-1)(y+2)} = \frac{A}{y} + \frac{B}{y-1} + \frac{C}{y+2}$$

$$\Rightarrow A = -\frac{1}{2},\ B = \frac{1}{3},\ C = \frac{1}{6}$$

$$-\frac{1}{2}\int \frac{1}{y}\,dy + \frac{1}{3}\int \frac{1}{y-1}\,dy + \frac{1}{6}\int \frac{1}{y+2}\,dy = -\frac{1}{2}\ln|y| + \frac{1}{3}\ln(e^x-1) + \frac{1}{6}\ln(e^x+2)$$

$$+ C = -\frac{1}{2}x + \frac{1}{6}\ln\,(e^x-1)^2(e^x+2) + C.$$

2. $\frac{R(\ln x)}{x}$

$y = \ln x$

Př.: $\int \frac{\ln^2 x + \ln x + 1}{x}\,dx \quad \left| \begin{array}{l} y = \ln x \\ dy = \frac{1}{x}dx \\ x>0 \end{array} \right| = \int (y^2 + y + 1)\,dy =$

$$= \frac{y^3}{3} + \frac{y^2}{2} + y + C = \frac{\ln^3 x}{3} + \frac{\ln^2 x}{2} + \ln x + C$$

3. Goniometrické substituce

1. $y = \operatorname{tg}\frac{x}{2}$ („univerzální") NEBO $t = \operatorname{tg}\frac{x}{2}$, ALE $x = 2\operatorname{arctg} y$ „blízko", tak radši y

2. $y = \operatorname{tg} x$ (sudé při současné záměně $\sin \to -\sin$, $\cos \to -\cos$)

Př.

$$\int \frac{\sin^2 x}{1+\sin^2 x}\,dx \;\left|\begin{array}{l} R(-\cos x,-\sin x)=R(\cos x,\sin x)\\ y=\operatorname{tg} x\;,\; x=\operatorname{arctg} y\\ y^2=\dfrac{\sin^2 x}{1-\sin^2 x}\Rightarrow \sin^2 x=\dfrac{y^2}{1+y^2}\\ dx=\dfrac{1}{1+y^2}\,dy \end{array}\right| =$$

$$= \int \frac{y^2}{1+y^2}\cdot\frac{1}{1+\frac{y^2}{1+y^2}}\cdot\frac{1}{1+y^2}\,dy = \int \frac{y^2}{(1+y^2)(1+2y^2)}\,dy$$

$$= \int\left(\frac{1}{1+y^2}-\frac{1}{1+2y^2}\right)dy = \operatorname{arctan} y - \int\frac{dy}{2y^2+1}\;\left|\begin{array}{l} z=\sqrt{2}\,y\\ dz=\sqrt{2}\,dy\end{array}\right.$$

$$= \operatorname{arctg} y - \int\frac{\frac{dz}{\sqrt{2}}}{z^2+1}\,dz = \operatorname{arctg}\operatorname{tg} x -$$

$$-\frac{1}{\sqrt{2}}\operatorname{arctg}\sqrt{2}\,y + C = x - \frac{1}{\sqrt{2}}\operatorname{arctg}\left(\sqrt{2}\operatorname{tg} x\right)+C$$

$x \in (-\pi,\pi)$

TABLET

Př.

$$\int \frac{1}{2\sin x-\cos x+5}\,dx \;\left|\begin{array}{ll} y=\operatorname{tg}\dfrac{x}{2} & \cos x=\dfrac{1-y^2}{1+y^2}\\ dx=\dfrac{2}{1+y^2} & \cos x=\dfrac{2y}{1+y^2}\end{array}\right| =$$

$$= \int \frac{1}{\frac{4y}{1+y^2}+\frac{y^2-1}{y^2+1}+5}\cdot\frac{2}{1+y^2}\,dy = \int\frac{2}{6y^2+4y+4}\,dy =$$

$$= \int\frac{1}{3y^2+2y+2}\,dy = \frac{1}{3}\int\frac{1}{y^2+\frac{2}{3}y+\frac{2}{3}}\,dy =$$

$$= \frac{1}{3}\int\frac{dy}{y^2+\underbrace{2\cdot\tfrac{1}{3}\,y}_{2ab}+\frac{1}{9}+\frac{2}{3}-\frac{1}{9}} = \frac{1}{3}\int\frac{dy}{\left(y+\frac{1}{3}\right)^2+\frac{5}{9}} =$$

$$= \frac{1}{3}\frac{9}{5}\int \frac{dy}{\frac{9}{5}\left(y+\frac{1}{3}\right)^2 + \underline{\underline{1}}} = \frac{3}{5}\int \frac{dy}{\underbrace{\left(\frac{3y+1}{\sqrt{5}}\right)^2+1}_{\text{lineární}}} \qquad \left| \begin{array}{l} z = \frac{3y+1}{\sqrt{5}} \\ dz = \frac{3}{\sqrt{5}}\,dy \end{array} \right|$$

1. subst.

$$= \frac{3}{5}\int \frac{\sqrt{5}}{3}\frac{dz}{z^2+1} = \frac{1}{\sqrt{5}}\operatorname{arctg}\frac{3y+1}{\sqrt{5}} = \frac{1}{\sqrt{5}}\operatorname{arctg}\left(\frac{3\operatorname{tg}\frac{x}{2}+1}{\sqrt{5}}\right)+C$$

④ Integrály*) $\int R\left(x, \left(\frac{ax+b}{cx+d}\right)^{1/s}\right)dx$, $t = \left(\frac{ax+b}{cx+d}\right)^{\frac{1}{s}}$

Př.: $\int \frac{\sqrt{2x+3}+x}{\sqrt{2x+3}-x}\,dx$ $\left| \begin{array}{l} t = \sqrt{2x+3} \\ t^2 = 2x+3 \\ x = \frac{1}{2}(t^2-3) \end{array} \quad dx = \frac{1}{2}(2t\,dt) = t\,dt \right|$

$2x+3>0$

$$= \int \frac{t+\frac{1}{2}(t^2-3)}{t-\frac{1}{2}(t^2-3)}\,t\,dt = \int \frac{t^2+2t-3}{-t^2+2t+3}\,t\,dt =$$

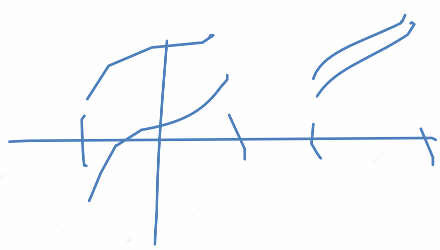
dělte a rozložte

$$= \int\left(-t-4-\frac{9}{t-3}+\frac{1}{t+1}\right)dt =$$

$$= -\frac{1}{2}t^2 - 4t - 9\ln|t-3| + \ln|t+1| + C =$$

$$= -\frac{1}{2}(2x+3)^2 - 4(2x+3) - 9\ln\left|\sqrt{2x+3}-3\right| + \ln\left|\sqrt{2x+3}+1\right| + C$$

✓



*) Mocninné subst.

Př.: $\int \sqrt{\frac{x-1}{x+2}}\, x\, dx$ $\Big|$ $t=\sqrt{\frac{x-1}{x+2}}$

$t^2x+2t^2=x-1 \nearrow$

$t^2x-x=-1-2t^2$

$x(t^2-1)=-1-2t^2$

$x=\frac{-1-2t^2}{t^2-1}=\frac{1+2t^2}{1-t^2}$

$$dx=\frac{4t-4t^3+2t+4t^3}{(1-t^2)^2}\,dt=\frac{6t}{(1-t^2)^2}\,dt$$

$$\int t\, \underline{\frac{1+2t^2}{1-t^2}\,\frac{6t}{(1-t^2)^2}}\,dt$$

. Dál parciální zlomky. Všechny kořeny reálné.

Př. $\int\sqrt{1-x^2}\,dx$ je také typu výše: $\int\sqrt{(1-x)(1+x)}\,dx=\int \underbrace{\sqrt{\frac{1-x}{1+x}}}_{=t}(1+x)\,dx$

Avšak lze počítat i substitucí $x=\sin t$

$$\int\sqrt{1-\sin^2 t}\,\cos t\,dt = \int\cos^2 t\,dt = \text{per-partes} \rightarrow \text{proveďte}$$

Omezme se na $(0,\pi)$

Anebo

$$\int\frac{\cos 2x+1}{2}\,dx=\frac{1}{2}\,\frac{\sin 2x}{2}+\frac{1}{2}x+C$$

$\cos 2x=\cos^2x-\sin^2x=$
$=\cos^2x-1+\cos^2x=$
$=2\cos^2x-1$

$$\int\cos 2x\,dx \;\Big|\; y=2x \;\Big| = \int\cos y\,\frac{dy}{2} =$$

$$=\frac{\sin y}{2}+C=\frac{\sin 2x}{2}$$

(5) Eulerovy substituce $\int R(x,y)\,dx$, $y=\sqrt{ax^2+bx+c}$

a) Dva reálné: $x_1=x_2 \Rightarrow a|x-x_1|$

$x_1\neq x_2 \Rightarrow \sqrt{a(x-x_1)(x-x_2)}=\sqrt{a\frac{x-x_1}{x-x_2}}\,(x-x_2)$

Vytknutím $a \rightsquigarrow$ typ minulý

b) $a>0$ $\sqrt{ax^2+bx+c}=\pm\sqrt{a}\,x+t$

c) $c>0$ $\sqrt{ax^2+bx+c}=\sqrt{c}\pm xt$

Př.

$$\int \frac{x^2}{2\sqrt{1-x^2}} = \int \frac{x^2}{2\sqrt{(1-x)(1+x)}}\,dx =$$

$$= \int \frac{x^2}{2}\sqrt{\frac{1+x}{1-x}}\,(1+x)$$. Máme minulý typ.

Př.:

$$\int \frac{dx}{x+\sqrt{x^2+x+1}} \qquad \sqrt{x^2+x+1} = x+t$$

$$\int \frac{1}{2\frac{t^2-1}{1-2t}+\cancel{t}} \qquad x = \frac{t^2-1}{1-2t}$$

$$\frac{dx}{dt} = \frac{2t(1-2t)+2(t^2-1)}{(1-2t)^2} =$$

$$\int \frac{-2\,\frac{t^2-t+1}{1-2t}\,dt}{2\frac{t^2-1}{1-2t}+t}$$

$$= \frac{2t-4t^2+2t^2-2}{(1-2t)^2} =$$

$$= \frac{-2t^2+2t-2}{1-2t} = -2\,\frac{t^2-t+1}{1-2t}$$

$$= \int \frac{-2(t^2-t+1)}{2\cancel{t^2}-2+t-2\cancel{t^2}}\,dt =$$

$$= \int \frac{-2t^2+2t-2}{t-2} = -2\int \frac{t^2-t+1}{t-2}$$

$$(t^2-t+1):(t-2) = t+1+\frac{3}{t-2}$$

$$\begin{array}{l} -(t^2-2t) \\ \hline \quad t+1 \\ \quad -(t-2) \\ \hline \qquad 3 \end{array}$$

$$-2\int\left(t+1+\frac{3}{t-2}\right)dt=-2\left[\frac{t^2}{2}+t+3\ln|t-2|\right]+C$$

$$=-t^2-2t-6\ln|t-2|+C=$$

$$=-\left(\sqrt{x^2+x+1}-x\right)^2-2\left(\sqrt{x^2+x+1}-x\right)-6\ln\left|\sqrt{x^2+x+1}-x-2\right|+C$$

Př.

$$\int\frac{2x+3}{(x-2)(x+5)}\,dx \qquad \Bigg| \qquad \frac{2x+3}{(x-2)(x+5)}=\frac{A}{x-2}+\frac{B}{x+5}$$

$$A=\frac{7}{7},\quad B=\frac{-7}{-7}=1$$

$$\ln|x-2|+\ln|x+5|$$

Doporučuji k písemce: Dopočíst příklady nespočtené, pak Kopáček, event. rel. složité příklady od M. Pokorného – „můj" web

<u>Lepení</u>: $a<b<c$ | F_1 na (a,b) prim k f

$f:(a,c)\to\mathbb{R}$ | F_2 na (b,c)

spojitá

$$\text{Pak } F(x)=\begin{cases} F_1(x) & x\in(a,b)\\ \lim\limits_{x\to b^-} F_1(x) & x=b\\ F_2(x)-\lim\limits_{x\to b^+}F_2(x)+\lim\limits_{x\to b^-}F_1(x), & x\in(b,c)\end{cases}$$

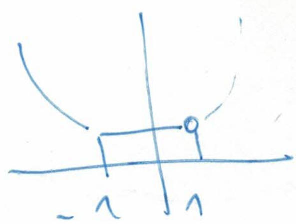
je prim k f na (a,c).

<u>Př.:</u> Spočtěte $\int \max\{1,x^2\}\,dx$ na $(-\infty,1)$

$x\in(-\infty,-1)$ $\quad f(x)=x^2$ $\quad \int f(x)=\frac{x^3}{3}+C_1=F_1(x)$

$x\in(-1,1)$ $\quad f(x)=1$ $\quad \int f(x)=x+C_2=F_2(x)$

$$\lim_{x\to-1^-}F_1(x)=-\frac{1}{3}+C_1$$



$$\lim_{x\to-1^+}F_2(x)=1+C_2$$

$$\int \max\{1,x^2\}\,dx = \begin{cases} \frac{x^3}{3}+C_1 & x<-1\\ -\frac{1}{3}+C_1 & x=-1\\ x+C_2-1-C_2+-\frac{1}{3}+C_1= & \\ = x-\frac{2}{3}+C_1-C_2+C_2=x-\frac{2}{3}+C_1 & x\in\langle-1,1)\end{cases}$$