Matematika pro ekonomy, 9.12.2020

ZTB-1

$$f(x)=\frac{e^{2x}}{e(1-x)}=\frac{e^{2x-1}}{1-x}$$

$D_f=\mathbb{R}\setminus\{1\}$, ani sudá ani lichá

$P_y=[0,e^{-1}]$, $f(0)=\dfrac{e^{-1}}{1-0}$

P_x nejsou, protože $e^y\neq 0$ vždy

$$\lim_{x\to 1+}\frac{e^{2x-1}}{1-x}=\frac{e^1}{0-}=\underline{\underline{-\infty}}$$

$$\lim_{x\to 1-}\frac{e^{2x-1}}{1-x}=\frac{e^1}{0+}=\underline{\underline{+\infty}}$$

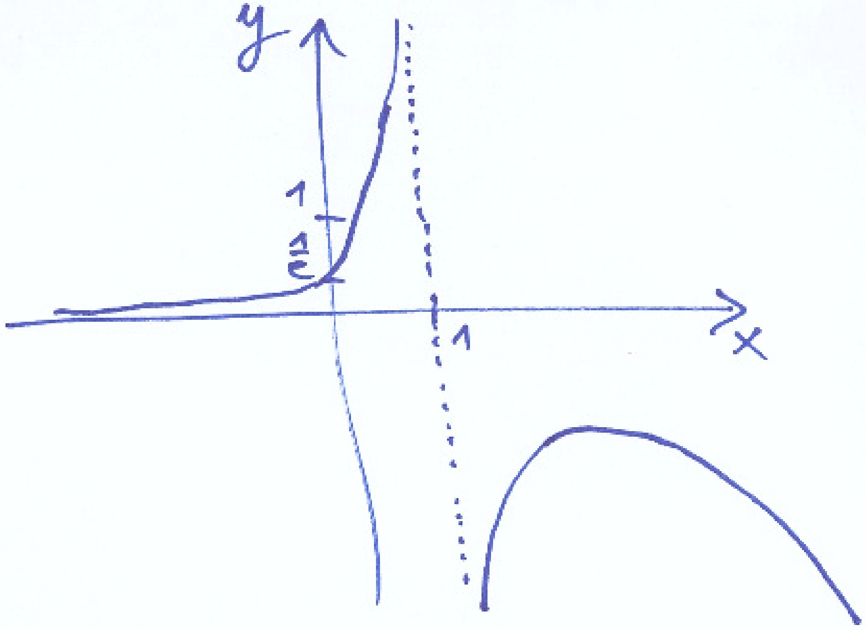
$$\lim_{x\to -\infty}\frac{e^{2x-1}}{1-x}=\frac{0}{+\infty}=\underline{\underline{0}}$$

čitatel: „$e^{-\infty}$" $=\lim\limits_{y\to-\infty}e^y=0$

$$\lim_{x\to +\infty}\frac{e^{2x-1}}{1-x}\left(=\frac{+\infty}{-\infty}\right)\overset{L'H}{=}$$

čitatel: „$e^{+\infty}$" $=\lim\limits_{y\to+\infty}e^y=+\infty$

$$=\lim_{x\to+\infty}\frac{2\cdot e^{2x-1}}{-1}=-2\cdot(+\infty)=\underline{\underline{-\infty}}$$



$e^{-1}=\frac{1}{e}\doteq 0{,}37$

$$f'(x) = \left(\frac{e^{2x-1}}{1-x}\right)' = \frac{2\cdot e^{2x-1}\cdot(1-x) + e^{2x-1}\cdot 1}{(1-x)^2} = \frac{e^{2x-1}\left[2-2x+1\right]}{(1-x)^2} =$$

$$= \frac{e^{2x-1}(3-2x)}{(1-x)^2} = 0 \iff 3-2x = 0$$

$$3 = 2x$$

$$x = \frac{3}{2} \text{ stacion. bod}$$

vždy $\neq 0$

$$f\left(\frac{3}{2}\right) = \frac{e^{2\cdot\frac{3}{2}-1}}{1-\frac{3}{2}} = \frac{e^2}{-\frac{1}{2}} = \underline{\underline{-2e^2}}$$

$$\doteq -14{,}78$$

$$V_f = \left\{-\infty, 1-, 1+, \frac{3}{2}, +\infty\right\}$$

$x \in V_f$	$-\infty$	$1-$	$1+$	$\frac{3}{2}$	$+\infty$
$f(x)$ / $\lim f(x)$	0	$+\infty$	$-\infty$	$-2e^2$	$-\infty$
		roste		roste	klesá

lok. max. $\left[\frac{3}{2}, -2e^2\right]$

$$H_f = (-\infty, -2e^2\rangle \cup (0, +\infty)$$

$$f''(x) = \left(\frac{e^{2x-1}\cdot(3-2x)}{(1-x)^2}\right)' =$$

$$= \frac{\left(e^{2x-1}\cdot(3-2x)\right)'\cdot(1-x)^2 - e^{2x-1}\cdot(3-2x)\cdot\left((1-x)^2\right)'}{(1-x)^4}$$

$$\left(e^{2x-1}\cdot(3-2x)\right)' = 2e^{2x-1}(3-2x) + e^{2x-1}\cdot(-2) =$$

$$= e^{2x-1}\left[2(3-2x)-2\right] = e^{2x-1}(4-4x)$$

$$\left((1-x)^2\right)' = 2(1-x)\cdot(-1) = 2(x-1)$$

$$\Rightarrow f''(x) = \frac{e^{2x-1}(4-4x)\cdot(1-x)^2 - e^{2x-1}(3-2x)\cdot 2(x-1)}{(1-x)^4} =$$

$$= e^{2x-1}\,\frac{(4-4x)(1-x)^{\cancel{2}\,1} + (3-2x)\cdot 2\cancel{(1-x)}}{(1-x)^{\cancel{4}\,3}} = e^{2x-1}\,\frac{(4-4x-4x+4x^2)+(6-4x)}{(1-x)^3} =$$

$$= \underbrace{e^{2x-1}}_{\neq 0}{}^{>0}\,\frac{\overbrace{4x^2-12x+10}^{>0}}{(1-x)^3} \stackrel{?}{=} 0 \Leftrightarrow 4x^2-12x+10=0$$

$$D = 144 - 4\cdot 4\cdot 10 < 0$$

$\Rightarrow$ f'' nikdy není 0 $\Rightarrow$ nejsou infl. body

$\Rightarrow$ znaménko f'' je určeno znaménkem jmenovatele

$x>1 \Rightarrow 1-x<0 \Rightarrow (1-x)^3<0 \Rightarrow f''(x)<0$ $\cap$ konk.

$x<1$ $\Rightarrow f''(x)>0$ $\cup$ konv.

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asymptoty: $\exists$ svislá $x=1$ protože $\lim_{x\to 1\pm} f(x) = \mp\infty$

v $-\infty$: as: $y=0$

v $+\infty$: $\lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \frac{e^{2x-1}}{\underbrace{x(1-x)}_{-x^2+x}} \stackrel{L'H}{=} \lim_{x\to+\infty} \frac{2e^{2x-1}}{-2x+1} \stackrel{L'H}{=} \lim_{x\to+\infty}\frac{4e^{2x-1}}{-2} = -\infty$

as. neex. $\Leftarrow$

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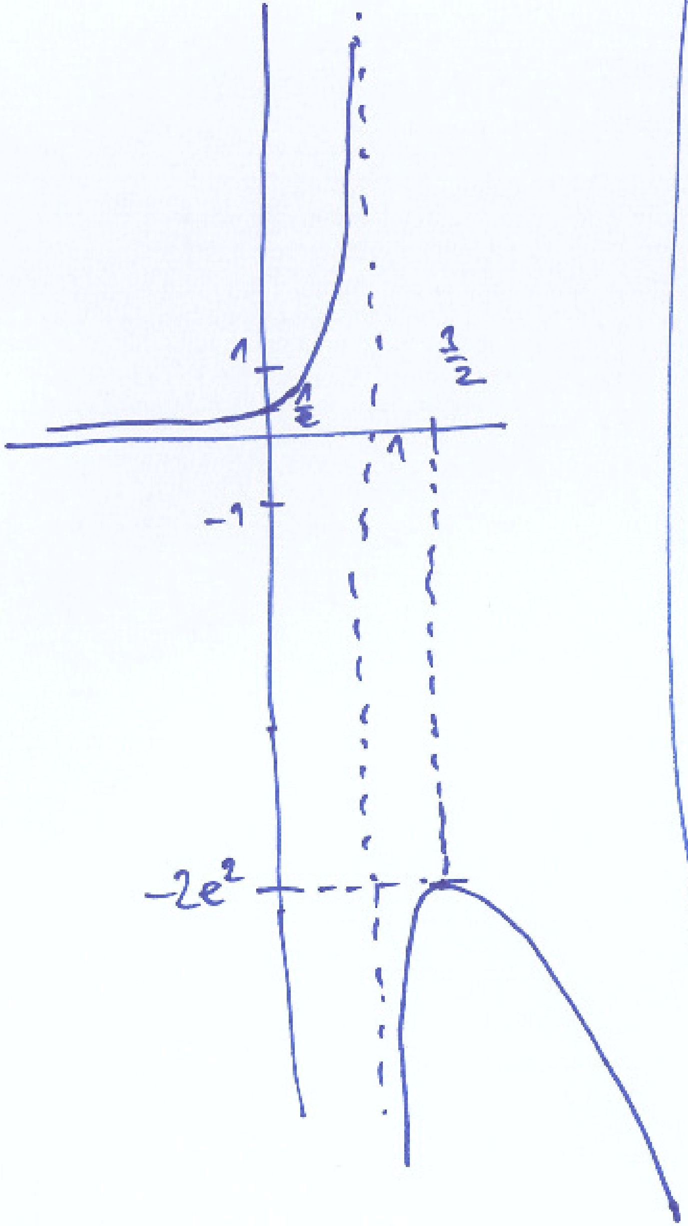
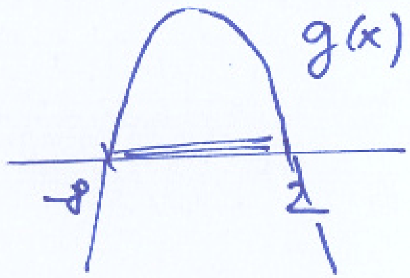


② ZTC $\qquad f(x)=\sqrt{16-6x-x^2}-3$

D_f: $\; g(x)= 16-6x-x^2 \ge 0 \quad /\cdot(-1)$

$x^2+6x-16\le 0$

$(x+8)(x-2)\le 0$



$D_f=\langle -8, 2\rangle$

P_y: $\; f(0)=\sqrt{16}-3=4-3=1 \quad P_y=[0,1]$

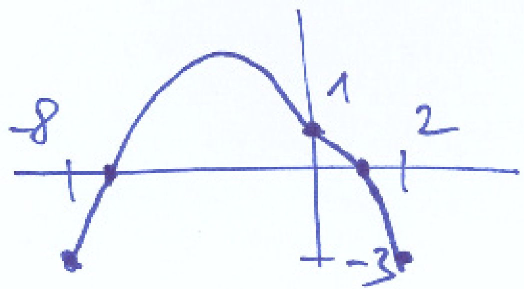
P_x: $\; \sqrt{16-6x-x^2}=3 \quad /(\;)^2$

$16-6x-x^2=9 \quad \dots \quad x_1=-7, \; x_2=1$

$P_x=[-7,0], [1,0]$

$\lim\limits_{x\to -8} f(x)=f(-8)=\sqrt{\underbrace{16+48-64}_{=0}}-3=-3$

$\lim\limits_{x\to 2} f(x)=f(2) = 0-3=-3$



$$f'(x) = \left(\sqrt{16-6x-x^2}-3\right)' = \frac{1}{2\sqrt{16-6x-x^2}}(-6-2x) = 0$$

f' je def. v $(-8,2)$

stac. bod: $-6-2x=0$

$-2x = 6$

$\boxed{x=-3}$

$V_f = \{-8,-3,2\}$

$f(-3) = \sqrt{16-6(-3)-9}-3 = \sqrt{25}-3 = \underline{\underline{2}}$

	-8	-3	2
$f(x)$	-3	2	-3
	↗		↘

glob. maximum: $[-3,2]$

glob. minima: $[-8,-3]$, $[2,-3]$

asymptoty – žádné nemohou být

$$f''(x) = \left(\frac{-6-2x}{2\sqrt{16-6x-x^2}}\right)' =$$

$$= \frac{1}{2}\,\frac{-2\cdot\sqrt{16-6x-x^2} - (-6-2x)\dfrac{-6-2x}{2\sqrt{16-6x-x^2}}}{16-6x-x^2} =$$

$$= \frac{1}{2}\,\frac{-2(16-6x-x^2)+(6+2x)(-3-x)}{(16-6x-x^2)^{3/2}} =$$

$$= \frac{\cancel{x^2}+\cancel{6x}-16-\cancel{x^2}-\cancel{6x}-9}{(\quad)^{3/2}} = \frac{-25}{\underbrace{(\quad)^{3/2}}_{>0 \text{ v } (-8,2)}}^{<0}$$

⇙

$\underline{f'<0 \text{ v } (-8,2)} \Rightarrow \underline{\text{konkávní v } D_f}$