Scenario Optimization and the Risk of Empirical Costs

Algo Carè
Centrum Wiskunde & Informatica (CWI)
Amsterdam, The Netherlands
algocare@gmail.com

Abstract

Many decision problems can be formulated as the problem of minimising a convex cost function that also depends on an uncertain variable. In this seminar, we take a data-based approach to deal with uncertainty. We will focus mainly on a worst-case approach, where we minimise the maximum of the cost functions corresponding to some previously observed instances of the uncertain variable. We call these instances *scenarios*, and the solution to the optimisation problem is the *scenario solution*. The empirical costs are defined as the cost values that the scenario solution incurs for the various scenarios that have been used in optimisation. The *risk* of an empirical cost is defined as the probability that the empirical cost will be exceeded tomorrow, i.e., when a new instance of the uncertain variable will be experienced. We present a theorem stating that the vector of the risks of the empirical costs has an ordered Dirichlet distribution independently of the distribution of the uncertainty, [1]. By virtue of this result, one can construct a probability box for the distribution of tomorrow's cost, without making any assumption on the specific distribution of the uncertainty and without resorting to new observations for validation. We will also mention some new research directions aimed at obtaining similar results for the average approach, where the average of the costs corresponding to the previously observed scenarios is minimised, [2]. Various application domains will be mentioned in the course of the presentation.

References

[1] A. Carè, S. Garatti, and M.C. Campi, Scenario Min-Max Optimization and the Risk of Empirical Costs. *SIAM Journal on Optimization*, 25(4):2061-2080, 2015.

[2] A. Carè, S. Garatti, and M.C. Campi, A Coverage Theory for Least Squares, *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, online-first [doi: 10.1111/rssb.12219], 2016.

[3] M.C. Campi, A. Carè, Random convex programs with L1-regularization: Sparsity and generalization. *SIAM Journal on Control and Optimization*, 51(5):3532-3557. 2013.