Mathematics for Economists I
Problems 13
Optimization problems – Lagrange multipliers

Find the extremes of the function $f(x, y)$ on the domain $M \subset \mathbb{R}^2$.

1.	$f(x,y) = 6x - 3y$	$M = \{[x,y] \in \mathbb{R}^2;\ x \in \langle -2, 2\rangle,\ x^2 - 4 \le y \le 0\}$
2.	$f(x,y) = x - y$	$M = \{[x,y] \in \mathbb{R}^2;\ x \in \langle 0, 2\rangle,\ 0 \le y \le e^{x-1}\}$
3.	$f(x,y) = 2x - 2y + 3$	$M = \{[x,y] \in \mathbb{R}^2;\ x^2 + y^2 \le 2\}$
4.	$f(x,y) = 2x + y - 5$	$M = \{[x,y] \in \mathbb{R}^2;\ x^2 + y^2 \le 5\}$
5.	$f(x,y) = 3x + 2y$	$M = \{[x,y] \in \mathbb{R}^2;\ x^2 + y^2 \le 13,\ y \ge 0\}$
6.	$f(x,y) = 7x + y$	$M = \{[x,y] \in \mathbb{R}^2;\ x^2 + y^2 \le 50,\ y \ge 0, x \ge y\}$
7.	$f(x,y) = x^2 + 10y$	$M = \{[x,y] \in \mathbb{R}^2;\ x^2 + y^2 \le 169;\ -x - 17 \le y \le x + 7\}$
8.	$f(x,y) = xy + x^3$	$M = \{[x,y] \in \mathbb{R}^2;\ -2x - 4 \le y \le 2 - x - x^2\}$
9.	$f(x,y) = x^2 + y^2 + 3x - 4y$	$M = \{[x,y] \in \mathbb{R}^2;\ x^2 + y^2 \le 100;\ x \ge -2y - 10\}$
10.	$f(x,y) = (x-1)^2 + (y-2)^2$	$M = \{[x,y] \in \mathbb{R}^2;\ x^2 + y^2 \le 8;\ 0 \le y \le x\}$

Solutions (all candidates):

1. $f(-2,0) = -12$ MIN, $f(2,0) = 12$, $f(1,-3) = 15$ MAX

2. $f(0,0) = 0$, $f(2,0) = 2$ MAX, $f(0,\frac{1}{e}) = -\frac{1}{e} \doteq -0,37$, $f(2,e) = 2 - e \doteq -0,72$ MIN, $f(1,1) = 0$

3. $f(-1,1) = -1$ MIN, $f(1,-1) = 7$ MAX

4. $f(-2,-1) = -10$ MIN, $f(2,1) = 0$ MAX

5. $f(-\sqrt{13},0) = -3\sqrt{13}$ MIN, $f(\sqrt{13},0) = 3\sqrt{13}$, $f(3,2) = 13$ MAX

6. $f(0,0) = 0$ MIN, $f(7,1) = 50$ MAX, $f(5,5) = 40$, $f(\sqrt{50},0) = 7\sqrt{50}$

7. $f(-12,-5) = 94$, $f(5,12) = 145$, $f(-5,-12) = -95$, $f(0,-13) = -130$ MIN, $f(12,5) = 194$ MAX, $f(-5,2) = 45$

8. $f(-2;0) = -8$ MIN, $f(3;-10) = -3$, $f(0;0) = 0$, $f(2;-8) = -8$ MIN, $f(-2/3;-8/3) = 24/27$, $f(1;0) = 1$ MAX

9. $f(-10,0) = 70$, $f(6,-8) = 150$ MAX, $f(-3/2,2) = -6,25$ MIN, $f(-4,-3) = 25$, $f(-6,8) = 50$

10. $f(3/2,3/2) = \frac{1}{2}$ MIN, $f(\sqrt{8},0) = (\sqrt{8}-1)^2 \doteq 3,34$, $f(2,2) = 1$, $f(-2,-2) = 25$ MAX