

Hilbert's Entscheidungsproblem

The problem (Lect. 7) asks for an algorithm that would decide whether a FO sentence φ is logically valid: $\models \varphi$.

Theorem (A. Church / A. Turing 1936)

No such alg. exists.

Proof: We know (from the proof of Gödel's theorem) that $G \vdash \varphi$ is algorithmically undecidable. But:

$$G \vdash \varphi \Leftrightarrow \vdash \bigwedge G \rightarrow \varphi \Leftrightarrow \models \bigwedge G \rightarrow \varphi$$

↓
Deduction
theorem

↓
Completeness
thm

Here $\bigwedge G$ is the conjunction of all (finitely many!) axioms of G .

□ + thm.

Remark: The thm holds if, for example, the language contains at least one binary relation / symbol (or function)