Introduction

The Model

Simple Market Maker Problem

Fight with Technical Analysis

Problems Met

Future

Arbitrage on Limit Order Markets

Martin Šmíd, Aleš Kuběna, Peter Hron,...

ÚTIA, Czech Acac. Sci., Pod Vodárenskou věží 4, Praha 8, 182 08 Czech Republic

Seminar SPA, October 21h, 2021

Market Prices Are no Mystery

Arbitrage on Limit Order Markets

Martin Šmíd, Aleš Kuběna, Peter Hron,. . .

Introduction

The Model

Simple Market Maker Problem

Fight with Technical Analysis

Problems Met

Future

Market Prices Are no Mystery

- price = F(behavior of traders)

Market Prices Are no Mystery

Introduction

The Model

Simple Market Maker Problem

Fight with Technical Analysis

Problems Met

Future

- price $= F$(behavior of traders)
- knowing traders' strategies $\Rightarrow$ knowing prices

Market Prices Are no Mystery

- price $= F$(behavior of traders)
- knowing traders' strategies $\Rightarrow$ knowing prices
  - strategies are random walk $\Rightarrow$ prices are random walk

Market Prices Are no Mystery

Arbitrage on Limit Order Markets
Martin Šmíd, Aleš Kuběna, Peter Hron,...
Introduction
The Model
Simple Market Maker Problem
Fight with Technical Analysis
Problems Met
Future

- price $= F(\text{behavior of traders})$
- knowing traders' strategies $\Rightarrow$ knowing prices
  - strategies are random walk $\Rightarrow$ prices are random walk
  - strategies are Markov $\Rightarrow$ prices are Markov

Market Prices Are no Mystery

- price $= F(\text{behavior of traders})$
- knowing traders' strategies $\Rightarrow$ knowing prices
  - strategies are random walk $\Rightarrow$ prices are random walk
  - strategies are Markov $\Rightarrow$ prices are Markov
  - strategies buy more/less than sell $\Rightarrow$ prices go up/down

Market Prices Are no Mystery

- price = F(behavior of traders)
- knowing traders' strategies ⇒ knowing prices
  - strategies are random walk ⇒ prices are random walk
  - strategies are Markov ⇒ prices are Markov
  - strategies buy more/less than sell ⇒ prices go up/down
- Two problems

Market Prices Are no Mystery

- price $=$ F(behavior of traders)
- knowing traders' strategies $\Rightarrow$ knowing prices
  - strategies are random walk $\Rightarrow$ prices are random walk
  - strategies are Markov $\Rightarrow$ prices are Markov
  - strategies buy more/less than sell $\Rightarrow$ prices go up/down
- Two problems
  - **Uncertainty.** We don't know the others' strategies

Market Prices Are no Mystery

- price = F(behavior of traders)
- knowing traders' strategies ⇒ knowing prices
  - strategies are random walk ⇒ prices are random walk
  - strategies are Markov ⇒ prices are Markov
  - strategies buy more/less than sell ⇒ prices go up/down
- Two problems
  - **Uncertainty.** We don't know the others' strategies
    - Sometimes they don't know

Market Prices Are no Mystery

- price $= F$(behavior of traders)
- knowing traders' strategies $\Rightarrow$ knowing prices
  - strategies are random walk $\Rightarrow$ prices are random walk
  - strategies are Markov $\Rightarrow$ prices are Markov
  - strategies buy more/less than sell $\Rightarrow$ prices go up/down
- Two problems
  - **Uncertainty.** We don't know the others' strategies
    - Sometimes they don't know
    - Sometimes we don't know our own one

Market Prices Are no Mystery

- price = F(behavior of traders)
- knowing traders' strategies ⇒ knowing prices
  - strategies are random walk ⇒ prices are random walk
  - strategies are Markov ⇒ prices are Markov
  - strategies buy more/less than sell ⇒ prices go up/down
- Two problems
  - **Uncertainty.** We don't know the others' strategies
    - Sometimes they don't know
    - Sometimes we don't know our own one
  - **Complexity.** Practically we cannot compute optimal strategy.

The Key are Preferences

Arbitrage on Limit Order Markets
Martin Šmíd, Aleš Kuběna, Peter Hron,. . .
Introduction
The Model
Simple Market Maker Problem
Fight with Technical Analysis
Problems Met
Future

Agents' Preferences

The Key are Preferences

Agents' Preferences

- Strategies are (should be) determined by the agents' preferences over (discounted) cashflows (or consumption) – may be described by a pure-jump real process d

The Key are Preferences

Agents' Preferences

- Strategies are (should be) determined by the agents' preferences over (discounted) cashflows (or consumption) – may be described by a pure-jump real process d
- Except for some obvious cases (e.g. when FSD holds), it is difficult to compare two cashflows

The Key are Preferences

Agents' Preferences

- Strategies are (should be) determined by the agents' preferences over (discounted) cashflows (or consumption) – may be described by a pure-jump real process d
- Except for some obvious cases (e.g. when FSD holds), it is difficult to compare two cashflows
- Generally, there are two trade-offs
  - return $\times$ risk
  - today $\times$ tomorrow

The Key are Preferences

Agents' Preferences

- Strategies are (should be) determined by the agents' preferences over (discounted) cashflows (or consumption) – may be described by a pure-jump real process d
- Except for some obvious cases (e.g. when FSD holds), it is difficult to compare two cashflows
- Generally, there are two trade-offs
  - return $\times$ risk
  - today $\times$ tomorrow
- If there is a risk measure $\rho(\int d)$ (functional on the space of random variables) defined, then the ordering is complete

The Key are Preferences

Agents' Preferences

- Strategies are (should be) determined by the agents' preferences over (discounted) cashflows (or consumption) – may be described by a pure-jump real process d
- Except for some obvious cases (e.g. when FSD holds), it is difficult to compare two cashflows
- Generally, there are two trade-offs
  - return $\times$ risk
  - today $\times$ tomorrow
- If there is a risk measure $\rho(\int d)$ (functional on the space of random variables) defined, then the ordering is complete
- Usual risk measures are either
  - "theoretically reasonable" (coherent, time-consistent, etc) - e.g. Mean-CVaR, worst case
  - consistent with empirical evidence - e.g. Cumulative Prospect Theory
  - computable (single stage Mean-CVaR, Exponential utility)

unfortunately not simultaneously.

Arbitrage

Arbitrage - Are there Free Lunches Served?

Introduction
The Model
Simple Market Maker Problem
Fight with Technical Analysis
Problems Met
Future

Arbitrage

- Strongest notion = a zero cost strategy with outcome first order stochastically dominating zero (i.e. $\mathbb{P}[\int c \geq 0] > 0$)
  - Rare in practice (if it existed, it would be exploited and thus vanished)

Arbitrage - Are there Free Lunches Served?

Arbitrage

- Strongest notion = a zero cost strategy with outcome first order stochastically dominating zero (i.e. $\mathbb{P}[\int c \geq 0] > 0$)
  - Rare in practice (if it existed, it would be exploited and thus vanished)
- Weakest notion: zero cost strategy positively evaluated by own risk measure $\rho(\int c) > 0$
  - Likely to exist (ff the risk measure is expectation, for instance)

Arbitrage - Are there Free Lunches Served?

Arbitrage

- Strongest notion = a zero cost strategy with outcome first order stochastically dominating zero (i.e. $\mathbb{P}[\int c \geq 0] > 0$)
  - Rare in practice (if it existed, it would be exploited and thus vanished)
- Weakest notion: zero cost strategy positively evaluated by own risk measure $\rho(\int c) > 0$
  - Likely to exist (ff the risk measure is expectation, for instance)

Q: Is it possible that $\rho_1(\int c) > 0 \wedge \rho_2(-\int c) > 0$ for reasonable R_1, R_2? (i.e. can be both players satisfied by a zero sum game? Probably not...)

Arbitrage - Are there Free Lunches Served?

Arbitrage

- Strongest notion = a zero cost strategy with outcome first order stochastically dominating zero (i.e. $\mathbb{P}[\int c \geq 0] > 0$)
  - Rare in practice (if it existed, it would be exploited and thus vanished)
- Weakest notion: zero cost strategy positively evaluated by own risk measure $\rho(\int c) > 0$
  - Likely to exist (ff the risk measure is expectation, for instance)

Q: Is it possible that $\rho_1(\int c) > 0 \wedge \rho_2(-\int c) > 0$ for reasonable R_1, R_2? (i.e. can be both players satisfied by a zero sum game? Probably not...)

Textbook argument: Irrational strategies run out of money and vanish from the market.

Arbitrage - Are there Free Lunches Served?

Arbitrage

- Strongest notion = a zero cost strategy with outcome first order stochastically dominating zero (i.e. $\mathbb{P}[\int c \geq 0] > 0$)
  - Rare in practice (if it existed, it would be exploited and thus vanished)
- Weakest notion: zero cost strategy positively evaluated by own risk measure $\rho(\int c) > 0$
  - Likely to exist (ff the risk measure is expectation, for instance)

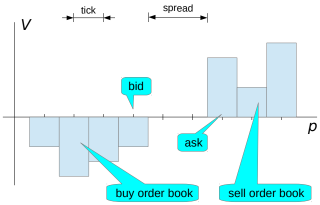
Q: Is it possible that $\rho_1(\int c) > 0 \wedge \rho_2(-\int c) > 0$ for reasonable R_1, R_2? (i.e. can be both players satisfied by a zero sum game? Probably not...)

Textbook argument: Irrational strategies run out of money and vanish from the market.

(Too ambitious) Q: If everyone acted rationally, would arbitrage opportunities exist?

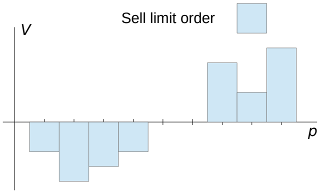
The F (how strategies are transformed to prices)

- On electronic markets (with a single commodity)
  - Offers = *limit orders*, containing
    - *limit price* - maximal/minimal price for which I am willing to buy/sell
    - *volume* - how much I want to buy/sell
  - Acceptances = *market orders*
    - *volume* - how much I want to buy/sell
    - naturally, the best limit orders are exploited
  - Market orders with plus/minus infinite price are equivalent to limit orders (we use this further)



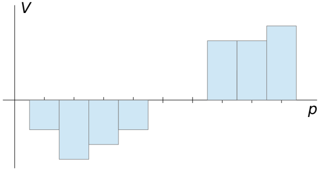
The F (how strategies are transformed to prices)

- On electronic markets (with a single commodity)
  - Offers = *limit orders*, containing
    - *limit price* - maximal/minimal price for which I am willing to buy/sell
    - *volume* - how much I want to buy/sell
  - Acceptances = *market orders*
    - *volume* - how much I want to buy/sell
    - naturally, the best limit orders are exploited
  - Market orders with plus/minus infinite price are equivalent to limit orders (we use this further)



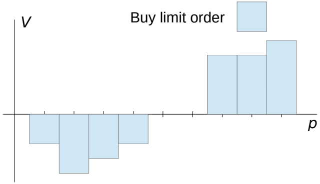
The F (how strategies are transformed to prices)

- On electronic markets (with a single commodity)
  - Offers = *limit orders*, containing
    - *limit price* - maximal/minimal price for which I am willing to buy/sell
    - *volume* - how much I want to buy/sell
  - Acceptances = *market orders*
    - *volume* - how much I want to buy/sell
    - naturally, the best limit orders are exploited
  - Market orders with plus/minus infinite price are equivalent to limit orders (we use this further)



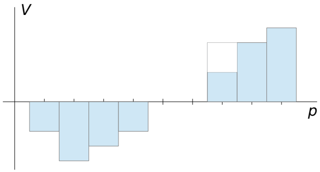
The F (how strategies are transformed to prices)

- On electronic markets (with a single commodity)
  - Offers = *limit orders*, containing
    - *limit price* - maximal/minimal price for which I am willing to buy/sell
    - *volume* - how much I want to buy/sell
  - Acceptances = *market orders*
    - *volume* - how much I want to buy/sell
    - naturally, the best limit orders are exploited
  - Market orders with plus/minus infinite price are equivalent to limit orders (we use this further)



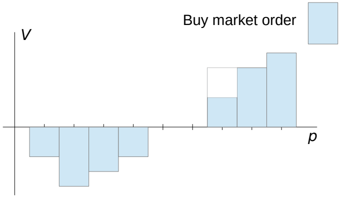
The F (how strategies are transformed to prices)

- On electronic markets (with a single commodity)
  - Offers = *limit orders*, containing
    - *limit price* - maximal/minimal price for which I am willing to buy/sell
    - *volume* - how much I want to buy/sell
  - Acceptances = *market orders*
    - *volume* - how much I want to buy/sell
    - naturally, the best limit orders are exploited
  - Market orders with plus/minus infinite price are equivalent to limit orders (we use this further)



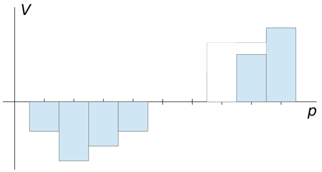
The F (how strategies are transformed to prices)

- On electronic markets (with a single commodity)
  - Offers = *limit orders*, containing
    - *limit price* - maximal/minimal price for which I am willing to buy/sell
    - *volume* - how much I want to buy/sell
  - Acceptances = *market orders*
    - *volume* - how much I want to buy/sell
    - naturally, the best limit orders are exploited
  - Market orders with plus/minus infinite price are equivalent to limit orders (we use this further)



The F (how strategies are transformed to prices)

- On electronic markets (with a single commodity)
  - Offers = *limit orders*, containing
    - *limit price* - maximal/minimal price for which I am willing to buy/sell
    - *volume* - how much I want to buy/sell
  - Acceptances = *market orders*
    - *volume* - how much I want to buy/sell
    - naturally, the best limit orders are exploited
  - Market orders with plus/minus infinite price are equivalent to limit orders (we use this further)



(Ambitious) Setting

- N agents

(Ambitious) Setting

Introduction

The Model

Simple Market Maker Problem

Fight with Technical Analysis

Problems Met

Future

- N agents
- single market with a single instrument, FIFO settlement (older orders preferred)

(Ambitious) Setting

- N agents
- single market with a single instrument, FIFO settlement (older orders preferred)
- prices in $\mathbb{N}$ (ticks)

(Ambitious) Setting

- N agents
- single market with a single instrument, FIFO settlement (older orders preferred)
- prices in $\mathbb{N}$ (ticks)
- infinite horizon (no final valuation of portfolios)

(Ambitious) Setting

- N agents
- single market with a single instrument, FIFO settlement (older orders preferred)
- prices in $\mathbb{N}$ (ticks)
- infinite horizon (no final valuation of portfolios)
- state space $\Xi \subset \mathbb{N}^{\mathbb{Z}}$ (+/- components are #s of sell/buy orders)

(Ambitious) Setting

- N agents
- single market with a single instrument, FIFO settlement (older orders preferred)
- prices in $\mathbb{N}$ (ticks)
- infinite horizon (no final valuation of portfolios)
- state space $\Xi \subset \mathbb{N}^{\mathbb{Z}}$ (+/- components are #s of sell/buy orders)
- at each jump τ_k, the i-th agent decides for
  - the cashflow $c_{i,k}$
  - the order profile $u_{i,k} \in \mathbb{N}^{\mathbb{Z}}$

(Ambitious) Setting

- N agents
- single market with a single instrument, FIFO settlement (older orders preferred)
- prices in $\mathbb{N}$ (ticks)
- infinite horizon (no final valuation of portfolios)
- state space $\Xi \subset \mathbb{N}^{\mathbb{Z}}$ (+/- components are #s of sell/buy orders)
- at each jump τ_k, the i-th agent decides for
    - the cashflow $c_{i,k}$
    - the order profile $u_{i,k} \in \mathbb{N}^{\mathbb{Z}}$
- the profile executed at $\tau_k + \theta_{i,k}$ (i.e. latency modelled)

(Ambitious) Setting

- N agents
- single market with a single instrument, FIFO settlement (older orders preferred)
- prices in $\mathbb{N}$ (ticks)
- infinite horizon (no final valuation of portfolios)
- state space $\Xi \subset \mathbb{N}^{\mathbb{Z}}$ (+/- components are #s of sell/buy orders)
- at each jump τ_k, the i-th agent decides for
  - the cashflow $c_{i,k}$
  - the order profile $u_{i,k} \in \mathbb{N}^{\mathbb{Z}}$
- the profile executed at $\tau_k + \theta_{i,k}$ (i.e. latency modelled)
- i-th agent minimizes $\rho_i(\sum_k \gamma_i^k c_{i,k})$

  γ_i - a discount factor

  ρ_i - a nested homogeneous risk measure (i.e. $\rho_i(\sum_k \gamma_i^k c_{i,k}) = \rho_i(c_{i,0} + \gamma_i \rho_i(c_{i,1} \dots))$ or $\rho_i(\sum_k \gamma_i^k c_{i,k}) = \rho_i(c_{i,0} \rho_i(c_{i,1} \dots)^{\gamma_i})$

(Ambitious) Setting

- N agents
- single market with a single instrument, FIFO settlement (older orders preferred)
- prices in $\mathbb{N}$ (ticks)
- infinite horizon (no final valuation of portfolios)
- state space $\Xi \subset \mathbb{N}^{\mathbb{Z}}$ (+/- components are #s of sell/buy orders)
- at each jump τ_k, the i-th agent decides for
  - the cashflow $c_{i,k}$
  - the order profile $u_{i,k} \in \mathbb{N}^{\mathbb{Z}}$
- the profile executed at $\tau_k + \theta_{i,k}$ (i.e. latency modelled)
- i-th agent minimizes $\rho_i(\sum_k \gamma_i^k c_{i,k})$

  γ_i - a discount factor

  ρ_i - a nested homogeneous risk measure (i.e. $\rho_i(\sum_k \gamma_i^k c_{i,k}) = \rho_i(c_{i,0} + \gamma_i \rho_i(c_{i,1} \dots))$ or $\rho_i(\sum_k \gamma_i^k c_{i,k}) = \rho_i(c_{i,0} \rho_i(c_{i,1} \dots)^{\gamma_i})$
- Constraints: money/instrument blocked when limit order posted

(Ambitious) Setting

- N agents
- single market with a single instrument, FIFO settlement (older orders preferred)
- prices in $\mathbb{N}$ (ticks)
- infinite horizon (no final valuation of portfolios)
- state space $\Xi \subset \mathbb{N}^{\mathbb{Z}}$ (+/- components are #s of sell/buy orders)
- at each jump τ_k, the i-th agent decides for
  - the cashflow $c_{i,k}$
  - the order profile $u_{i,k} \in \mathbb{N}^{\mathbb{Z}}$
- the profile executed at $\tau_k + \theta_{i,k}$ (i.e. latency modelled)
- i-th agent minimizes $\rho_i(\sum_k \gamma_i^k c_{i,k})$

  γ_i - a discount factor

  ρ_i - a nested homogeneous risk measure (i.e. $\rho_i(\sum_k \gamma_i^k c_{i,k}) = \rho_i(c_{i,0} + \gamma_i \rho_i(c_{i,1} \dots))$ or $\rho_i(\sum_k \gamma_i^k c_{i,k}) = \rho_i(c_{i,0} \rho_i(c_{i,1} \dots)^{\gamma_i})$
- Constraints: money/instrument blocked when limit order posted

$\Rightarrow$ $V_i(m, n, \Xi) = \max_{(u,c)\in X}[c + \gamma\rho_i(V_i(m_u, n_n, \Xi_u))]$ (or mult.)

Back to the Earth (MŠ 2019)

Market

- 16 price ticks
- unit order volumes

Two "agents"

- A Zero Intelligence Trader
  - constant unit intensity of order arrivals, uniform on price space and types (buy/sell)
  - unit cancellation intensity
- A Market Maker
  - Simultaneously posts buy and sell limit orders (quotes), earns on spread
  - (very often) posts quotes – one buy– and one sell limit order, may avoid quoting
  - risk measure: nested Mean-CVaR or exponential utility function

Our Goal

- Optimal strategy of a market maker

computation

Restrictions

- max 6 pieces of instrument,
- max 50 units of cash (aggregated by 2)
- max spread = 3

$\doteq$ 43000 states

$\Rightarrow$ about 20 non-zero transition probabilities

- $c_i \in \{0, 2\}$
- max 2 pieces of currency consumed

$\Rightarrow$ max 26 actions

Solution method: Approximate Dynamic Programming

Solution

Procedure

1. Construct initial "naive" strategy (quote if possible, consume if money account overruns a threshold). By simulation, find the optimal threshold
2. Initialize $V(m, n, X)$ by values, achieved by naive strategy with the optimal threshold
3. Apply ADP (with 200,000 iterations)
4. Evaluate the resulting policy by simulation for initial state with 30 units of cash (25,000 times)

Solution details

- Coded in C++
- Solved for Mean-CVaR and exponential utility with various risk-aversion parameters
- A solution taking $\sim$ 2 hrs, each policy evaluation $\sim$ 30 min, on Core I7

Results

Criterion	R.A.	Mean	CVaR	Min
Naive	0	174.835	156.006	
Mean-CVaR	0	222.526	187.25	
	0.05	208.322	176.11	
	0.1	181.234	145.98	
	0.25	2.230	-0.36	
Exp u.f.	0.001	220.928	187.249	
	0.01	221.992	187.713	
	0.1	184.050	144.247	
	1	8.354	-0.43	

Results

Criterion	R.A.	Mean	CVaR	Min
Naive	0	174.835	156.006	
Mean-CVaR	0	222.526	187.25	
	0.05	208.322	176.11	
	0.1	181.234	145.98	
	0.25	2.230	-0.36	
Exp u.f.	0.001	220.928	187.249	
	0.01	221.992	187.713	
	0.1	184.050	144.247	
	1	8.354	-0.43	

Surprise:

- No tradeoff between return and risk.
- Moreover, the "more risk-averse" variants are dominated except for the first two exp's

Results

Criterion	R.A.	Mean	CVaR	Min
Naive	0	174.835	156.006	
Mean-CVaR	0	222.526	187.25	
	0.05	208.322	176.11	
	0.1	181.234	145.98	
	0.25	2.230	-0.36	
Exp u.f.	0.001	220.928	187.249	
	0.01	221.992	187.713	
	0.1	184.050	144.247	
	1	8.354	-0.43	

Surprise:

- No tradeoff between return and risk.
- Moreover, the "more risk-averse" variants are dominated except for the first two exp's

Problem behind: Mean-CVaR collapses to worst case with decreasing time granularity.

COVID intermission (MŠ 2020)

Arbitrage on Limit Order Markets

Martin Šmíd, Aleš Kuběna, Peter Hron,. . .

Introduction

The Model

Simple Market Maker Problem

Fight with Technical Analysis

Problems Met

Future

- General simulation framework developed (`https://github.com/cyberklezmer/marketsim`)

Fight with Technical Analysis (MŠ & P. Hron 2021)

Research Q: Can "Machine learning" beat technical analysis?

TA - trading strategy based on analysing price trends

Market agents

Liquidity Taker: Buys/Sells randomly for the current price(s) (perhaps a pension fund)

Market Maker: Keeps quoting bid and ask according to Stoll [1978] (classics of MM decisions)

MACD: Agent following a certain TA strategy

ADP: Risk neutral ($\rho = \mathbb{E}$) "Machine learning" agent (uses ADP) maximizing profit from speculation (buying and selling at equidistant times)

Fight with Technical Analysis (MŠ & P. Hron 2021)

Research Q: Can "Machine learning" beat technical analysis?

TA - trading strategy based on analysing price trends

Market agents

Liquidity Taker: Buys/Sells randomly for the current price(s) (perhaps a pension fund)

Market Maker: Keeps quoting bid and ask according to Stoll [1978] (classics of MM decisions)

MACD: Agent following a certain TA strategy

ADP: Risk neutral ($\rho = \mathbb{E}$) "Machine learning" agent (uses ADP) maximizing profit from speculation (buying and selling at equidistant times)

Results (by repeated evaluation of the strategies)

strategy	ADP	MACD	MM	LT
profit	120820	3161.01	-139221	15239.7
std.err.	36964.1	891.022	41760.3	6837.08

Fight with Technical Analysis (MŠ & P. Hron 2021)

Research Q: Can "Machine learning" beat technical analysis?

Results (by repeated evaluation of the strategies)

strategy	ADP	MACD	MM	LT
profit	120820	3161.01	-139221	15239.7
std.err.	36964.1	891.022	41760.3	6837.08

Discussion

- ADP was able to learn from market prices (did not "see" the strategies)
- ADP out-performed TA
- Textbook MM failed to work (due to lack of knowledge of "fair price")
- ? Purely random LT earned significantly, maybe due to price increase (the last market price of the instrument was included into the cash-flow while in practice there is no guarantee it can be sold for that price)

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \dots + X_i + \sigma(X_{i+1} + \dots + X_{2i} + \sigma(\dots)))$$

where σ is the conditional mean-α-CVaR mapping

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)

Problems Met on the Road I: Risk Mean-CVaR scaling

Arbitrage on Limit Order Markets

Martin Šmíd, Aleš Kuběna, Peter Hron,...

Introduction

The Model

Simple Market Maker Problem

Fight with Technical Analysis

Problems Met

Future

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Problems Met on the Road I: Risk Mean-CVaR scaling

Arbitrage on Limit Order Markets

Martin Šmíd, Aleš Kuběna, Peter Hron,...

Introduction

The Model

Simple Market Maker Problem

Fight with Technical Analysis

Problems Met

Future

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\ldots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\ldots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

Arbitrage on Limit Order Markets

Martin Šmíd, Aleš Kuběna, Peter Hron,...

Introduction

The Model

Simple Market Maker Problem

Fight with Technical Analysis

Problems Met

Future

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\ldots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\ldots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\ldots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\ldots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

where σ is the conditional mean-α-CVaR mapping

- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

Arbitrage on Limit Order Markets

Martin Šmíd, Aleš Kuběna, Peter Hron,...

Introduction

The Model

Simple Market Maker Problem

Fight with Technical Analysis

Problems Met

Future

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \dots + X_i + \sigma(X_{i+1} + \dots + X_{2i} + \sigma(\dots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\ldots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\ldots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\ldots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\ldots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\ldots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\ldots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\ldots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

Arbitrage on Limit Order Markets
Martin Šmíd, Aleš Kuběna, Peter Hron,...
Introduction
The Model
Simple Market Maker Problem
Fight with Technical Analysis
Problems Met
Future

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\ldots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Problems Met on the Road I: Risk Mean-CVaR scaling

Arbitrage on Limit Order Markets

Martin Šmíd, Aleš Kuběna, Peter Hron,...

Introduction

The Model

Simple Market Maker Problem

Fight with Technical Analysis

Problems Met

Future

- Consider $X = \sum_{\tau=1}^{T} X_i$, $X_i \in \mathcal{F}_i$ for some filtration $\mathcal{F}_\bullet$, $T = n!$ for some n
- ρ_i the nested Mean-CVaR with step i (i.e.

$$\rho_i = \sigma(X_1 + \cdots + X_i + \sigma(X_{i+1} + \cdots + X_{2i} + \sigma(\dots)))$$

  where σ is the conditional mean-α-CVaR mapping
- Clearly, the less i, the greater the "overall risk aversion" (tail of tails is smaller than the tail)
- Pichler and Schlotter (2020?) address this issue for normal independent summands and adjust α with the decreasing time interval

Q: Can be something done for the stationary case (CLT exist)?

Q: Can be nested risk measure extended to infinite horizon (given discounting)?

Because then we could use the Rockafellar and Uryasev trick and incorporate the CVaR to the expectation

Problems Met II: Curses of Dimensionality

Arbitrage on Limit Order Markets
Martin Šmíd, Aleš Kuběna, Peter Hron,. . .
Introduction
The Model
Simple Market Maker Problem
Fight with Technical Analysis
Problems Met
Future

Problems Met II: Curses of Dimensionality

Arbitrage on Limit Order Markets
Martin Šmíd, Aleš Kuběna, Peter Hron,...
Introduction
The Model
Simple Market Maker Problem
Fight with Technical Analysis
Problems Met
Future

- With– or without risk aversion, we have

$$V(s) = \max_{(c,x)\in A(S)} [c + \gamma \mathbb{E}_Y V(S(x, c, Y))]$$

Problems Met II: Curses of Dimensionality

- With– or without risk aversion, we have

$$V(s) = \max_{(c,x)\in A(S)} [c + \gamma \mathbb{E}_Y V(S(x, c, Y))]$$

- $B(V) = \max(c + \gamma \mathbb{E} V)$ is a contraction operator (in sup norm) with fixed point

Problems Met II: Curses of Dimensionality

- With– or without risk aversion, we have

$$V(s) = \max_{(c,x)\in A(S)} [c + \gamma \mathbb{E}_Y V(S(x, c, Y))]$$

- $B(V) = \max(c + \gamma \mathbb{E} V)$ is a contraction operator (in sup norm) with fixed point

$\Rightarrow$ gradual approximation of V leads to (nearly) optimal solution, so what is the problem?

Problems Met II: Curses of Dimensionality

- With– or without risk aversion, we have

$$V(s) = \max_{(c,x)\in A(S)} [c + \gamma \mathbb{E}_Y V(S(x,c,Y))]$$

- $B(V) = \max(c + \gamma \mathbb{E} V)$ is a contraction operator (in sup norm) with fixed point

$\Rightarrow$ gradual approximation of V leads to (nearly) optimal solution, so what is the problem?

Problems Met II: Curses of Dimensionality

- With– or without risk aversion, we have

$$V(s) = \max_{(c,x)\in A(S)} [c + \gamma \mathbb{E}_Y V(S(x, c, Y))]$$

- $B(V) = \max(c + \gamma \mathbb{E} V)$ is a contraction operator (in sup norm) with fixed point

$\Rightarrow$ gradual approximation of V leads to (nearly) optimal solution, so what is the problem?

However, we have all the 3 curses of dimensionality

- Large action space (order profiles)
- Large probability space (limit order may appear anywhere)
- Large state space (prices are transient $\Rightarrow$ has to be [nearly] infinite

Problems Met II: Curses of Dimensionality

- With– or without risk aversion, we have

$$V(s) = \max_{(c,x)\in A(S)} [c + \gamma \mathbb{E}_Y V(S(x, c, Y))]$$

- $B(V) = \max(c + \gamma \mathbb{E} V)$ is a contraction operator (in sup norm) with fixed point

$\Rightarrow$ gradual approximation of V leads to (nearly) optimal solution, so what is the problem?

However, we have all the 3 curses of dimensionality

- Large action space (order profiles)
- Large probability space (limit order may appear anywhere)
- Large state space (prices are transient $\Rightarrow$ has to be [nearly] infinite

Moreover,

- γ is close to 1 $\Rightarrow$ convergence is slow

Problems Met III and IV:

Introduction

The Model

Simple Market Maker Problem

Fight with Technical Analysis

Problems Met

Future

Unknown Distribution

- As a resultant of dozens of strategies, the market is a quasi-random system.

Problems Met III and IV:

Unknown Distribution

- As a resultant of dozens of strategies, the market is a quasi-random system.
- (in our simulator, the only randomness stems from latency)

Problems Met III and IV:

Introduction

The Model

Simple Market Maker Problem

Fight with Technical Analysis

Problems Met

Future

Unknown Distribution

- As a resultant of dozens of strategies, the market is a quasi-random system.
- (in our simulator, the only randomness stems from latency)

$\Rightarrow$ No chance for knowledge of distribution

Problems Met III and IV:

Unknown Distribution

- As a resultant of dozens of strategies, the market is a quasi-random system.
- (in our simulator, the only randomness stems from latency)

$\Rightarrow$ No chance for knowledge of distribution

- We have to estimate the distribution or directly $\mathbb{E}V$

Problems Met III and IV:

Unknown Distribution

- As a resultant of dozens of strategies, the market is a quasi-random system.
- (in our simulator, the only randomness stems from latency)

$\Rightarrow$ No chance for knowledge of distribution

- We have to estimate the distribution or directly $\mathbb{E}V$

$\Rightarrow$ Bye bye ADP, hello machine learning

Problems Met III and IV:

Arbitrage on Limit Order Markets

Martin Šmíd, Aleš Kuběna, Peter Hron,...

Introduction
The Model
Simple Market Maker Problem
Fight with Technical Analysis
Problems Met
Future

Unknown Distribution

- As a resultant of dozens of strategies, the market is a quasi-random system.
- (in our simulator, the only randomness stems from latency)

$\Rightarrow$ No chance for knowledge of distribution

- We have to estimate the distribution or directly $\mathbb{E}V$

$\Rightarrow$ Bye bye ADP, hello machine learning

? Does it still converge?

Problems Met III and IV:

Introduction

The Model

Simple Market Maker Problem

Fight with Technical Analysis

Problems Met

Future

Unknown Distribution

- As a resultant of dozens of strategies, the market is a quasi-random system.
- (in our simulator, the only randomness stems from latency)

$\Rightarrow$ No chance for knowledge of distribution

- We have to estimate the distribution or directly $\mathbb{E}V$

$\Rightarrow$ Bye bye ADP, hello machine learning

? Does it still converge?

- "Art" of approximation needed

Problems Met III and IV:

Unknown Distribution

- As a resultant of dozens of strategies, the market is a quasi-random system.
- (in our simulator, the only randomness stems from latency)

$\Rightarrow$ No chance for knowledge of distribution

- We have to estimate the distribution or directly $\mathbb{E}V$

$\Rightarrow$ Bye bye ADP, hello machine learning

? Does it still converge?

- "Art" of approximation needed

Initialization of the Market

Problems Met III and IV:

Unknown Distribution

- As a resultant of dozens of strategies, the market is a quasi-random system.
- (in our simulator, the only randomness stems from latency)

$\Rightarrow$ No chance for knowledge of distribution

- We have to estimate the distribution or directly $\mathbb{E}V$

$\Rightarrow$ Bye bye ADP, hello machine learning

? Does it still converge?

- "Art" of approximation needed

Initialization of the Market

- Market microstructure works with notion of *fair price*

Problems Met III and IV:

Unknown Distribution

- As a resultant of dozens of strategies, the market is a quasi-random system.
- (in our simulator, the only randomness stems from latency)

$\Rightarrow$ No chance for knowledge of distribution

- We have to estimate the distribution or directly $\mathbb{E}V$

$\Rightarrow$ Bye bye ADP, hello machine learning

? Does it still converge?

- "Art" of approximation needed

Initialization of the Market

- Market microstructure works with notion of *fair price*

? But what is it when liquidity takers are price takers so market makers are free to determine midpoint price.

Problems Met III and IV:

Unknown Distribution

- As a resultant of dozens of strategies, the market is a quasi-random system.
- (in our simulator, the only randomness stems from latency)

⇒ No chance for knowledge of distribution

- We have to estimate the distribution or directly $\mathbb{E}V$

⇒ Bye bye ADP, hello machine learning

? Does it still converge?

- "Art" of approximation needed

Initialization of the Market

- Market microstructure works with notion of *fair price*

? But what is it when liquidity takers are price takers so market makers are free to determine midpoint price.

- (in practice, other markets and/or overnight auctions stabilize the price)

Last chance for ADP: Market Makers's Problem

- Recall: MM is obliged to keep two quotes (b - bid, a - ask)

Last chance for ADP: Market Makers's Problem

- Recall: MM is obliged to keep two quotes (b - bid, a - ask)

$\Rightarrow$ At least action space simpler:
$A(m, n) = \{(c, b, a) : b < a, m - b - c \geq 0, n \geq 0\}$ where m/n is the amount of money/instrument held.

Last chance for ADP: Market Makers's Problem

- Recall: MM is obliged to keep two quotes (b - bid, a - ask)

$\Rightarrow$ At least action space simpler: $A(m,n) = \{(c,b,a) : b < a, m - b - c \geq 0, n \geq 0\}$ where m/n is the amount of money/instrument held.

- Bellman equation

$$V(m,n) = \max_{(c,b,a)\in A(m,n)} c + \gamma \mathbb{E}_{D,C} V(m - c - Db + Ca, n + D - C)$$

where C/D is the fact of selling/purchase.

Last chance for ADP: Market Makers's Problem

- Recall: MM is obliged to keep two quotes (b - bid, a - ask)

$\Rightarrow$ At least action space simpler:
$A(m,n) = \{(c,b,a) : b < a, m - b - c \geq 0, n \geq 0\}$ where m/n is the amount of money/instrument held.

- Bellman equation

$$V(m,n) = \max_{(c,b,a)\in A(m,n)} c + \gamma \mathbb{E}_{D,C} V(m - c - Db + Ca, n + D - C)$$

where C/D is the fact of selling/purchase.

- Simplification: distribution of $C, D|$state dependent only on $\Delta a = a - A$ and $\Delta b = b - B$ where A and B are best quotes of the others

Last chance for ADP: Market Makers's Problem

- Recall: MM is obliged to keep two quotes (b - bid, a - ask)

$\Rightarrow$ At least action space simpler: $A(m,n) = \{(c,b,a) : b < a, m - b - c \geq 0, n \geq 0\}$ where m/n is the amount of money/instrument held.

- Bellman equation

$$V(m,n) = \max_{(c,b,a) \in A(m,n)} c + \gamma \mathbb{E}_{D,C} V(m - c - Db + Ca, n + D - C)$$

where C/D is the fact of selling/purchase.

- Simplification: distribution of $C, D|$state dependent only on $\Delta a = a - A$ and $\Delta b = b - B$ where A and B are best quotes of the others
- Approximations of V forced to be increasing in m and n (ADP still converges by Powel et al. 2015)

Last chance for ADP: Market Makers's Problem

- Recall: MM is obliged to keep two quotes (b - bid, a - ask)

$\Rightarrow$ At least action space simpler: $A(m,n) = \{(c,b,a) : b < a, m - b - c \geq 0, n \geq 0\}$ where m/n is the amount of money/instrument held.

- Bellman equation

$$V(m,n) = \max_{(c,b,a)\in A(m,n)} c + \gamma \mathbb{E}_{D,C} V(m - c - Db + Ca, n + D - C)$$

where C/D is the fact of selling/purchase.

- Simplification: distribution of $C, D|$state dependent only on $\Delta a = a - A$ and $\Delta b = b - B$ where A and B are best quotes of the others
- Approximations of V forced to be increasing in m and n (ADP still converges by Powel et al. 2015)

! At least two these agents have to be present to compete.

Last chance for ADP: Market Makers's Problem

- Recall: MM is obliged to keep two quotes (b - bid, a - ask)

⇒ At least action space simpler: $A(m,n) = \{(c,b,a) : b < a, m - b - c \geq 0, n \geq 0\}$ where m/n is the amount of money/instrument held.

- Bellman equation

$$V(m,n) = \max_{(c,b,a)\in A(m,n)} c + \gamma \mathbb{E}_{D,C} V(m - c - Db + Ca, n + D - C)$$

  where C/D is the fact of selling/purchase.

- Simplification: distribution of $C, D|$state dependent only on $\Delta a = a - A$ and $\Delta b = b - B$ where A and B are best quotes of the others
- Approximations of V forced to be increasing in m and n (ADP still converges by Powel et al. 2015)

! At least two these agents have to be present to compete.

- Work in progress.

Future

Arbitrage on Limit Order Markets

Martin Šmíd, Aleš Kuběna, Peter Hron,...

Introduction

The Model

Simple Market Maker Problem

Fight with Technical Analysis

Problems Met

Future

Near future

- Redesign the simulator
  - The present version is not real-time, so it gives unlimited computation time to strategies

  $\Rightarrow$ Has to be refactored via threads

Future

Introduction
The Model
Simple Market Maker Problem
Fight with Technical Analysis
Problems Met
Future

Near future

- Redesign the simulator
  - The present version is not real-time, so it gives unlimited computation time to strategies

  $\Rightarrow$ Has to be refactored via threads
- Implement intelligent Market Makers (similar to above)

Future

Arbitrage on Limit Order Markets
Martin Šmíd, Aleš Kuběna, Peter Hron,...
Introduction
The Model
Simple Market Maker Problem
Fight with Technical Analysis
Problems Met
Future

Near future

- Redesign the simulator
  - The present version is not real-time, so it gives unlimited computation time to strategies
  - $\Rightarrow$ Has to be refactored via threads
- Implement intelligent Market Makers (similar to above)
- Implement an intelligent arbitrageur

Future

Arbitrage on Limit Order Markets
Martin Šmíd, Aleš Kuběna, Peter Hron,...
Introduction
The Model
Simple Market Maker Problem
Fight with Technical Analysis
Problems Met
Future

Near future

- Redesign the simulator
  - The present version is not real-time, so it gives unlimited computation time to strategies
  - ⇒ Has to be refactored via threads
- Implement intelligent Market Makers (similar to above)
- Implement an intelligent arbitrageur
- Observe his/her profitability

Future

Introduction

The Model

Simple Market Maker Problem

Fight with Technical Analysis

Problems Met

Future

Near future

- Redesign the simulator
  - The present version is not real-time, so it gives unlimited computation time to strategies
  - $\Rightarrow$ Has to be refactored via threads
- Implement intelligent Market Makers (similar to above)
- Implement an intelligent arbitrageur
- Observe his/her profitability
- (maybe also some technical analyst and observe if he looses)

Introduction

The Model

Simple Market Maker Problem

Fight with Technical Analysis

Problems Met

Future

Future

Near future

- Redesign the simulator
  - The present version is not real-time, so it gives unlimited computation time to strategies
  - ⇒ Has to be refactored via threads
- Implement intelligent Market Makers (similar to above)
- Implement an intelligent arbitrageur
- Observe his/her profitability
- (maybe also some technical analyst and observe if he looses)

More distant future

Future

Near future

- Redesign the simulator
  - The present version is not real-time, so it gives unlimited computation time to strategies

  $\Rightarrow$ Has to be refactored via threads
- Implement intelligent Market Makers (similar to above)
- Implement an intelligent arbitrageur
- Observe his/her profitability
- (maybe also some technical analyst and observe if he looses)

More distant future

- Implement a Python frontend

Future

Near future

- Redesign the simulator
  - The present version is not real-time, so it gives unlimited computation time to strategies
  - $\Rightarrow$ Has to be refactored via threads
- Implement intelligent Market Makers (similar to above)
- Implement an intelligent arbitrageur
- Observe his/her profitability
- (maybe also some technical analyst and observe if he looses)

More distant future

- Implement a Python frontend
- Compare "classical" and "deep" methods

Future

Near future

- Redesign the simulator
  - The present version is not real-time, so it gives unlimited computation time to strategies
  - $\Rightarrow$ Has to be refactored via threads
- Implement intelligent Market Makers (similar to above)
- Implement an intelligent arbitrageur
- Observe his/her profitability
- (maybe also some technical analyst and observe if he looses)

More distant future

- Implement a Python frontend
- Compare "classical" and "deep" methods
- Maybe invite anyone to compete with the existing strategies

Future

Arbitrage on Limit Order Markets

Martin Šmíd, Aleš Kuběna, Peter Hron,...

Introduction

The Model

Simple Market Maker Problem

Fight with Technical Analysis

Problems Met

Future

Near future

- Redesign the simulator
  - The present version is not real-time, so it gives unlimited computation time to strategies
  - $\Rightarrow$ Has to be refactored via threads
- Implement intelligent Market Makers (similar to above)
- Implement an intelligent arbitrageur
- Observe his/her profitability
- (maybe also some technical analyst and observe if he looses)

More distant future

- Implement a Python frontend
- Compare "classical" and "deep" methods
- Maybe invite anyone to compete with the existing strategies
- ...

Future

Near future

- Redesign the simulator
  - The present version is not real-time, so it gives unlimited computation time to strategies

  $\Rightarrow$ Has to be refactored via threads
- Implement intelligent Market Makers (similar to above)
- Implement an intelligent arbitrageur
- Observe his/her profitability
- (maybe also some technical analyst and observe if he looses)

More distant future

- Implement a Python frontend
- Compare "classical" and "deep" methods
- Maybe invite anyone to compete with the existing strategies
- ...

Thanks for Attention!