1)
$$\lim_{x\to 2}\frac{\ln(x^2+x-5)}{x^2-5x+6}\overset{\text{"}\frac{0}{0}\text{"}}{\underset{(L.H)}{=}}\lim_{x\to 2}\frac{\frac{2x+1}{x^2+x-5}}{2x-5}=\lim_{x\to 2}\frac{2x+1}{(x^2+x-5)(2x-5)}$$

$$=\frac{5}{1\cdot(-1)}=\underline{\underline{-5}}$$

2)
$$f(x)=e^{2x}\sqrt{x^2+2x+3}$$

$\boxed{D_f=\mathbb{R}}$ (pretože $x^2+2x+3\geq 0\ \forall x\in\mathbb{R}$
- rovnica $x^2+2x+3=0$ má záporný diskriminant a koeficient $a>0\to$ smerujo nahor a nepretína os x.)

$$f'(x)=2e^{2x}\sqrt{x^2+2x+3}+e^{2x}\cdot\frac{2x+2}{2\sqrt{x^2+2x+3}}$$

derivacia súčinu

$$=e^{2x}\left(2\sqrt{x^2+2x+3}+\frac{\cancel{2}(x+1)}{\cancel{2}\sqrt{x^2+2x+3}}\right)$$

$$=e^{2x}\left(\frac{2(x^2+2x+3)+x+1}{\sqrt{x^2+2x+3}}\right)=e^{2x}\frac{2x^2+5x+7}{\sqrt{x^2+2x+3}}$$

3)
$$f(x)=\frac{3x+1}{x-1}\qquad k=-1$$

$\boxed{D_{f'}=\mathbb{R}}$ (rovnaký argument)

$D_f=\mathbb{R}\setminus\{1\}\to$ asymptota $x=1$

Dotyčnica
$$\boxed{k=f'(x_0)}=\frac{3(x_0-1)-(3x_0+1)\cdot 1}{(x_0-1)^2}=\boxed{\frac{-4}{(x_0-1)^2}}$$

$$-1=\frac{-4}{(x_0-1)^2}\quad /\cdot(x-1)$$

$$-(x_0-1)^2=-4$$

$$-x_0^2+2x-1=-4$$

$$x_0^2-2x_0-3=0$$

$$(x_0-3)(x_0+1)=0$$

$\boxed{\begin{matrix}x_1=3\\x_2=-1\end{matrix}}\longrightarrow$ body dotyku $\boxed{\begin{matrix}T_1=[3,5]\\T_2=[-1,1]\end{matrix}}$

Rovnice dotyčnic: $y=kx+q$

t_1: $5=-1\cdot 3+q\Rightarrow q=8\Rightarrow\boxed{y=-x+8}$

t_2: $1=-1(-1)+q\Rightarrow q=0\Rightarrow\boxed{y=-x}$

Priesečníky dotyčnic: P_{x_t}: $y=0\Rightarrow 0=-x+8\Rightarrow x=8$

$0=-x\Rightarrow x=0$

P_{y_t}: $x=0\Rightarrow y=-0+8\Rightarrow y=8$

tiež aj priesečník

$$\boxed{\begin{matrix}P_{x_{t_1}}=[8,0]\\P_{x_{t_2}}=[0,0]=P_{y_{t_2}}\\P_{y_{t_1}}=[0,8]\end{matrix}}$$

Hyperbola:

$P_x: \; y=0 \Rightarrow 0=\frac{3x+1}{x-1}$ $\quad x=-\frac{1}{3} \quad \Rightarrow \boxed{P_x=[-\frac{1}{3},0]}$

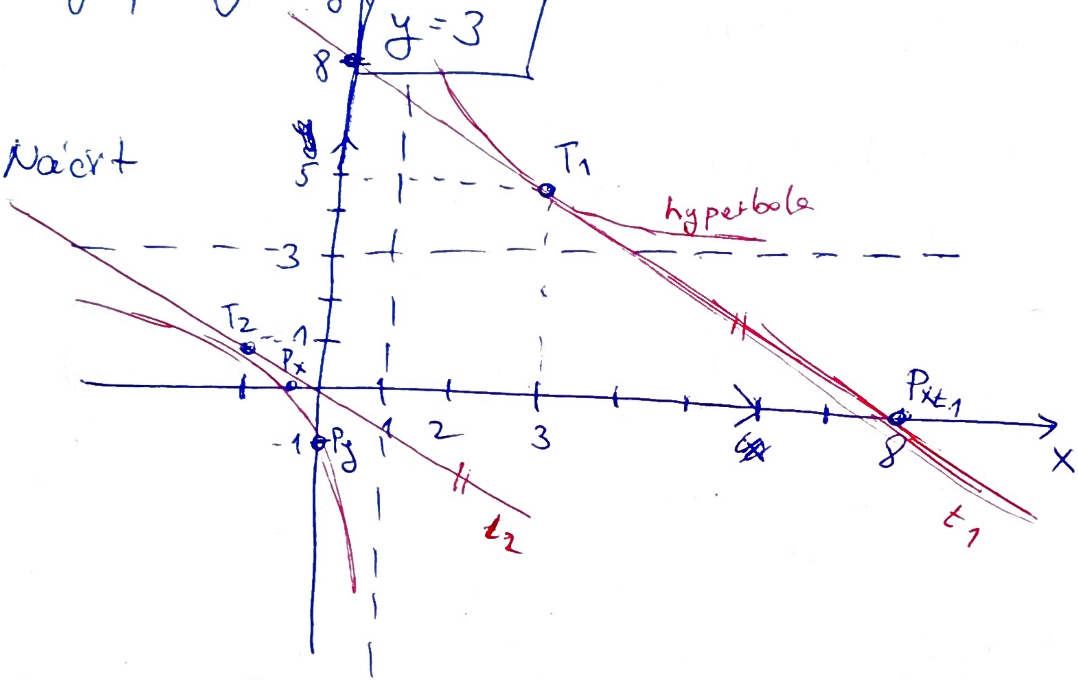
$P_y: \; x=0 \Rightarrow y=-1 \quad \boxed{P_y=[0,-1]}$

Stred: $(3x+1):(x-1)=3+\frac{4}{x-1} \quad \boxed{S=[1,3]}$

$$\begin{array}{l} \ominus 3x \oplus 3 \\ \hline 4 \end{array}$$

Asymptoty $\boxed{\begin{array}{l} x=1 \\ y=3 \end{array}}$

Náčrt



4) $f(x)=\frac{x^2+8x}{1-x}$

- $D_f=\mathbb{R}\setminus\{1\}$
- parita → D_f nie je symetrický → nie je ani párna ani nepárna
- $P_x: \; y=0 \Rightarrow 0=x^2+8x$ $\quad P_y: \; x=0 \Rightarrow y=0 \quad \boxed{P_y=[0,0]=P_{x_1}}$

$0=x(8+x)$

$x=0$

$x=-8$ $\quad \boxed{\begin{array}{l} P_{x_1}=[0,0] \\ P_{x_2}=[-8,0] \end{array}}$

- ⊕/⊖:

$$\begin{array}{ccccc} \oplus & & \ominus & & \oplus \\ \hline & -8 & & 0 & \end{array}$$

Fcia je kladná pre $x\in(-\infty,-8)\cup(0,\infty)$.

Fcia je záporná pre $x\in(-8,0)$

$$\lim_{x\to+\infty}\frac{x^2+8x}{1-x}=\lim_{x\to+\infty}\frac{x^{\cancel{2}}(1+\frac{8}{x})}{(\frac{1}{x}-1)\cancel{x}}=\lim_{x\to+\infty}x\cdot\lim_{x\to+\infty}\frac{1+\frac{8}{x}}{\frac{1}{x}-1}=+\infty(-1)=-\infty$$

$$\lim_{x\to-\infty}\frac{x^2+8x}{1-x}=\dots=\lim_{x\to-\infty}x\cdot\lim_{x\to-\infty}\frac{1+\frac{8}{x}}{\frac{1}{x}-1}=-\infty\cdot(-1)=+\infty$$

$$\lim_{x\to 1^+} \frac{x^2+8x}{1-x} = \frac{9}{0^-} = -\infty \qquad \lim_{x\to 1^-} \frac{x^2+8x}{1-x} = \frac{9}{0^+} = +\infty$$

=> neexistujú glob. extrémy a máme vertikálnu asymptotu

- $$f' = \frac{(2x+8)(1-x)-(x^2+8x)\cdot(-1)}{(1-x)^2} = \frac{2x-2x^2+8-\cancel{8x}+x^2+\cancel{8x}}{(1-x)^2}$$

$$= \frac{-x^2+2x+8}{(1-x)^2} \qquad D_{f'} = R \setminus \{1\}$$

- ↗ | ↘ :

$\ominus$ | $\oplus$ | $\oplus$ | $\ominus$

↘ −2 ↗ 1 ↗ 4 ↘

stac. body: $-x^2+2x+8=0$

$x^2-2x-8=0$

$(x-4)(x+2)=0$

$x=4$

$x=-2$

$f(4) = -16$

$f(-2) = -4$

Fcia klesá na $(-\infty,-2) \cup (4,\infty)$

Fcia rastie na $(-2,1) \cup (1,4)$

V bode $[4,-16]$ je lok. max

V bode $[-2,-4]$ je lok. min

- Asymptoty: $+\infty$:

$$k = \lim_{x\to+\infty} \frac{x^2+8x}{(1-x)x} = \lim_{x\to+\infty} \frac{x^2(1+\frac{8}{x})}{x^2(\frac{1}{x}-1)} = -1$$

$$q = \lim_{x\to-\infty}\left(\frac{x^2+8x}{1-x}+x\right) = \lim_{x\to-\infty} \frac{x^2+8x+x(1-x)}{1-x} = \lim_{x\to-\infty} \frac{9x}{1-x} \overset{\frac{-\infty}{+\infty}}{\underset{L.H}{=}}$$

$$= \lim_{x\to-\infty} \frac{9}{0-1} = -9$$

=> šikmá asymptota v $+\infty$: $\boxed{y=-x-9}$

analogickým postupom v $-\infty$

$\boxed{\text{Navyše zvislá asymptota } x=1}$ (dokázané limitami v 1)

- $$f''(x) = \frac{(-2x+2)\cdot(1-x)^{\cancel{4}} - (-x^2+2x+8)2(1-x)\cdot(-1)}{(1-x)^{\cancel{4}3}}$$

$$= \frac{-\cancel{2x}+\cancel{2x^2}+2-\cancel{2x}-\cancel{2x^2}+\cancel{4x}+16}{(1-x)^3} = \frac{18}{(1-x)^3}$$

Nemá inflexný bod

∩ | ∪ :

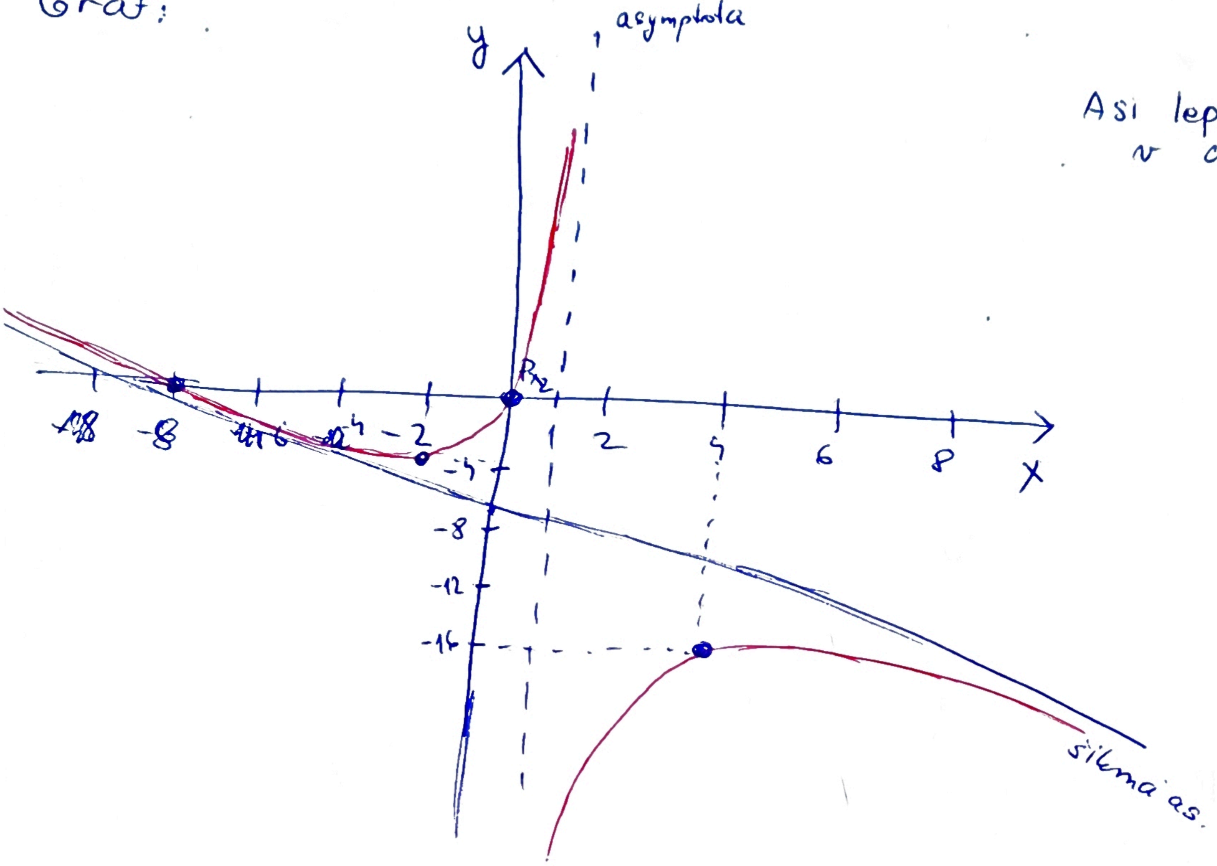
$\oplus$ | $\ominus$

1

Fcia je konvexná na $(-\infty,1)$

Fcia je konkávna na $(1,\infty)$

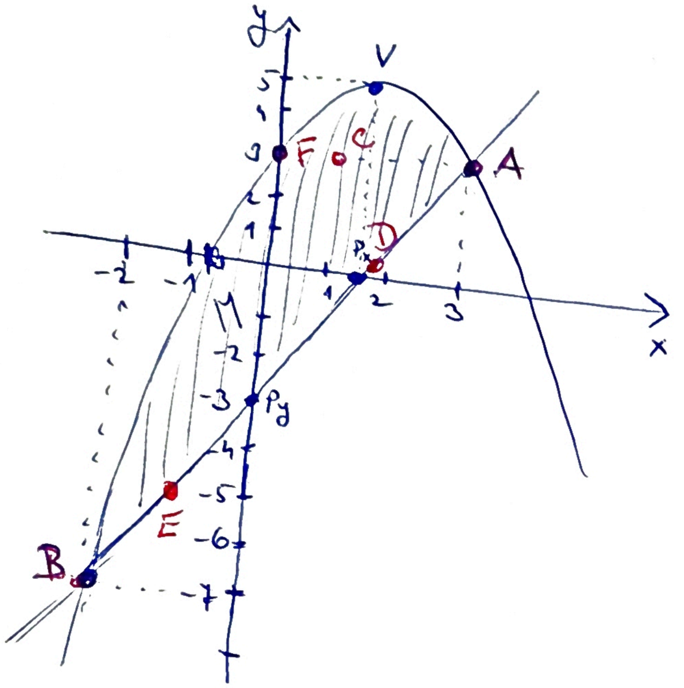
Graf:



Asi lepší obrázok
v desmose...

④ $f(x,y) = x^3 - 3x^2 + xy - y$ $M = \{[x,y] \in \mathbb{R}^2 : 2x-3 \leq y \leq -x^2+3x+3\}$

Náčrt množiny M: $M = M^\circ \cup \partial M$ $y \geq 2x-3$ → polrovina (rovnica s priamkou)



$P_x = [\frac{3}{2}, 0]$ $P_y = [0, -3]$

$y \leq -x^2+3x+3$ → parabola

$y = -x^2+3x+3$

hľadám vrchol cez extrém →

$y' = -2x+3$

$0 = -2x+3$ $x = \frac{3}{2}$

vrchol $V = [\frac{3}{2}, -(\frac{3}{2})^2 + 3 \cdot \frac{3}{2} + 3]$

$V = [\frac{3}{2}, \frac{-9+18+12}{4}]$

$V = [\frac{3}{2}, \frac{21}{4}]$

Priesečník oboch fcií:

$2x - 3 = y$

$-x^2+3x+3 = y$

$-x^2+3x+3 = 2x-3$

$x^2 - x - 6 = 0$

$(x+2)(x-3) = 0$ < $x_1 = -2$ $y = -7$; $x_2 = 3$ $y = 3$

$\boxed{A = [3,3] \quad B = [-2,-7]}$ } kandidáti na extrém lebo sú to vrcholy.

•Na M°:

$$\frac{\partial F}{\partial x} = 3x^2 - 6x + y = 0$$

$$\frac{\partial F}{\partial y} = x - 1 = 0 \qquad x = 1$$

$y = 3 \Rightarrow \boxed{C = [1,3]}$ → kandidát na extrém v M°.

•Na ∂M:

Na priamke: $F(x, 2x-3) = x^3 - 3x^2 + x(2x-3) - (2x-3)$

$g(x) = x^3 - 3x^2 + 2x^2 - 3x - 2x + 3 = x^3 - x^2 - 5x + 3$

$g'(x) = 3x^2 - 2x - 5 = 0$

$D = 4 + 4 \cdot 3 \cdot 5 = 64$

$x_{1,2} = \frac{2 \pm 8}{6}$ < $\frac{5}{3}$ → $y = 2 \cdot \frac{5}{3} - 3 = \frac{1}{3}$; -1 → $y = 2 \cdot (-1) - 3 = -5$

$\boxed{D = [\frac{5}{3}, \frac{1}{3}] \quad E = [-1,-5]}$ → kandidáti na priamke

Na parabole: cez LM:

$$L(x,y,\lambda) = x^3 - 3x^2 + xy - y + \lambda(-x^2 + 3x + 3 - y)$$

$$\frac{\partial L}{\partial x} = 3x^2 - 6x^2 + y - 2\lambda x = 0$$

$$\frac{\partial L}{\partial y} = x - 1 - \lambda = 0 \Rightarrow x = 1 + \lambda$$ dosadíme

$$\frac{\partial L}{\partial \lambda} = -x^2 + 3x + 3 - y = 0 \Rightarrow y = -x^2 + 3x + 3$$ dosadíme

... vyjde:

$$\boxed{\begin{matrix} x = 0 & \\ y = 3 & \lambda = -1 \end{matrix}}$$

$F = [0, 3]$

Vyhodnotenie:

$F(3,3) = 6 \rightarrow$ MAX

$F(-2,-7) = 1$

$F(1,3) = -2$

$F(\frac{5}{3}, \frac{1}{3}) = -\frac{94}{27} \longrightarrow$ MIN

$F(-1,-5) = 6 \rightarrow$ MAX

$F(0,3) =$ ~~[illegible]~~ -3