Mathematics for Economists I
Problems 10
Course of the function I

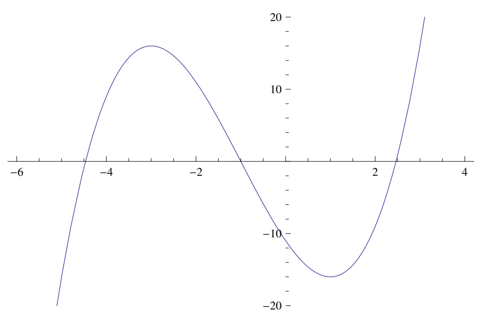
Examine the course of the function, i.e. find its domain, intersections with axes, limits at the extreme points of D_f, the derivative of the function and its zero points, local and global extrema, intervals of monotony, draw the graph. Justify everything properly.

1. $x^3+3x^2-9x-11$	**6.** $e^2e^{-x^2}$	**11.** $(3-x)e^x$
2. x^4-4x^3	**7.** $x\sqrt{1-x^2}$	**12.** x^3+2x^2-15x
3. $\frac{1-2x}{3x^2}$	**8.** $\frac{x^2-x-2}{x-3}$	**13.** $\sqrt{x^2+6x-16}$
4. $\frac{3x-1}{1-x}$	**9.** $\frac{1}{x^2-x-2}$	**14.** $\frac{x^2-5x+4}{x+1}$
5. $\frac{1}{1+e^{-x}}$	**10.** $\frac{\ln(3+2x-x^2)}{\ln 3}$	**15.** $\ln(1-x^2)$

Solutions:

1. $D_f=\mathbb{R}$, roots: $-1,-1\pm 2\sqrt{3}$, $\lim\limits_{x\to\pm\infty}=\pm\infty$, increases in $(-\infty,-3\rangle, \langle 1,+\infty)$, decreases in $\langle -3,1\rangle$.

2. $D_f=\mathbb{R}$, roots: 0 (triple), 4, $\lim\limits_{x\to\pm\infty}=+\infty$, decreases in $(-\infty,3\rangle$, increases in $\langle 3,+\infty)$.



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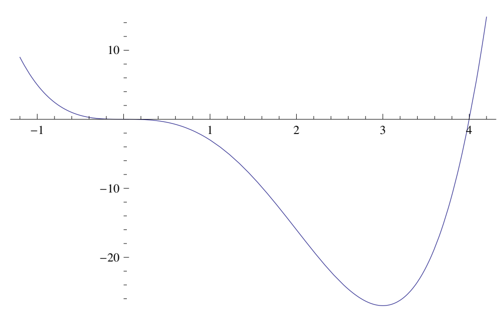
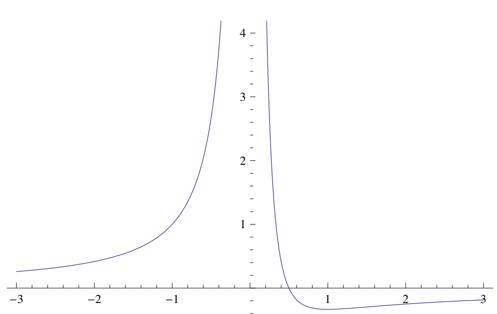


2.

3. $D_f=\mathbb{R}_-\cup\mathbb{R}_+$, root: $\frac{1}{2}$, $\lim\limits_{x\to 0\pm}=+\infty$, $\lim\limits_{x\to\pm\infty}=0$, increases in $(-\infty,0)$, $\langle 1,+\infty)$, decreases in $(0,1\rangle$.

4. $D_f=(-\infty,1)\cup(1,+\infty)$, root: $\frac{1}{3}$, $\lim\limits_{x\to 1\pm}=\mp\infty$, $\lim\limits_{x\to\pm\infty}=-3$, increases in $(-\infty,1)$, $(1,+\infty)$.



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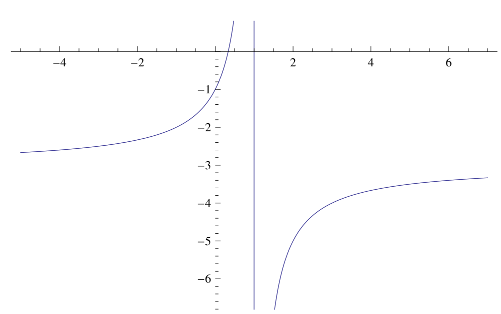
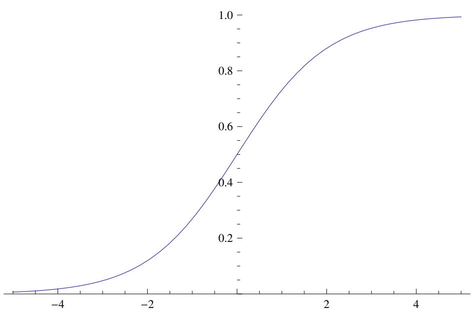


4.

5. $D_f = \mathbb{R}$, $f(0) = \frac{1}{2}$, $f(x) > 0$ v $\mathbb{R}$, $\lim\limits_{x\to-\infty} = 0$, $\lim\limits_{x\to+\infty} = 1$, increases in $\mathbb{R}$.

6. $D_f = \mathbb{R}$, $f(0) = e^2$, $f(x) > 0$ v $\mathbb{R}$, $\lim\limits_{x\to\pm\infty} = 0$, increases in $\mathbb{R}_-$, decreases in $\mathbb{R}_+$.



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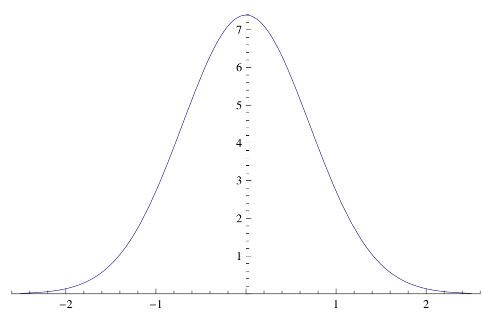
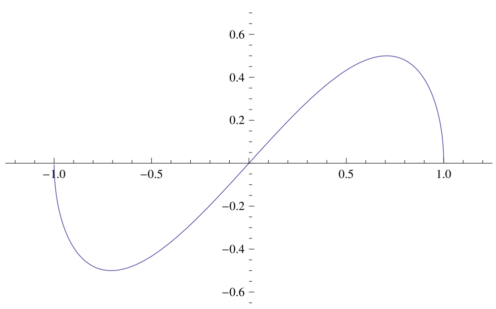


6.

7. $D_f = \langle -1, 1\rangle$, $f(-1) = f(0) = f(1) = 0$, decreases in $\langle -1, -\frac{1}{\sqrt{2}}\rangle, \langle \frac{1}{\sqrt{2}}, 1\rangle$, increases in $\langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\rangle$.

8. $D_f = (-\infty, 3) \cup (3, +\infty)$, $f(0) = \frac{2}{3}$, roots: $-1, 2$, $\lim\limits_{x\to3\pm} = \pm\infty$, $\lim\limits_{x\to\pm\infty} = \pm\infty$, increases in $(-\infty, 1), (5, +\infty)$, decreases in $(1, 3), (3, 5)$.



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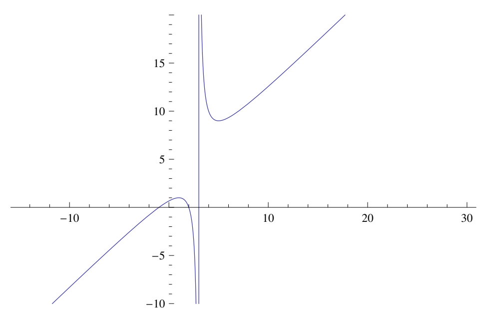
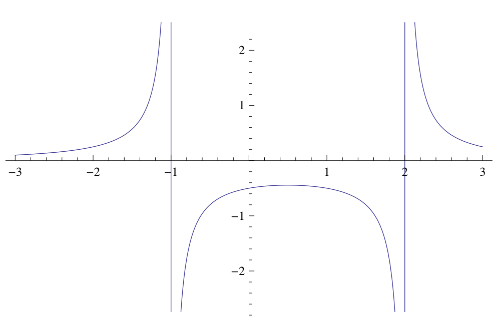


8.

9. $D_f = (-\infty, -1) \cup (1, 2) \cup (2, +\infty)$, $f(0) = -\frac{1}{2}$, $f(x) \neq 0$ v $\mathbb{R}$, $\lim\limits_{x\to-1\pm} = \mp\infty$, $\lim\limits_{x\to2\pm} = \pm\infty$, $\lim\limits_{x\to\pm\infty} = 0$, increases in $(-\infty, -1), (-1, \frac{1}{2})$, decreases in $(\frac{1}{2}, 2), (2, +\infty)$.

10. $D_f = (-1, 3)$, $f(0) = 1$, roots $1 \pm \sqrt{3}$, $\lim\limits_{x\to-1+} = -\infty$, $\lim\limits_{x\to3-} = -\infty$, increases in $(-1, 1)$, decreases in $(1, 3)$.



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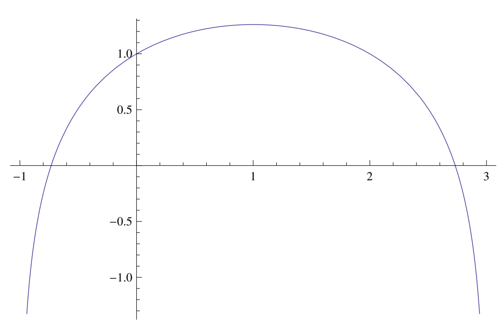

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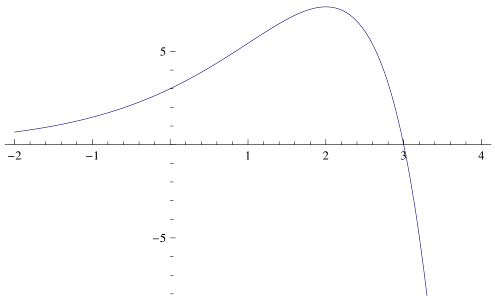
11. $D_f = \mathbb{R}$, root: 3, $\lim\limits_{x\to-\infty} = 0$, $\lim\limits_{x\to+\infty} = -\infty$, increases in $(-\infty, 2\rangle$, decreases in $\langle 2, +\infty)$.

12. $D_f = \mathbb{R}$, roots: $-5, 0, 3$, $\lim\limits_{x\to\pm\infty} = \pm\infty$, increases in $(-\infty, -3\rangle, \langle\frac{5}{3}, +\infty)$, decreases in $\langle -3, \frac{5}{3}\rangle$.



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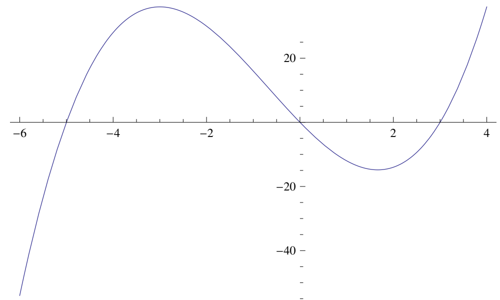

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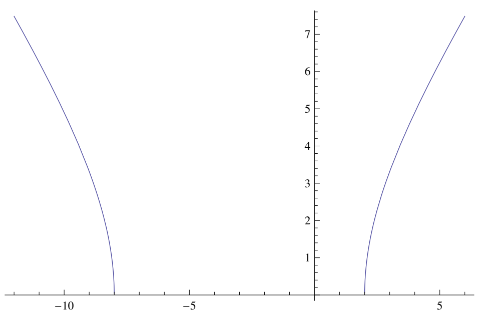
13. $D_f = (-\infty, -8\rangle \cup \langle 2, +\infty)$, roots: $-8, 2$, $\lim\limits_{x\to\pm\infty} = +\infty$, increases in $\langle 2, +\infty)$, decreases in $\langle -\infty, -8\rangle$.

14. $D_f = (-\infty, -1) \cup (-1, +\infty)$, roots: $1, 4$, $\lim\limits_{x\to\pm\infty} = \pm\infty$, $\lim\limits_{x\to -1\pm} = \pm\infty$, increases in $(-\infty, -1-\sqrt{10}\rangle$ and in $\langle -1+\sqrt{10}, +\infty)$, decreases in $\langle -1-\sqrt{10}, -1)$ and in $(-1, -1+\sqrt{10}\rangle$.

15. $D_f = (-1, 1)$, root: 0, $\lim\limits_{x\to\pm 1} = -\infty$, increases in $(-\infty, 0\rangle$, decreases in $\langle 0, \infty)$.



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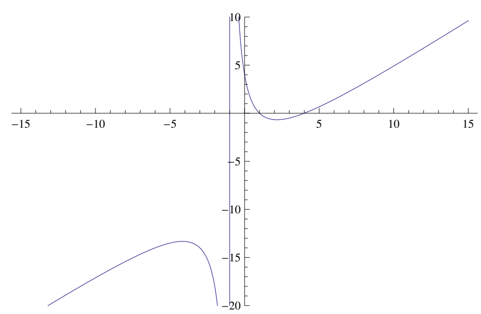

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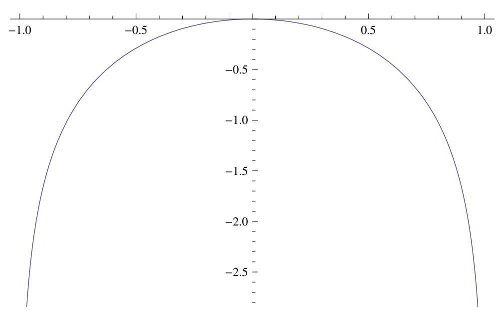

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