Mathematical programs with marketing decisions: Demand-based management

Seminar *Stochastic Programming and Approximation*
(at DPMS, Charles University, Prague, Czech Republic)

by
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About author

- **Current employment** (2015-present): Faculty of Applied Informatics, Tomas Bata University in Zlín, Czech Republic

- **Education**

	Title	*University*	*Branch*
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** **PhD topic**: Mathematical Programs for Dynamic Pricing - Demand Based Management*
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- 1. Introduction (and motivation)
- 2. Underlying Demand Based Problems
  - 2.1 Classical newsvendor problem
  - 2.2 Transportation network design problem (TNDP)
- 3. Pricing
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  - 3.2 TNDP with pricing
- 4. Newsvendor Problem with Advertising
- 5. Newsvendor Problem with Joint Pricing and Advertising
- 6. Waste Processing Facility Location Problem with Stochastic Programming
- 7. Conclusions and Further Research

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- About author
- I. PhD research and publication (time and topic) scheme
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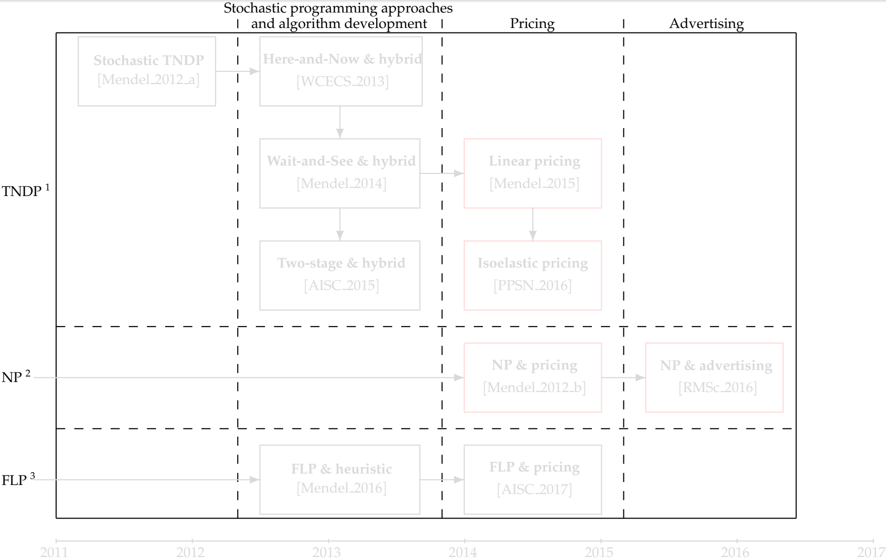

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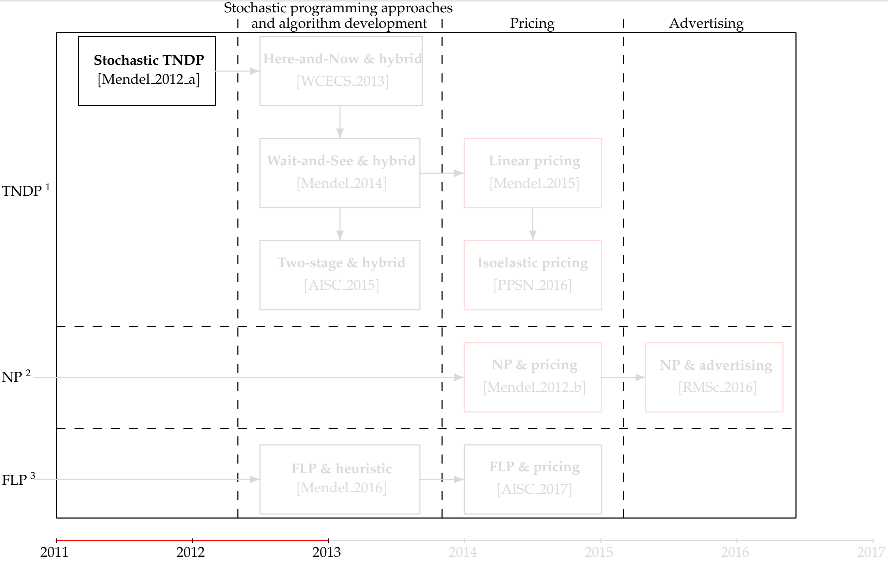

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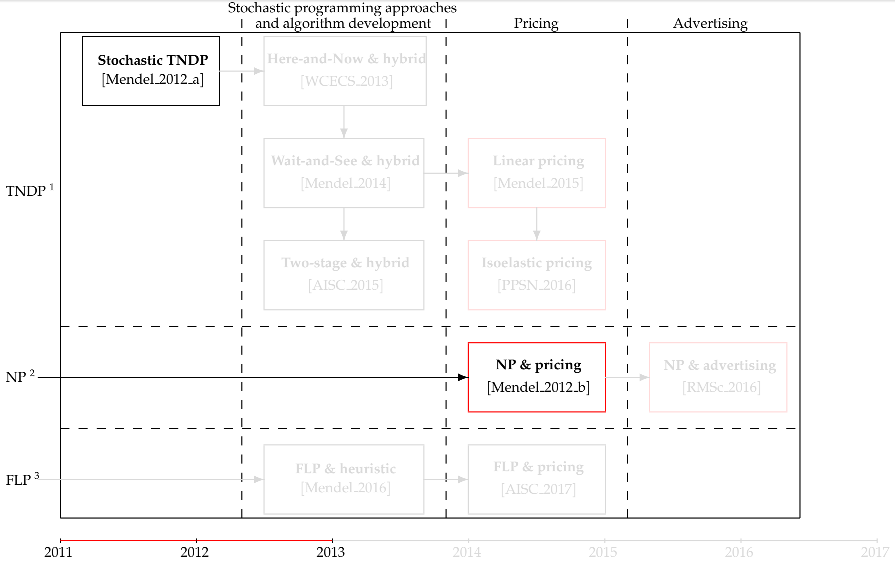

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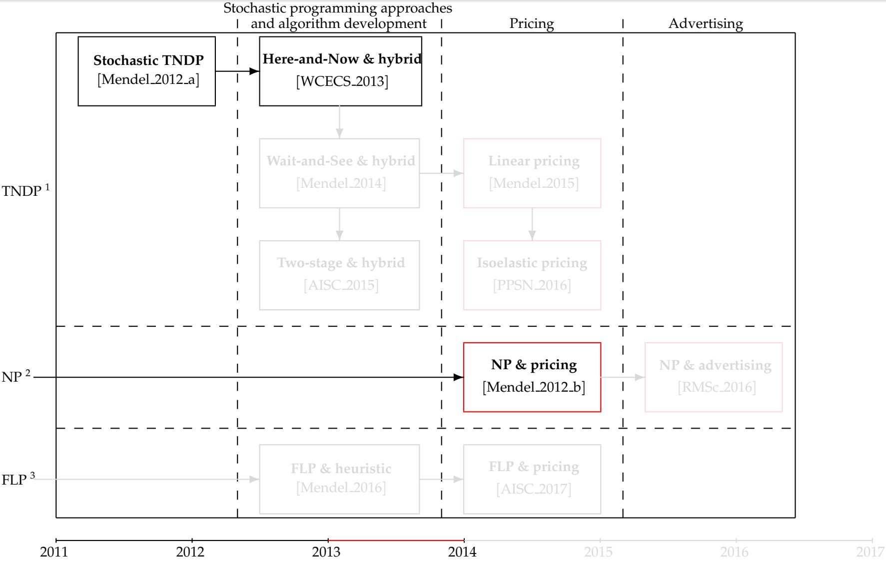

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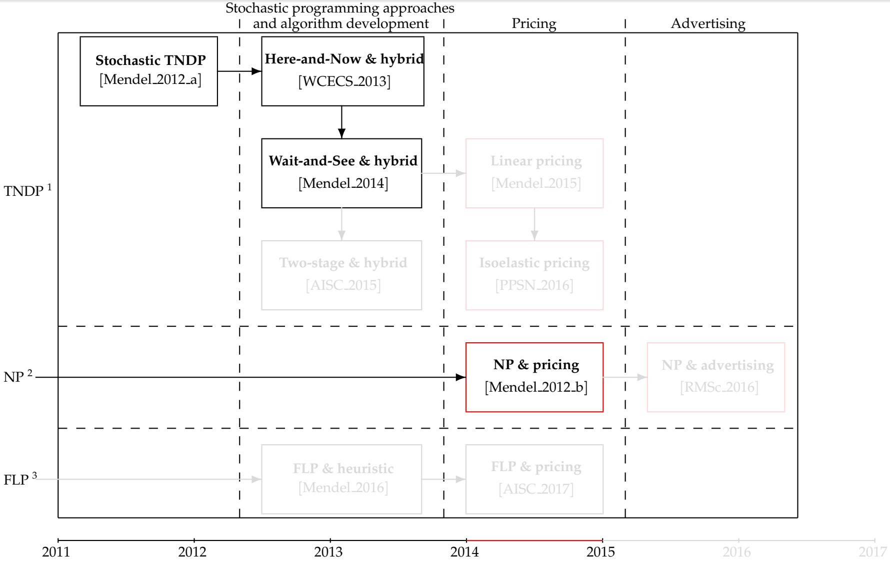

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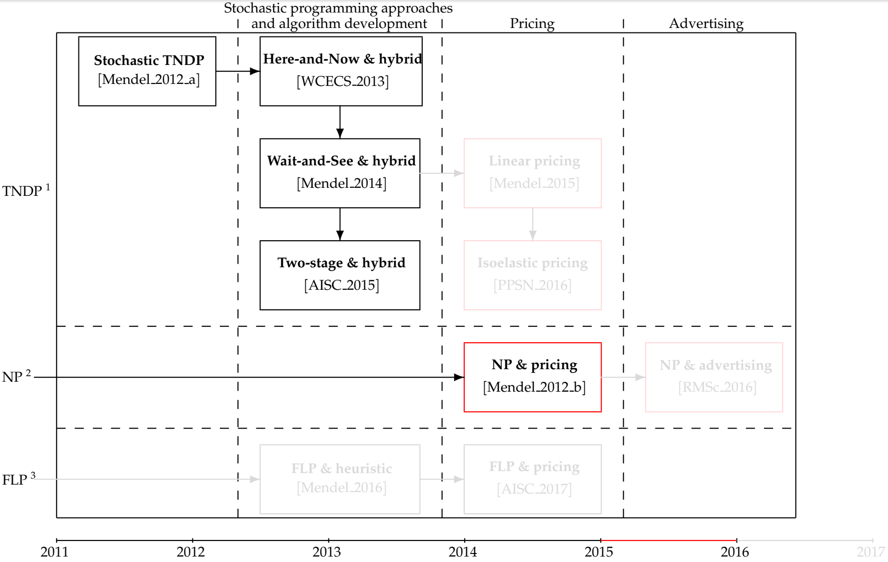

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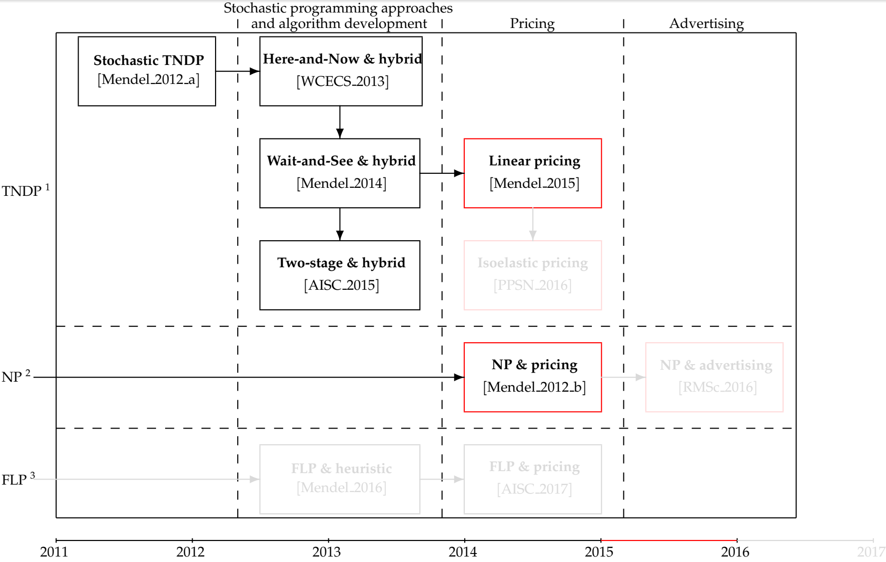

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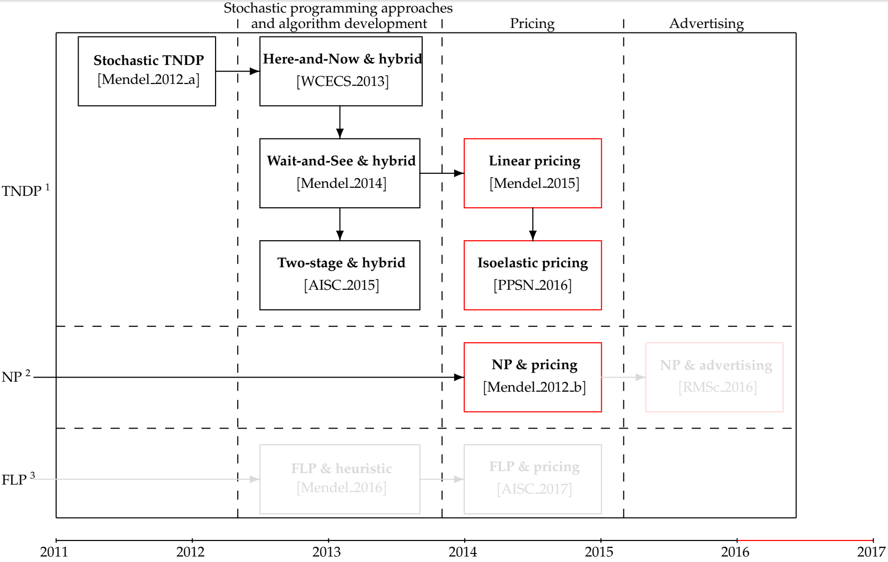

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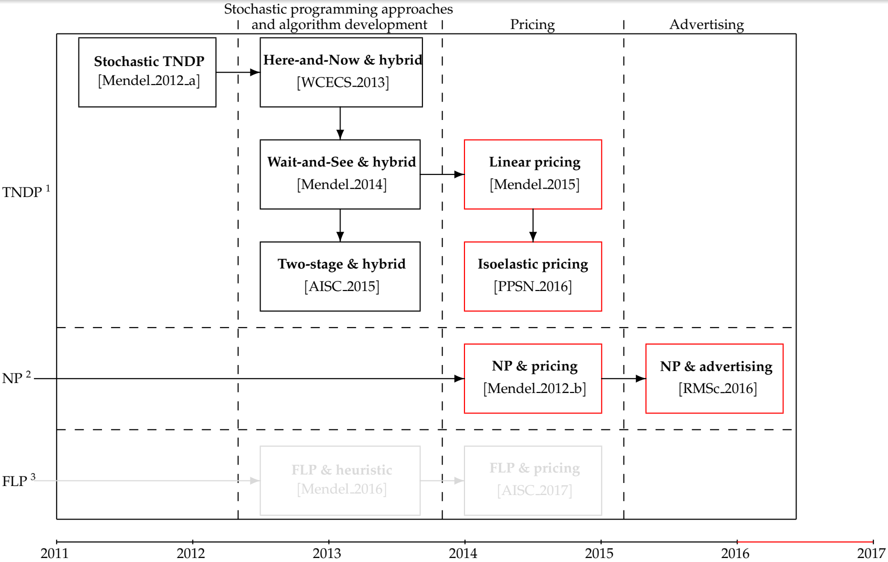

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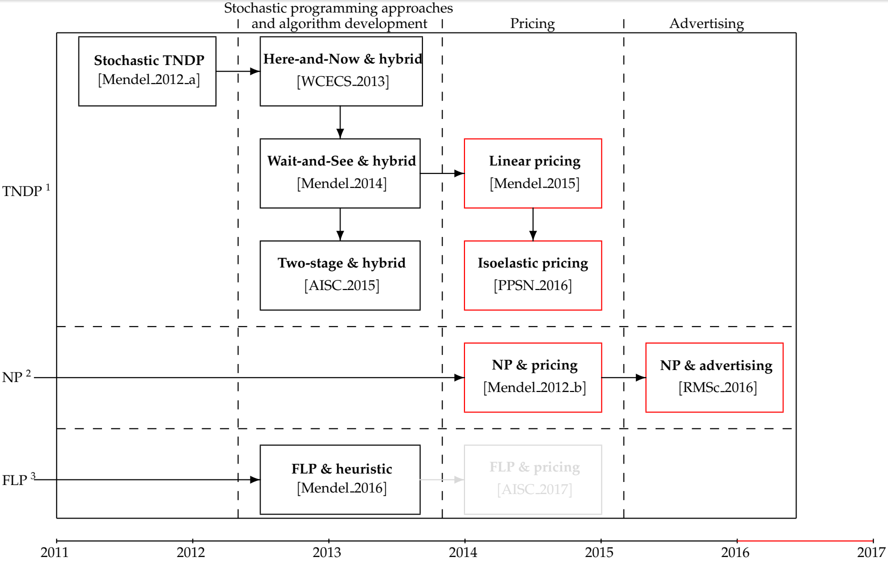

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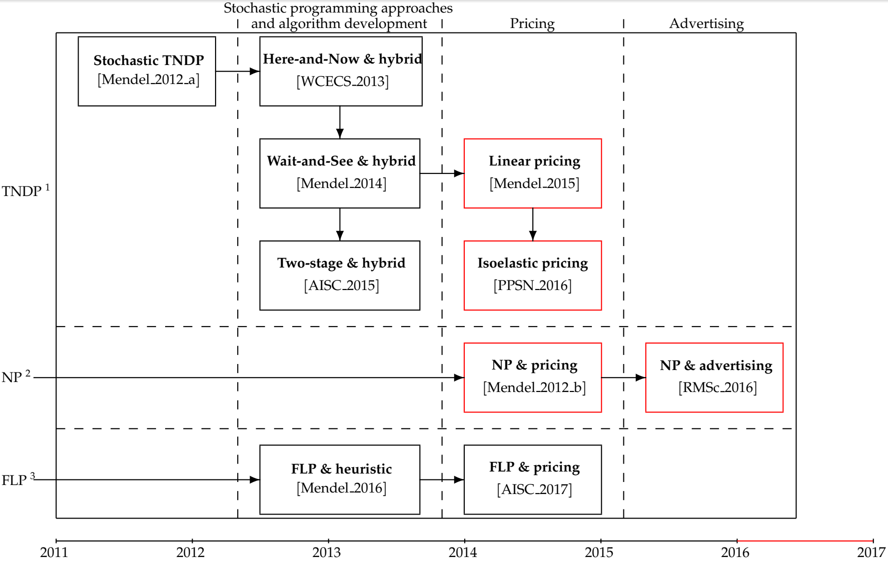

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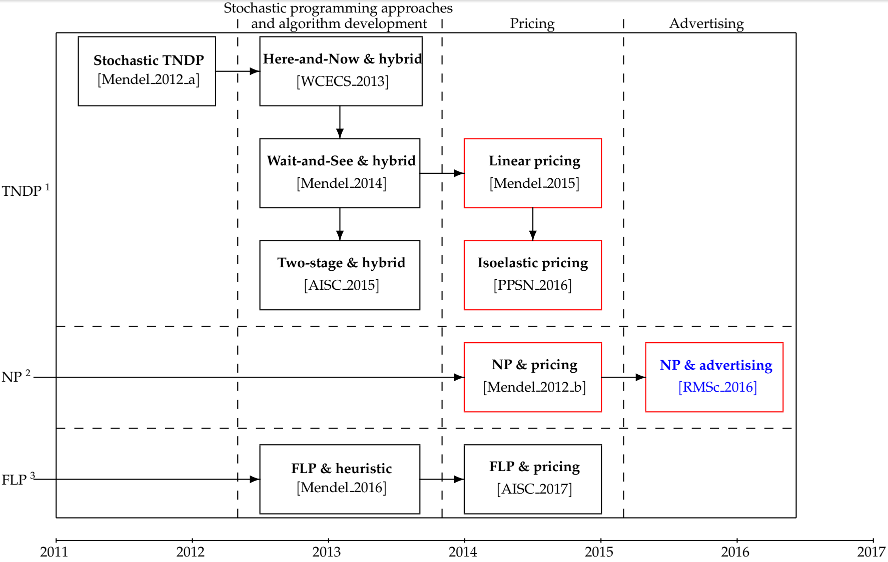

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II. Simple newsvendor problem: An introduction

Figure: Newsvendor (alternatively newsboy).

Simple newsvendor problem

This problem may be simply explained through the following example by [Hill]:

"Early each morning, the owner of a corner newspaper stand needs to order newspapers for that day. If the owner orders too many newspapers, some papers will have to be thrown away or sold as scrap paper at the end of the day. If the owner does not order enough newspapers, some customers will be disappointed and sales and profit will be lost. The newsvendor problem is to find the best (optimal) amount of newspapers to buy that will maximize the expected (average) profit given that the demand distribution and cost parameters are known".

Remark on history: *"the newsvendor problem has a long and interesting history. The original newsvendor model appears back in 1888, when Edgeworth in developed an idea which deals with a bank cash-flow problem."*

(See author's PhD thesis [Hrabec_PhD] available online)

II. Simple newsvendor problem: Mathematical model

- First, the newsvendor decides on the amount to buy and so he stocks x units of the product for a unit cost c. Then, the selling period begins. If demand ξ is greater than x, all stocked units are sold for revenue px, where p is a unit price.
-
-

Then, the objective (profit) function is denoted by $\pi(x, \xi)$ defined as follows:

Objective function of the simple/classical newsvendor problem

$$\pi(x, \xi) = \begin{cases} px - cx & \text{for } x < \xi, \\ p\xi - cx & \text{for } x \geq \xi. \end{cases} \tag{1}$$

Decision variable: x (order amount/quantity).
Random variable: ξ (demand)
Parameters: c (buying cost), p (selling price, $p > c$)

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$$\pi(x,\xi) \;=\; p\min\{x,\xi\} - cx - s\max\{\xi - x, 0\} + v\max\{x - \xi, 0\}.$$

• Let the expected profit be denoted as $\Pi(x) = E_\xi[\pi(x,\xi)]$, then:

$$\Pi(x) = p\left(\int_0^x tf(t)\mathrm{d}t + x\int_x^\infty f(t)\mathrm{d}t\right) - cx - s\int_x^\infty (t-x)f(t)\mathrm{d}t + v\int_0^x (x-t)f(t)\mathrm{d}t.$$

III. Newsvendor problem with advertising (NPA)

Objective function of the simple newsvendor problem: repetition of (1)

$$\pi(x,\xi) = \begin{cases} px - cx - s(\xi - x), & \text{for } x < \xi, \\ p\xi - cx + v(x - \xi), & \text{for } x \geq \xi. \end{cases}$$

Advertising-dependent demand: Let the demand be denoted as D and let it satisfy

$$D(a, \xi_a, \xi_m) = d(a)\xi_m + \xi_a, \tag{2}$$

where ξ_a, ξ_m are independent continuous random variables and $d(a)$ is the so-called advertising response function.

Objective function of the newsvendor problem with advertising

$$\pi(a, x, \xi_a, \xi_m) = \begin{cases} px - cx - s[D(a, \xi_a, \xi_m) - x] - a, & x < D(a, \xi_a, \xi_m), \\ pD(a, \xi_a, \xi_m) - cx + v[x - D(a, \xi_a, \xi_m)] - a, & x \geq D(a, \xi_a, \xi_m). \end{cases} \tag{3}$$

Two decisions are assumed:

- the retailer has to decide about an amount a to advertise for a product to be sold
- and simultaneously has to buy and stock x units of the product for a unit cost c.

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Demand function and advertising response function

Demand function $D(a, \xi_a, \xi_m)$

Two special cases of demand function (2), which was defined as

$$D(a, \xi_a, \xi_m) = d(a)\xi_m + \xi_a,$$

are considered:

a) *multiplicative demand case*: $\mathrm{P}(\xi_a = 0) = 1$, $\xi_m \in [A_m, B_m]$ and satisfy $\mathrm{E}[\xi_m] = 1$;

b) the *additive demand case*: $P(\xi_m = 1) = 1$, $\xi_a \in [A_a, B_a]$ and satisfy $\mathrm{E}[\xi_a] = 0$.

Then, for both cases, the expectation of D is specified as:

$$\mathrm{E}[D(a, \xi_a, \xi_m)] = d(a).$$

From this point, we will only deal with the *multiplicative demand case*, i.e.

$$D(a, \xi_a, \xi_m) \equiv D(a, \xi_m) = d(a)\xi_m.$$

Response function $d(a)$

Let the response function $d(a)$ be continuous, nonnegative, twice-differentiable and increasing on its domain $[0, a_{max}]$ in the advertising expenditure a. Moreover, since $d(0) > 0$, $d(a)$ is positive.

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Let the response function $d(a)$ be continuous, nonnegative, twice-differentiable and increasing on its domain $[0, a_{max}]$ in the advertising expenditure a. Moreover, since $d(0) > 0$, $d(a)$ is positive.

Demand function and advertising response function

Demand function $D(a, \xi_a, \xi_m)$

Two special cases of demand function (2), which was defined as

$$D(a, \xi_a, \xi_m) = d(a)\xi_m + \xi_a,$$

are considered:

a) *multiplicative demand case*: $\mathrm{P}(\xi_a = 0) = 1$, $\xi_m \in [A_m, B_m]$ and satisfy $\mathrm{E}[\xi_m] = 1$;

b) the *additive demand case*: $P(\xi_m = 1) = 1$, $\xi_a \in [A_a, B_a]$ and satisfy $\mathrm{E}[\xi_a] = 0$.

Then, for both cases, the expectation of D is specified as:

$$\mathrm{E}[D(a, \xi_a, \xi_m)] = d(a).$$

From this point, we will only deal with the *multiplicative demand case*, i.e.

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Advertising response function examples

Three function types/classes will further be applied:

a) concave response function without threshold in demand;

b) concave response function with threshold in demand;

c) S-shaped response function.

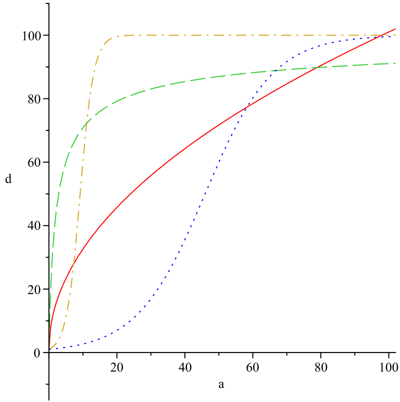

Figure: 4 examples of response function $d(a)$: a), b) and 2x c).

Multiplicative demand case

Let $F(\cdot)$ denote a cumulative distribution function (cdf) and $f(\cdot)$ be a probability density function (pdf) of ξ_m. In order to assure that demand is positive, we require that $A_m > 0$.

The objective function (3) can be rewritten by substituting $D(a, \xi_m) = d(a)\xi_m$ and utilizing the 'stocking factor' defined as

$$z = \frac{x}{d(a)}, \tag{4}$$

which provides an alternative interpretation of the stocking decision: if the choice of z is greater than the realized value of random variable ξ_m, then leftovers occur, otherwise shortages occur.

Objective function of the newsvendor problem with advertising

$$\pi(a, z, \xi_m) = \begin{cases} pzd(a) \quad - czd(a) - sd(a)[\xi_m - z] - a, & \text{for } z < \xi_m, \\ p\xi_m d(a) - czd(a) + vd(a)[z - \xi_m] - a, & \text{for } z \geq \xi_m. \end{cases} \tag{5}$$

Expected objective, expected quantities and optimal z^*

Using expected objective reformulation we can get

$$\Pi(a, z) = \Psi(a) - L(a, z) = d(a)[p - c - l(z)] - a, \qquad (6)$$

where

- $\Psi(a) = (p - c)d(a) - a$ is the *riskless profit* that occurs in the absence of uncertainty,
- $L(a, z) = d(a)l(z)$ is the *expected loss* that occurs as a result of the presence of uncertainty and
- $l(z) = (c - v)\Lambda(z) + (p + s - c)\Theta(z)$ is the *expected loss per unit*.

Note that $d(a)\Lambda(z)$ denotes *expected leftovers* and $d(a)\Theta(z)$ *expected shortages*, where

- $\Lambda(z) = E[\max\{z - \xi_m, 0\}] = \int_A^z (z - t)F_m(t)\mathrm{d}t,$
- $\Theta(z) = E[\max\{\xi_m - z, 0\}] = \int_z^B (t - z)F_m(t)\mathrm{d}t.$

Assumption 1 (from (6))

The per-unit expected benefit must be positive, i.e., $p - c - l(z^*) > 0$.

Optimal stocking quantity

Let us take partial derivative $\frac{\partial \Pi(a,z)}{\partial z}$ and let us solve equation $\frac{\partial \Pi(a,z)}{\partial z} = 0$.
Under some assumptions (e.g., assuming that F is invertible), the optimal and unique z^* leads to the same quantity as in the NP (optimal x) as well as in the NPP (optimal z):

$$z^* = F^{-1}\left(\frac{p + s - c}{p + s - v}\right).$$

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Optimal advertising expenditure a^*

Optimal advertising expenditure

Let us take partial derivative $\frac{\partial \Pi(a,z^*)}{\partial a}$ and let us solve equation $\frac{\partial \Pi(a,z^*)}{\partial a} = 0$.

Remark 1: The optimal advertising expenditure a^* must satisfy the (necessary) optimality condition:

$$\frac{\mathrm{d}d(a^*)}{\mathrm{d}a} = \frac{1}{p - c - l(z^*)}. \tag{7}$$

Comparison with riskless problem

Theorem 1 *For the multiplicative demand model, the optimal advertising a^* is always less than or equal to the optimal riskless advertising a^*_Ψ.*

Theorem 2 *For the additive demand model, the optimal advertising a^* is always equal to the optimal riskless advertising a^*_Ψ.*

Why? *The difference in observations can be mainly explained by their variances and coefficients of variation:*

- while in the additive case the variance of the demand is constant (independent of a), i.e., $\sigma^2[D_A(a,\xi_a)] = \sigma_A^2$ that is the *constant variance case*,
- in the multiplicative case the variance is a function of the response function, i.e., $\sigma^2[D_M(a,\xi_m)] = [d(a)]^2\sigma_M^2$, and the coefficient of variation is constant, i.e., $c_v[D_M(a,\xi_m)] = \sigma_M$ that is the *constant coefficient of variation case*.

See [Khouja] for a similar variance analysis.

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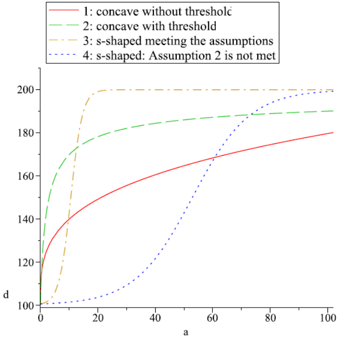
Optimal order amount x^* and illustrations

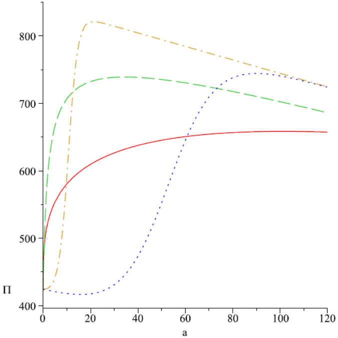
Optimal order amount

Substituting back to the stocking quantity substitution/definition (4), we get:

$$x^* = z^* \cdot d(a^*).$$



(a) Response function examples $d_1(a)$ - $d_4(a)$.



(b) Expected profit functions $\Pi(a, z^*)$.

Comparison with NPP (i.e., pricing vs. advertising)

Remark 2: *The optimal price for multiplicative uncertain demand is not less than the riskless price in the NPP.*

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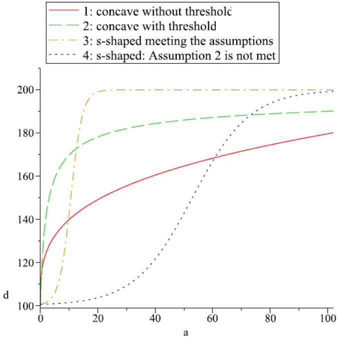

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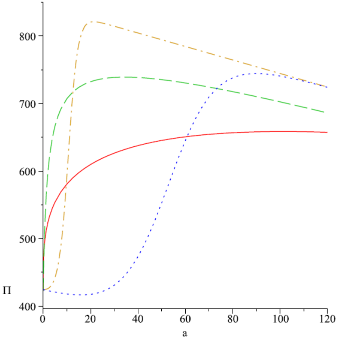

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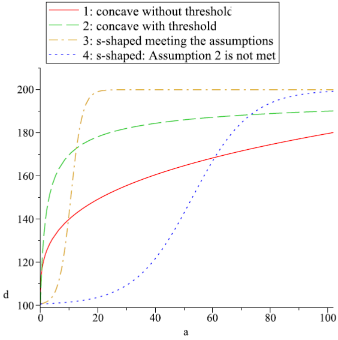

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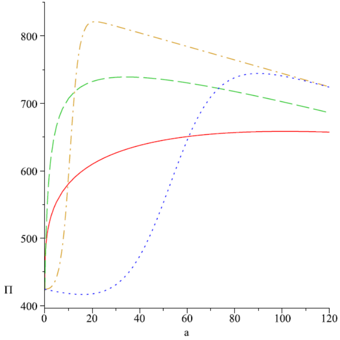

(b) *Expected profit functions* $\Pi(a, z^*)$.

Assumption 2

The demand function $d(a)$ satisfies that $\lim_{\Delta a \to 0_+} \frac{d(\Delta a) - d(0)}{\Delta a} > \frac{1}{p - c - l(z^*)}$ and $\lim_{\Delta a \to 0_+} \frac{d(a_{max}) - d(a_{max} - \Delta a)}{\Delta a} < \frac{1}{p - c - l(z^*)}$.

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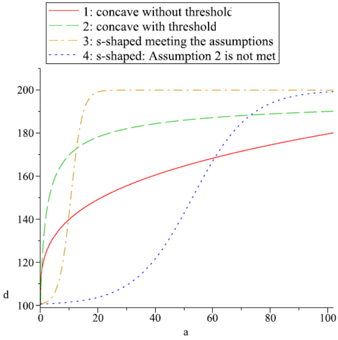

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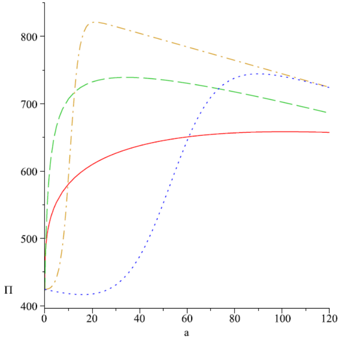

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Theorem 3

If the response function $d(a)$ is strictly concave, then, under assumptions 1 and 2, the expected profit $\Pi(a, z^*)$ is strictly concave in a and so the globally optimal advertising expenditure a^* is unique and is given by solution of (7) with respect to decision variable a.

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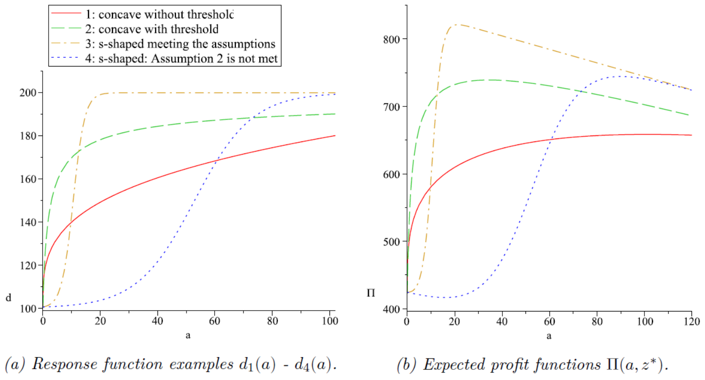

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Theorem 4

If the response function $d(a)$ is S-shaped, then, under assumptions 1 and 2, the expected profit $\Pi(a, z^*)$ is strictly quasi-concave in a and so the globally optimal advertising expenditure is unique and is given by (7).

Numerical example for the multiplicative demand case

Example:

Let us consider following situation (i.e., concrete parameters):

- **Product:** *consider a product with* $p = 15$, $c = 10$, $v = 8$ *and* $s = 2$.
- **Distribution of random variable:** *let* $\xi \sim U(A, B)$, *where* $[A, B] = [0.5, 1.5]$.
- **Advertising response function:** *and let* $d_0 = 100$ *and* $a_{max} = 150$.

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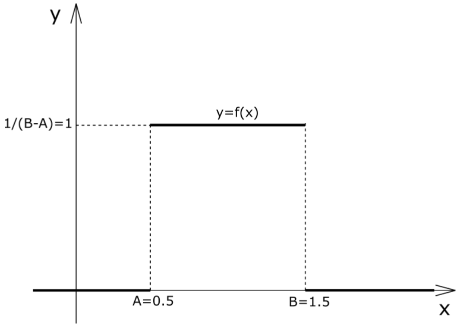

Figure: Uniform distribution example (pdf).

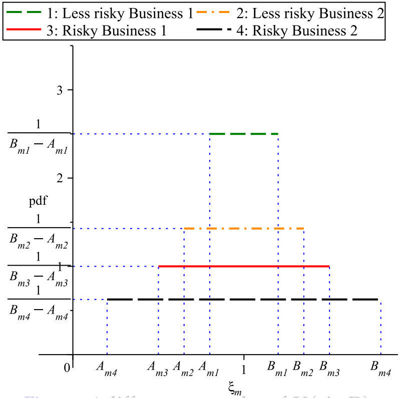

Figure: 4 different examples of $U(A, B)$.

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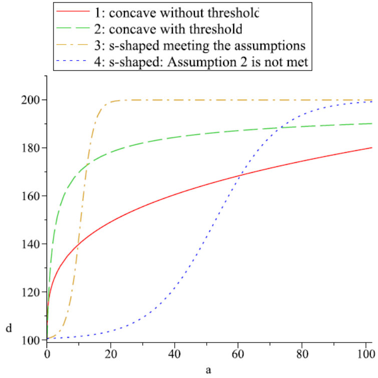

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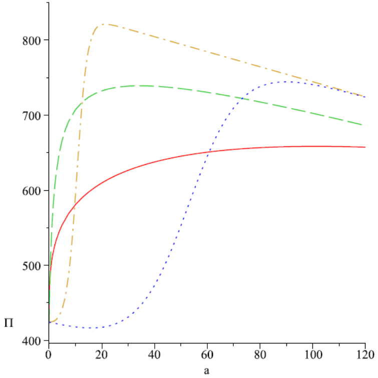

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Numerical example: optimal values

Uniform distribution

Let the random variable ξ_m be uniformly distributed, i.e., $\xi_m \sim U(A, B)$. Then, we get

$$z^* = A + \frac{(p+s-c)(B-A)}{p+s-v}$$

$$\Rightarrow l(z^*) = (z^* - A_m)\frac{c-v}{2} = \frac{B_m - A_m}{2}(c-v)\frac{p+s-c}{p+s-v}.$$

Problem	Multiplicative NPA				Riskless NPA				NP
	$d_1(a)$	$d_2(a)$	$d_3(a)$	$d_4(a)$	$d_1(a)$	$d_2(a)$	$d_3(a)$	$d_4(a)$	-
a^*	101.2	34.5	21.3	89.9	128.9	38.7	21.6	91.6	-
$d(a^*)$	179.9	183.2	199.5	197.6	185.9	184.1	199.6	198.0	-
x^*	229.9	234.1	254.9	252.5	187.2	185.4	200.9	199.2	127.8
$\Pi(a^*, x^*)$	658.4	739.1	821.2	744.3	799.9	881.2	975.6	897.4	444.4

Table: Numerical results for the uniform distribution for two cases: multiplicative and riskless; numerical results of the equivalent NP

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Numerical example: Impact of price and cost changes

Impact on optimal advertising

Let us take the first derivative w.r.t. a of the expression $\Pi(a, z^*) = d(a)[p - c - l(z^*)] - a$ and let us substitute z^* (and $l(z^*)$ respectively) for the uniform distribution. Then, taking derivative w.r.t. p, we get

$$\frac{\partial(\frac{\partial \Pi(a,z^*)}{\partial a})}{\partial p} = \frac{\partial d(a)}{\partial a} \left[1 - \frac{B_m - A_m}{2} \frac{(c-v)^2}{(p+s-v)^2}\right] > 0.$$

Therefore, $\Pi(a, z^*)$ is strictly supermodular in (a, p) (see [Wang]) and the optimal advertising is strictly increasing in selling price p.

With the same procedure but w.r.t. c, we get:

$$\frac{\partial(\frac{\partial \Pi(a,z^*)}{\partial a})}{\partial c} = d(a) \left[\frac{B_m - A_m}{2} \frac{2c - p - s - v}{p + s - v} - 1\right] < 0,$$

which means that $\Pi(a, z^*)$ is strictly submodular in (a, c) and the optimal advertising is strictly decreasing in buying cost c.

Therefore, any increase in the unit profit margin, i.e. range $p - c$ (see [Khouja]), leads to higher optimal advertising expenditure a^*.

Numerical example: effects of cost on expected unit profit

U_i	A_{mi}	B_{mi}	σ^2	z^*	$l(z^*)$	$p-c-l(z^*)$	a^*	x^*
U_1	0.8	1.2	0.0133	1.111	0.311	4.689	117.6	204.0
U_2	0.65	1.35	0.0408	1.194	0.544	4.456	109.3	217.1
U_3	0.5	1.5	0.0833	1.278	0.778	4.222	101.2	229.9
U_4	0.2	1.8	0.2133	1.444	1.244	3.756	85.6	254.2

Table: Numerical examples of various uniform distributions U_1 - U_4 for $d_1(a)$.

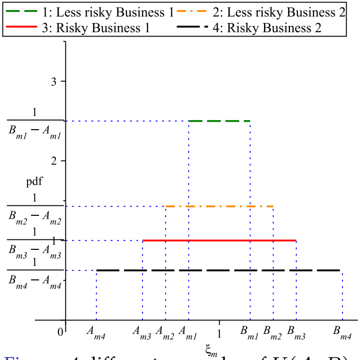


Figure: 4 different examples of $U(A, B)$.

What we can see from this:

- with increasing variance of the random element, the optimal z^* as well as $l(z^*)$ increases;
- a higher $l(z^*)$ leads to a lower optimal advertising a^*, which corresponds to a lower expected demand $d(a^*)$, the optimal strategy is to buy a higher amount x^* of the product, although a less profit $\Pi(a^*, x^*)$ is expected.

Numerical example: effects of cost on expected unit profit

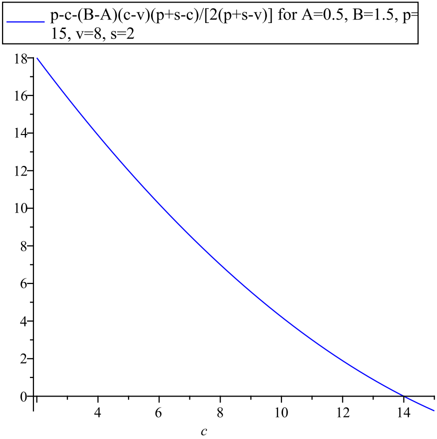


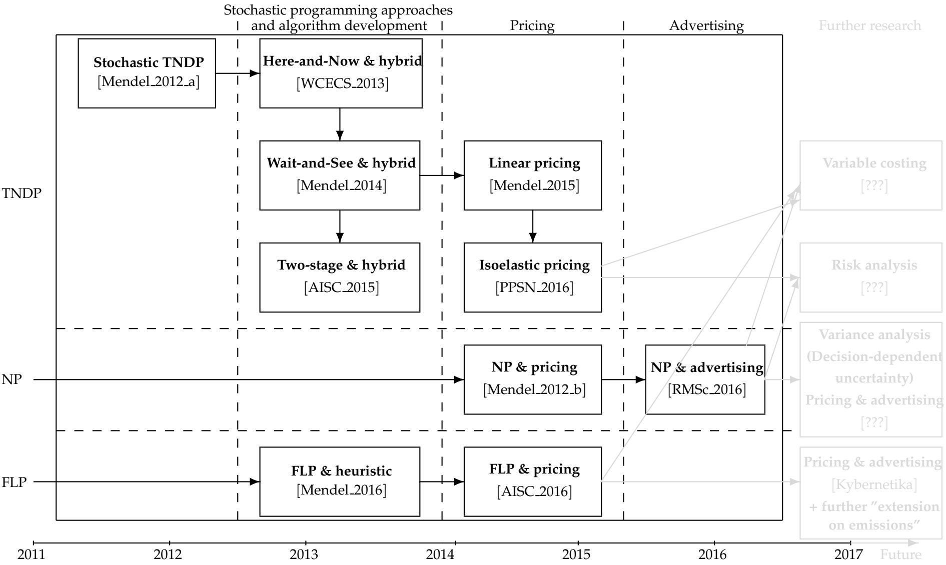
Figure: Assumption 1: expected unit profit/loss $p - c - l(z^*)$ as function of the cost c.

IV. Conclusions and further research

Conclusions:

- Review of of NP, NPA, NPP, its decisions and modifications.
- Assumptions and theorems that guarantee existence and uniqueness of optimal decisions.
- Comparisons of decisions (within NPA, NPA vs NPP and NP, etc.)
- Advertising response functions examples and extensions on S-shaped response functions.
- Analysis of effects of parameters on optimal decisions (sensitivity analysis).

Further research ideas and suggestions



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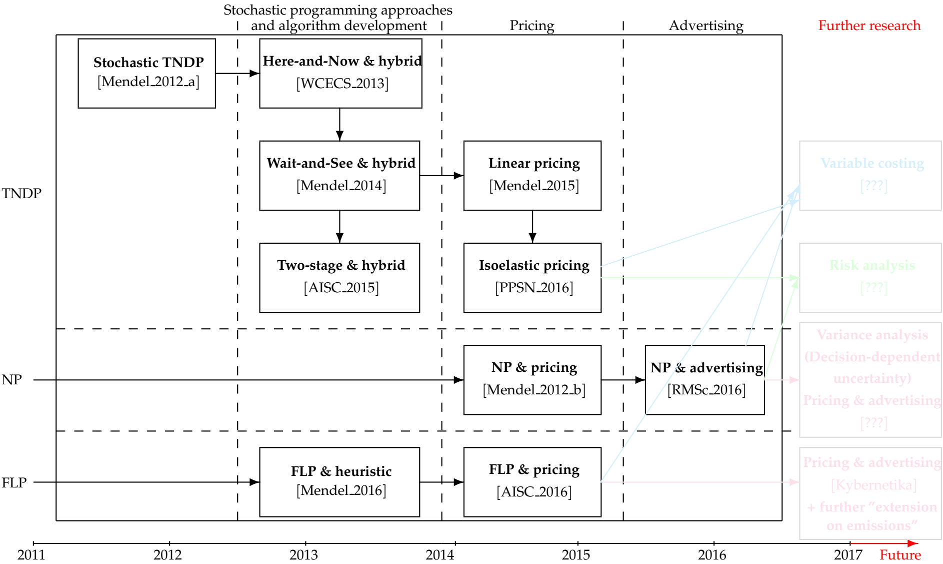

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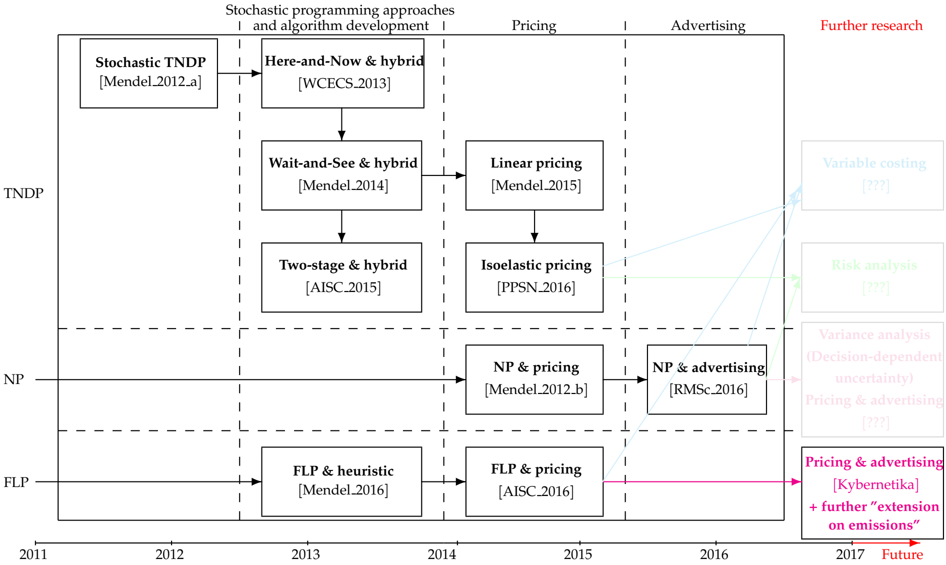

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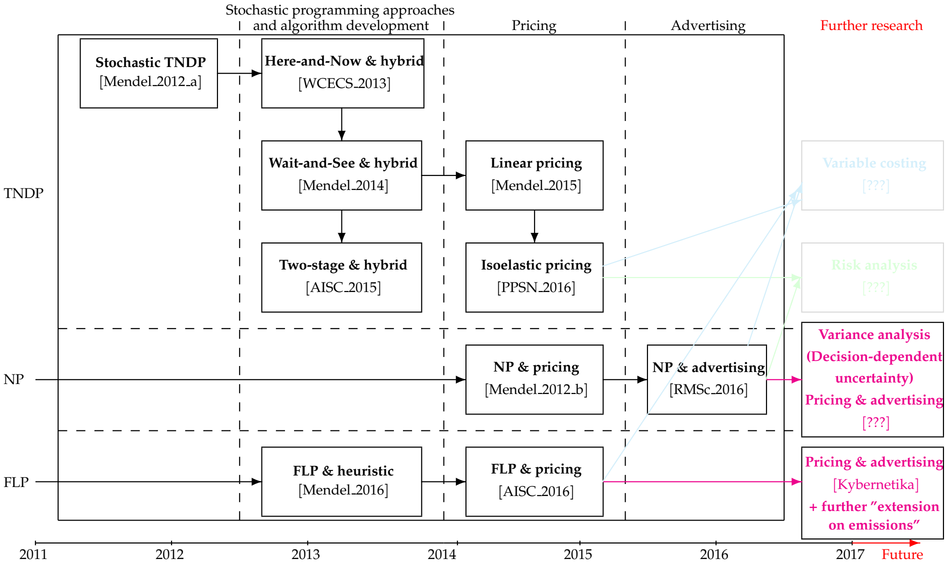

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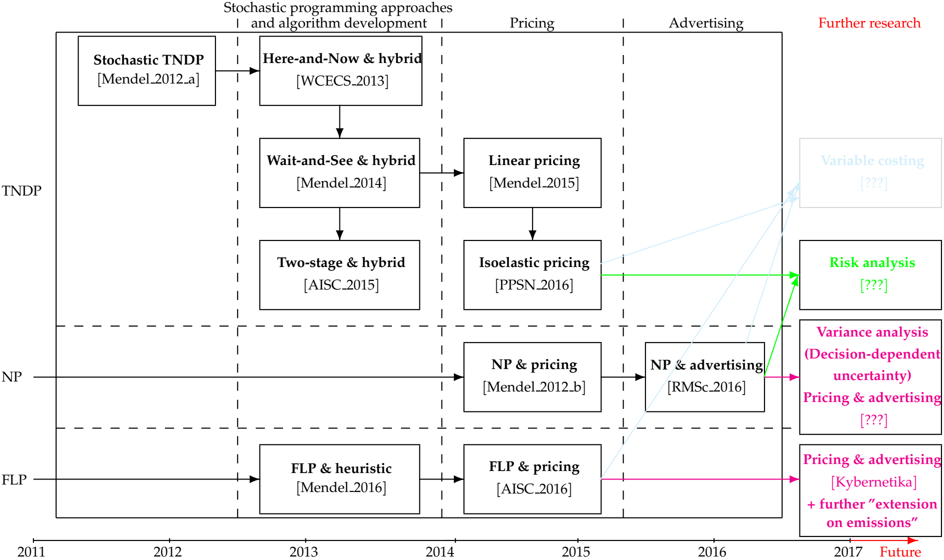

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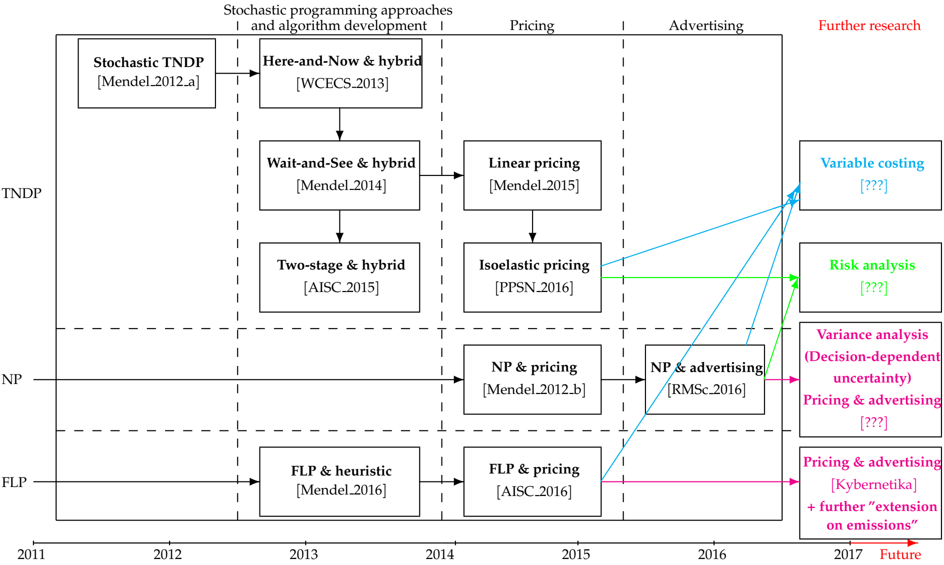

Further research I

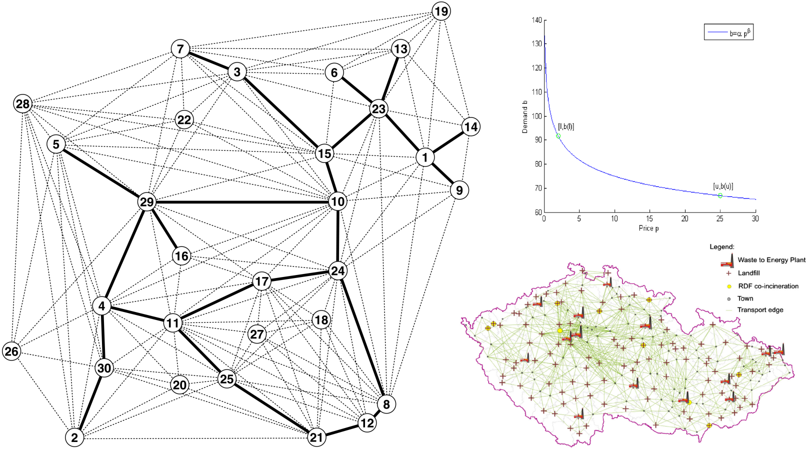

Figure: Motivation for TNDP with pricing (a joint work by Roupec, Hrabec, Popela, Šomplák, Nevrlý, Kůdela, Novotný, etc.)

Further research II

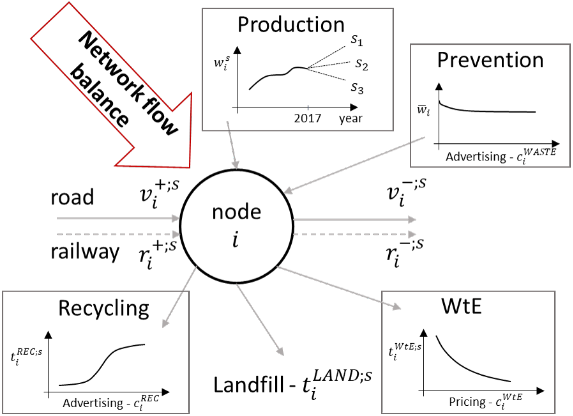

Figure: A scheme of a new joint pricing and advertising application (by [Hrabec, Šomplák, Nevrlý, Kůdela, Popela])

So, what do I really work on now? (or What am I going to work on?)

Production and logistic applications

• A cooperation with **Molde University College - University Specialized in Logistics**, Norway:
Kjetil K. Haugen (game theory), Asmund Olstad (pricing, lot-sizing), Lars M. Hvattum (shipping and routing), Arild Hoff (hub location problems and routing), etc.

Waste management applications

• A cooperation with **Institute of Process Engineering, Brno University of Technology**:
Radovan Šomplák, Vlastimír Nevrlý, Pavel Popela, Jakub Kůdela, etc.

Newsvendor problem (or "Stochastic single-preiod problem") with multiple marketing decisions, e.g. the variance analysis

My own. No cooperation yet :) (alternatively, Kjetil Haugen suggested some game-theory related ideas)

Crossroad optimization (optimization of lights setting)

CROSS, a.s. - a potential new topic for me (a need of mathematical modelling and algorithmic solutions)

Summary keywords (or intersections) of the work:

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References: Author's related publications

Mendel_2012_b HRABEC, D., POPELA, P., NOVOTNÝ, J., HAUGEN, K.K., OLSTAD, A., 2012. *A note on the newsvendor problem with pricing.* In Proceedings of the 18th International Conference on Soft Computing MENDEL 2012, Vol. 18, pp. 410-415.

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Thank you for your attention.