Matematika pro ekonomy, 16. 12. 2020

(4A) $f(x,y,z) = x^4 - 2z^2$

$M = \{[x,y,z] \in \mathbb{R}^3 :$

$x^4 + (y-1)^2 + z^2 = 81 \}$

musíme použít Lagr. mult.

$L(x,y,z,\lambda) = x^4 - 2z^2 + \cancel{A}$

$+ \lambda \cdot (x^4 + (y-1)^2 + z^2 - 81)$

$\partial_x L = 4x^3 + 4\lambda x^3 = 0$ (1)

$\partial_y L = 2\lambda(y-1) = 0$ (2)

$\partial_z L = -4z + 2\lambda z = 0$ (3)

$x^4 + (y-1)^2 + z^2 = 81$ (4)

(1) : $4x^3(1+\lambda) = 0 \quad < \begin{matrix} x=0 \\ \lambda=-1 \end{matrix}$

(2) : $2\lambda(y-1) = 0 \quad < \begin{matrix} \lambda=0 \\ y=1 \end{matrix}$

(3) : $2z(-2+\lambda) = 0 \quad < \begin{matrix} z=0 \\ \lambda=2 \end{matrix}$

a) (1) $\lambda = -1 \overset{(2)}{\Rightarrow} y = 1$

$\overset{(3)}{\Rightarrow} z = 0$

(4) $x^4 + 0^2 + 0^2 = 81$

$x = \pm 3 \Rightarrow A_\pm = [\pm 3, \overset{1}{\cancel{0}}, 0]$

b) $\overset{(2)}{\lambda = 0} \overset{(1)}{\Rightarrow} x = 0$

$\overset{(3)}{\Rightarrow} z = 0$

(4) $0^4 + (y-1)^2 + 0^2 = 81$

$y^2 - 2y + 1 = 81$

$y^2 - 2y - 80 = 0$

$(y-10)(y+8) = 0 \Rightarrow \begin{cases} y = 10 \\ y = -8 \end{cases}$

$B = [0, 10, 0], \quad C = [0, -8, 0]$

c) (3) $d=2 \overset{(1)}{\Rightarrow} x=0$

$\overset{(2)}{\Rightarrow} y=1$

$\boxed{f=x^4 2z^2}$

(4) $0^4+0^2+z^2=81$

$z=\pm 9$

$D_\pm=[0,1,\pm 9]$

d) (1) $x=0$, (2) $y=1$, (3) $z=0$

(4): $0^4+0^2+0^2 \neq 81$

není řešení

$f(A_\pm)=f(\pm 3,1,0)=81$ MAX

$f(B)=f(0,10,0)=0$

$f(C)=f(0,-8,0)=0$

$f(D_\pm)=-2\cdot 81=-162$ MIN

4B $f(x,y,z)=xy+2z$

$g_1: x^2+y^2=20$

$g_2: x+y-z=0$

3 prom., 2 vazby ⇒ lze užít Jac.

$$J=\begin{pmatrix} y & x & 2 \\ 2x & 2y & 0 \\ 1 & 1 & -1 \end{pmatrix}$$

$$|J|=-2y^2+0+4x-4y-0+2x^2=$$

$$=2x^2-2y^2+4x-4y=$$

$$=2(x-y)(x+y+2)=0 \quad (1)$$

$$x^2+y^2=20 \quad (2)$$

$$x+y-z=0 \quad (3)$$

(1) a) $(x-y)=0$

b) $(x+y+2)=0$

a) $x - y = 0$

$x = y$

(2) $\Rightarrow x^2 + x^2 = 20$

$x^2 = 10$

$y = x = \pm\sqrt{10}$

(3) $x + y - z = 0$

$z = x + y = 2x$

$\Rightarrow$ kand. $A_\pm = \pm[\sqrt{10}, \sqrt{10}, 2\sqrt{10}]$

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b) $x + y + 2 = 0$ (1)

$x^2 + y^2 = 20$ (2)

$x + y - z = 0$ (3)

(1)-(3) $2 + z = 0$

$\underline{z = -2}$

(1) $x = -y - 2$ dosadíme do (2)

(2) $(-y-2)^2 + y^2 = 20$

$y^2 + 4y + 4 + y^2 = 20$

$2y^2 + 4y - 16 = 0$ $/:2$

$y^2 + 2y - 8 = 0$

$(y+4)(y-2) = 0$

α) $y = -4 \Rightarrow x = 2 \Rightarrow B = [2, -4, -2]$

β) $y = 2 \Rightarrow x = -4 \Rightarrow C = [-4, 2, -2]$

$f(x, y, z) = xy + 2z$

Pomůcka: $4\sqrt{10} \doteq 12{,}65$

$f(A_+) = 10 + 4\sqrt{10} \doteq 22{,}65$ MAX

$f(A_-) = 10 - 4\sqrt{10} \doteq -2{,}65$

$f(B) = -12$

$f(C) = -12$ MIN

Totéž pomocí Lagr. mult.

$$L = xy + 2z + \lambda_1(x^2+y^2-20) + \lambda_2(x+y-z)$$

$$\partial_x L = y + 2\lambda_1 x + \lambda_2 = 0 \quad (1)$$
$$\partial_y L = x + 2\lambda_1 y + \lambda_2 = 0 \quad (2)$$
$$\partial_z L = 2 - \lambda_2 = 0 \quad (3)$$
$$x^2+y^2=20 \quad (4)$$
$$x+y-z=0 \quad (5)$$

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(3) $\lambda_2 = 2 \Rightarrow$ (1) $y + 2\lambda_1 x + 2 = 0$

(2) $x + 2\lambda_1 y + 2 = 0$

Chceme se zbavit λ_1, vyjádříme si λ_1 z (1) a (2) pomocí x, y, porovnáme

(1) $y + 2\lambda_1 x + 2 = 0$

$$2\lambda_1 x = -y-2 \quad /:2x$$
$$\lambda_1 = \frac{-y-2}{2x} \quad \text{pro } x \neq 0$$

(2) ...... $\lambda_1 = \frac{-x-2}{2y}$ pro $y \neq 0$

$$\Rightarrow \lambda_1 = \frac{-y-2}{2x} = \frac{-x-2}{2y} \quad /\cdot 2xy$$
$$-y^2-2y = -x^2-2x$$
$$x^2-y^2+2x-2y = 0$$
$$(x-y)(x+y) + (x-y)\cdot 2 = 0$$
$$(x-y)(x+y+2) = 0$$

a dále stejně jako u Jac

Může se stát $x=0$ nebo $y=0$?

$x=0 \Rightarrow$ (1) $y+2=0$, $y=-2$ $\Rightarrow$ není splněna (4)

$\Rightarrow x=0$ nemůže nastat!

Obdobně $y=0$ —"—!

4D

$f(x,y,z) = x - z$

$g_1: \; x + (y-1)^2 - z = 0$

$g_2: \; x^2 + y^2 = 16$

Jacobián:

$$J = \begin{pmatrix} 1 & 0 & -1 \\ 1 & 2(y-1) & -1 \\ 2x & 2y & 0 \end{pmatrix}$$

$$|J| = 0 + 0 + (-2y) + 4x(y-1) + 2y - 0 = 4x(y-1) = 0 \quad (1)$$

$$x + (y-1)^2 - z = 0 \quad (2)$$

$$x^2 + y^2 = 16 \quad (3)$$

(1) a) $x = 0$

b) $y = 1$

a) $x = 0 \overset{(3)}{\Rightarrow} y^2 = 16$

$y = \pm 4$

$y = 4$: (2) $x + (4-1)^2 - z = 0$

$9 - z = 0$

$z = 9$

$y = -4$: (2) $x + (-4-1)^2 - z = 0$

$25 - z = 0$

$z = 25$

$\Rightarrow A = [0, 4, 9], \; B = [0, -4, 25]$

b) $y = 1 \overset{(3)}{\Rightarrow} x^2 + 1 = 16$

$x^2 = 15$

$x = \pm\sqrt{15}$

(2): $x - z = 0$

$z = x$

$\Rightarrow C_{\pm} = [\pm\sqrt{15}, 1, \pm\sqrt{15}] \; (x = z)$

$f(A) = -9$

$f(B) = -25$ MIN

$f(C_{\pm}) = 0$ MAX