**Mathematics for Economists I**
**Problems 8**

## Tangent to the graph of a function

**1.** Consider the function $f(x) = -2x^2 + 3x + 2$. Find the equation of its tangent at the point $x_0 = 1$. Sketch the parabola with intercepts with the axes, the vertex, and the specified tangent.

**2.** The parabola is given as the graph of the function $f(x) = 3x^2 + x - 2$. Determine the points $x_0 \in \mathbb{R}$ at which the tangent to the graph of $f$ has the equation $y = kx + q$ with $k = -5$. At each such point, calculate the value of the coefficient $q$ and write the equation of the corresponding tangent. Sketch this parabola with the intersections marked with the axes, the vertex, and the specified tangent including the point of contact with the parabola.

**3.** The hyperbola is given as the graph of the function

$$f(x) = \frac{2x + 1}{x - 2}.$$

Determine the equation of the tangent to the graph of $f$ at the point $x_0 = 3$. Sketch this hyperbola with intercepts with the axes, center, asymptotes and with the specified tangent. For the tangent, determine and mark its intersections with the axes and the point of contact with the hyperbola.

**Voluntary problems:**

**4.** Let $f(x) = x^2 + bx + c$. Determine the values of the coefficients $b, c$ so that the function $f$ has a tangent $y = -2x + 3$ at the point $x_0 = 3$. For these calculated values of $b, c$, sketch the corresponding parabola including the intersections with the axes, the vertex of the parabola and the specified tangent.

**5.** The function $f(x) = x^3 - 6x^2 + 10x$ is given. Determine the points $x_0$ in which the tangent to the graph of the function $f$ has the equation $y = ax + b$ with direction $a = 1$. At each such point, then calculate the value of the coefficient $b$ and write the equation of the corresponding tangent.

**6.** In the function $f(x) = x^3 + cx^2 - 7x + d$, determine the coefficients $c, d$ so that $f$ has a tangent $y = x + 2$ at the point $x_0 = 2$ .

**7.** Let $f(x) = x^3 - 2x^2 - 13x + d$. Determine for which $d \in \mathbb{R}$ $f$ has a tangent $y = 2x + 2$ and at which points it happens.

**Solutions:**

**1.** $y = -x + 4$

**2.** $x_0 = -1, y = -5x - 5$

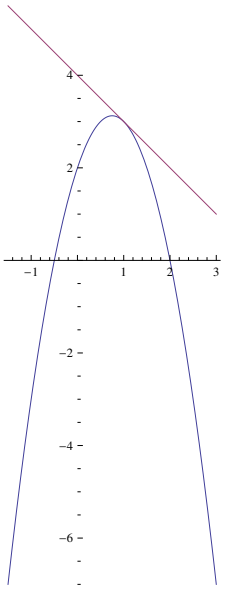
**3.** $y = -5x + 22$, contact point $[3, 7]$, center $[2, 2]$, intercepts $[0, -1/2], [-1/2, 0]$

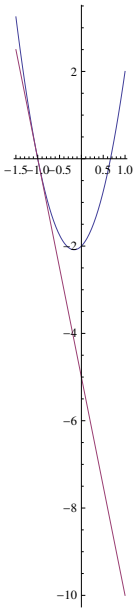
**4.** $b = -8, c = 12$

**5.** two solutions: $x_0 = 1, b = 4$, tangent $y = x + 4$; $x_0 = 3, b = 0$, tangent $y = x$.

**6.** $c = -1, d = 14$.

**7.** two solutions: $x = 3, d = 38$; $x = -\frac{5}{3}, d = -\frac{346}{27}$ .

1. 


2. 


4.