Matematika pro ekonomy, 7.12.2020

Úlohy z testů ZS 2018/19:

$\infty-\infty$ není def.

PTA-1:

$$\lim_{n\to\infty}\left(\sqrt{n^2+2n+4}-\sqrt{n^2-2n-6}\right)\frac{\sqrt{n^2+2n+4}+\sqrt{n^2-2n-6}}{\sqrt{\;''\;}+\sqrt{\;''\;}}=$$

$$=\lim_{n\to\infty}\frac{(\cancel{n^2}+2n+4)-(\cancel{n^2}+2n+6}{\sqrt{n^2+2n+4}+\sqrt{n^2-2n-6}}=\lim_{n\to\infty}\frac{\cancel{4n+10}\;\cancel{n}\left(4+\frac{10}{n}\right)}{\cancel{n}\left(\underbrace{\sqrt{1+\frac{2}{n}+\frac{4}{n^2}}}_{\downarrow\,1}+\underbrace{\sqrt{1-\frac{2}{n}-\frac{6}{n^2}}}_{\downarrow\,1}\right)}$$

$$=\frac{4}{2}=\underline{\underline{2}}$$

$$\sqrt{n^2\left(1+\frac{2}{n}+\frac{4}{n^2}\right)}=$$

$$\sqrt{n^2}\cdot\sqrt{1+\frac{2}{n}+\frac{4}{n^2}}$$

$$\sqrt{n^2}=n$$

$$\lim_{n\to\infty} n\left(\sqrt{1+\frac{2}{n}+\frac{4}{n^2}}-\sqrt{1-\frac{2}{n}-\frac{6}{n^2}}\right)$$

$n\to\infty$, $\sqrt{1+\frac{2}{n}+\frac{4}{n^2}}\to 1$, $\sqrt{1-\frac{2}{n}-\frac{6}{n^2}}\to 1$, difference $\to 0$

$\infty\cdot 0$ není def.

PTA-2: $f(x) = \frac{1}{2}x^2 + 4x + 6$

tečnu v bodě $x_0 = -8$

\+ vše nakreslit

$y = kx + q$

$k = f'(x_0)$

$f'(x) = x + 4$

$f'(-8) = -8 + 4 = \underline{\underline{-4 = k}}$

$f(-8) = \frac{1}{2} \cdot 64 + 4 \cdot (-8) + 6 =$

$= 32 - 32 + 6 = 6 = y_0$

dosadíme do $y = k \cdot x + q$:

$6 = (-4) \cdot (-8) + q$

$6 = 32 + q$

$\underline{\underline{-26 = q}}$

$\Rightarrow$ tečna $\boxed{y = -4x - 26}$

průsečíky, vrchol, graf:

$P_y: [0,6]$

$P_x: \frac{1}{2}x^2 + 4x + 6 = 0 \quad / \cdot 2$

$x^2 + 8x + 12 = 0$

$(x+2)(x+6) = 0 \Rightarrow x_1 = -2,$

$x_2 = -6$

$V: f(x) = \frac{1}{2}(x^2 + 8x + 12) =$

a) doplnění na □

$= \frac{1}{2}\left((x+4)^2 - 16 + 12\right) =$

$= \frac{1}{2}(x+4)^2 - 2$

$\Downarrow$ $x_v = -4$ $\qquad$ $\Downarrow$ $y_v = -2$

b) $x_v = \frac{x_1 + x_2}{2} = \frac{-2-6}{2} = -4$

$+ f(-4) = -2$

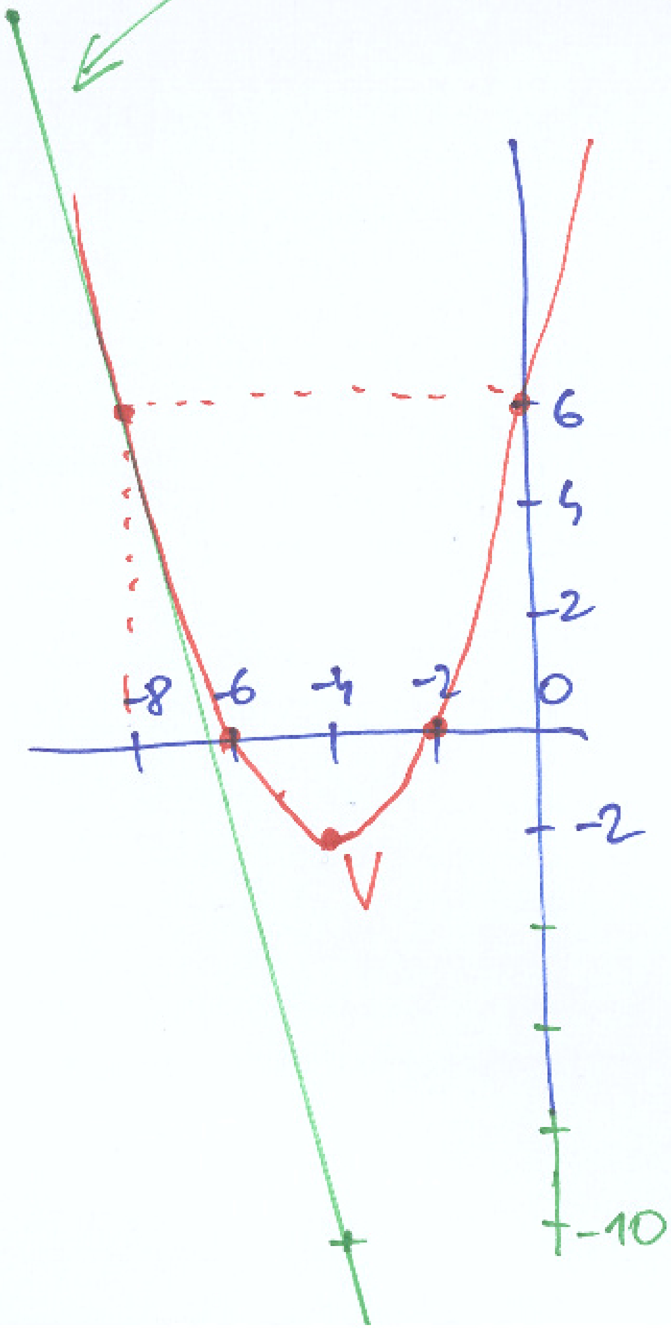
c) $f'(x) = x + 4 = 0 \Rightarrow x_v = -4$

d) $x_v = \frac{-b}{2a} = -4$

$y_v =$ $c - \frac{b^2}{4a} = -2$

tečna $y = -4x - 26$

$x = -4 \Rightarrow y = 16 - 26 = -10$



PTA-3 Průběh fce

$$f(x) = x^3 - 8x^2 + 5x + 14$$

$D_f = \mathbb{R}$, není sudá ani lichá

$P_y = [0, 14]$, P_x: $x = 1 \dots y = 12$

$\boxed{x_1 = -1} \dots y = -1 - 8 - 5 + 14 = 0$

dělení:

$$(x^3 - 8x^2 + 5x + 14) : (x+1) = x^2 - 9x + 14$$

$$\frac{-(x^3 + x^2)}{\quad -9x^2 + 5x + 14}$$

$$-(-9x^2 - 9x)$$

$$14x + 14$$

$\Downarrow$

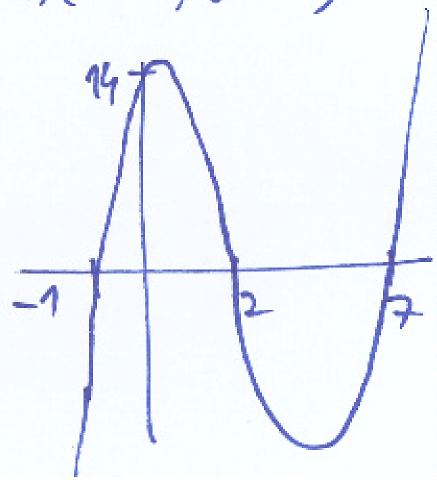
$\boxed{x_2 = 2, \quad x_3 = 7}$

$\Downarrow$

$$f(x) = (x+1)(x-2)(x-7)$$

$$\lim_{x \to +\infty} f(x) = +\infty \qquad (\text{F.č.1})$$

$$\lim_{x \to -\infty} f(x) = -\infty$$



3]

$$f(x) = x^3 - 8x^2 + 5x + 14$$

$$f'(x) = 3x^2 - 16x + 5 = 0$$

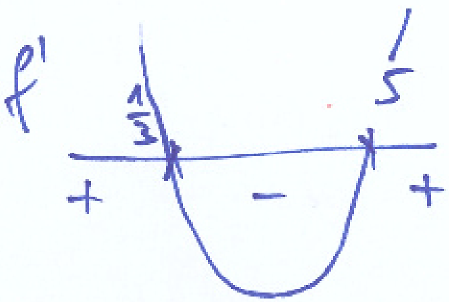
$$D = 256 - 4 \cdot 3 \cdot 5 = 196$$

$$x_{1,2} = \frac{16 \pm 14}{6} = \left\langle \begin{array}{l} \underline{\underline{5}} \\ \frac{2}{6} = \underline{\underline{\frac{1}{3}}} \end{array} \right.$$

stacion. body

	$-\infty$		$\frac{1}{3}$		5		$+\infty$
zn. $f'(x)$		$+$		$-$		$+$	

$\Downarrow$



f roste v $\left(-\infty, \frac{1}{3}\right)$

a v $(5, +\infty)$

f klesá v $\left(\frac{1}{3}, 5\right)$

$\Rightarrow$ lok. maximum $\left[\frac{1}{3}, \frac{400}{27}\right]$

$$f\left(\frac{1}{3}\right) = \frac{1}{27} - \frac{8}{9} + \frac{5}{3} + 14 = \frac{400}{27} \doteq$$

$$\doteq 14{,}81$$

lok. min. $[5, 36]$

$$f(5) = 125 - 8 \cdot 25 + 5 \cdot 5 + 14 = -36$$

$$f''(x) = 6x - 16 = 0$$

$$6x = 16$$

$$x = \frac{16}{6} = \frac{8}{3} \doteq 2{,}67$$

kandidát na inflexní bod

$f''(x) < 0$ pro $x < \frac{8}{3} \Rightarrow f$ konk. $\cap$

> 0 pro $x > \frac{8}{3} \Rightarrow f$ konv. $\cup$

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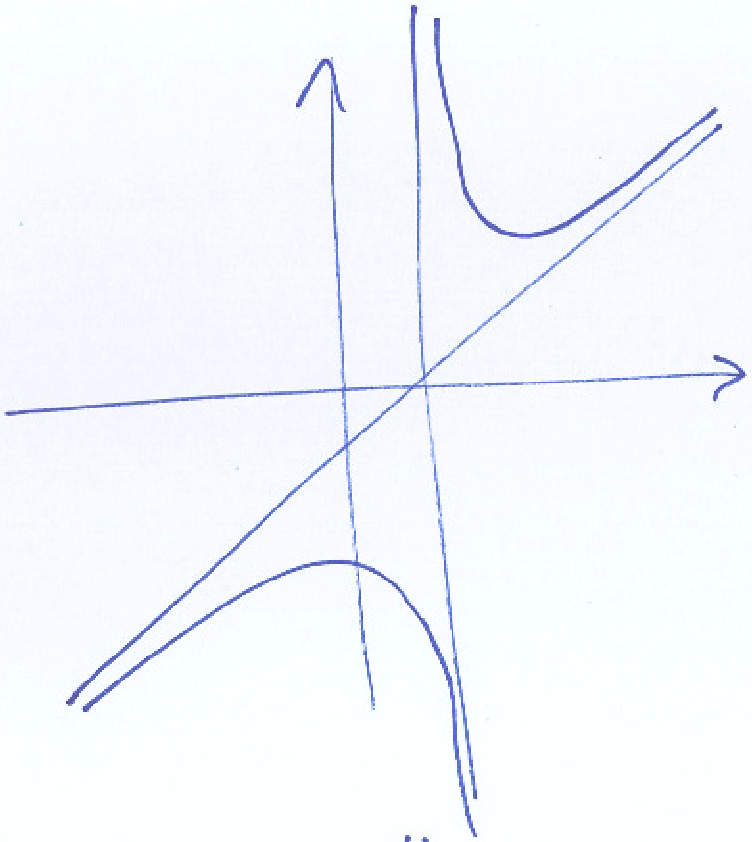
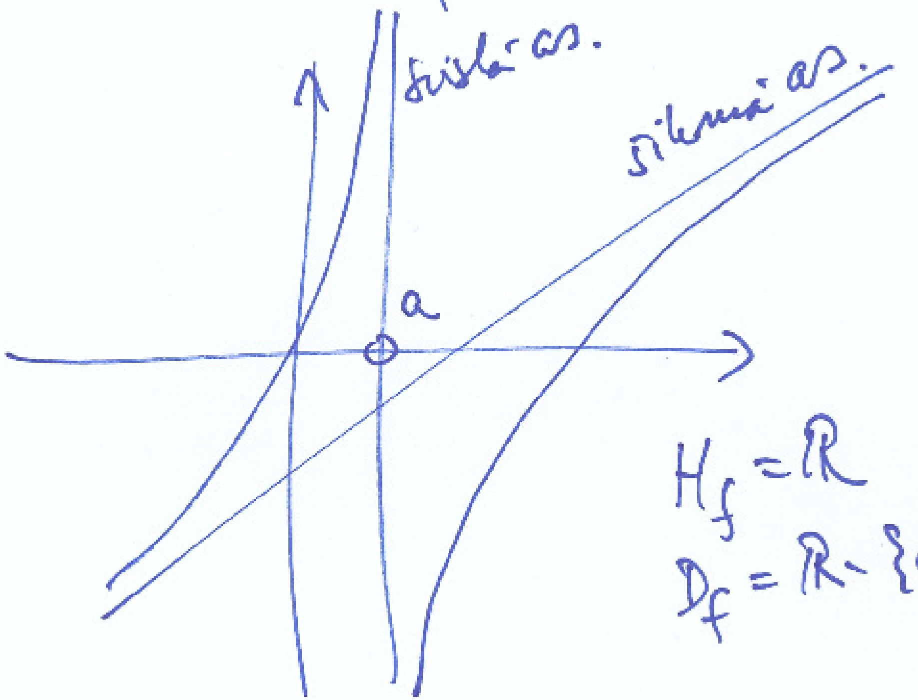
asymptoty:

$\lim\limits_{\pm\infty} f(x) = \pm\infty \Rightarrow$ as. v $\pm\infty$

nemusí a nemusí existovat





$$\frac{x^2+\dots+\dots}{x-a}$$

$H_f = \mathbb{R}$

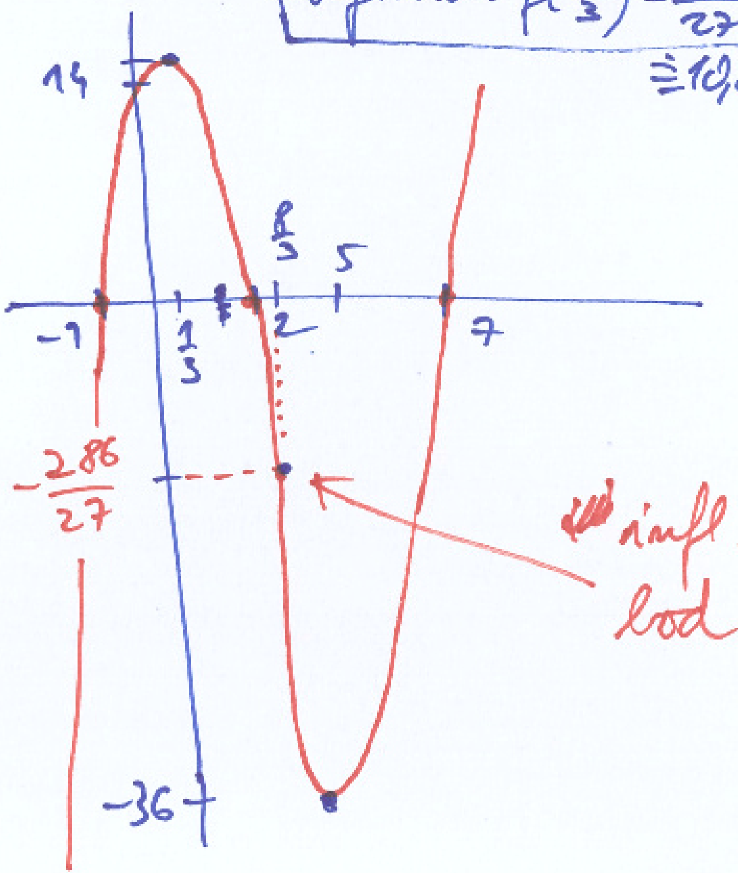
$D_f = \mathbb{R} \setminus \{a\}$

$$\lim_{x\to\pm\infty}\frac{f(x)}{x} = \lim_{x\to\pm\infty}\frac{x^3-8x^2+5x+14}{x} =$$

$$= \lim_{x\to\pm\infty}\frac{\cancel{x}\left(x^2-8x+5+\frac{14}{x}\right)}{\cancel{x}} = +\infty$$

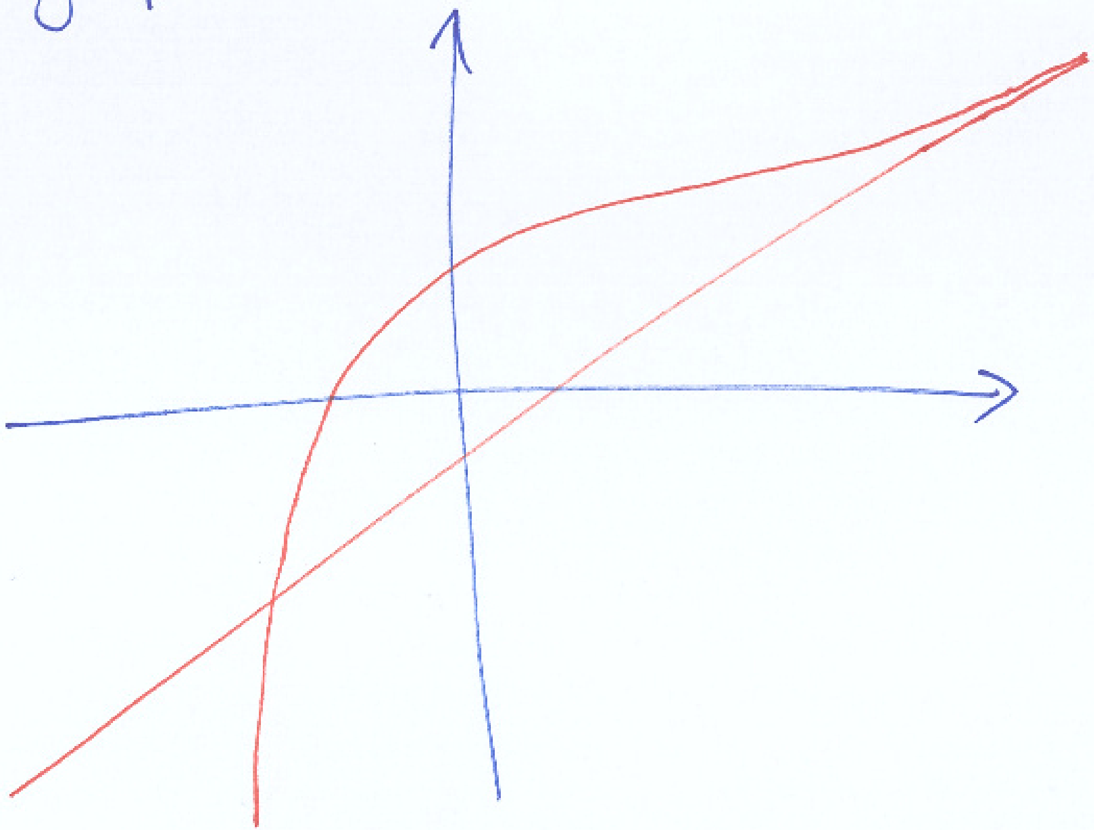
as. v $\pm\infty$ neexistuje

infl. bod: $f\left(\frac{8}{3}\right) = \frac{286}{27} \doteq 10{,}59$



5)

Asymptota:



Infl. bod:

$$f\left(\frac{8}{3}\right) = \left(\frac{8}{3}\right)^3 - 8\cdot\left(\frac{8}{3}\right)^2 + 5\cdot\frac{8}{3} + 14$$

$$= \frac{-286}{27} \doteq -10{,}59$$