SLP-IOR

Building and solving SLP models with SLP-IOR

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Content

Current availibility (development) of software tools for SP.

J.J. Bisshop et al.	Paragon Decision Technology	AIMMS
A Meeraus et al.	GAMS	**GAMS**
B Kristjansson	Maximal Software	MPL
R. Fourier et al.	Northwestern University	AMPL
M. A. H. Dempster et al.	Cambridge University	STOCHGEN
E. Fragniere et al.	University of Geneva	SETSTOCH
A King et al.	IBM COIN-OR	**OSL/SE** SMI
H. I. Gassmann et al.	Dalhousie University	**MSLiP**
G. Infanger et al.	Statford University	DECIS
P. Kall et al.	University of Zürich	**SLP-IOR**
G. Mitra et al.	Brusel University	**SPInE**
A. Gaivoronsky	Norwegian University of Science and Technology	SQG

SPInE

= Stochastic Programming Integrated Environment (for multistage SLP).

- E. Mesisina, G. Mitra (1997). *Modelling and analysis of multistage stochastic programming problems: A software environment*. European Journal of Operational Research 101, pp 343-359.
- MPL (Mathematical Programming Language)
- Solvers - FortMP (deterministic), MSLiP (stochastic problems).
- MDDB (MultiDimensional DataBases) - Express, Oracle Corp.
- Procedures for generating scenarios, for building the deterministic equivalent model to the stochastic one.

SLP-IOR

= Stochastic Linear Programming System of the Institute for Operations Research of the University of Zürich.

The main features

- SUPPORT of the entire life cycle of a model:
  - model formulation,
  - analysis of the model instance,
  - model solving,
  - solution analysis.
- CONNECTION to an algebraic modeling system (GAMS).
  - availability of the powerful general-purpose solvers connected to GAMS for solving deterministic equivalents of SLP problems,
  - import and stochastic formulation of models written in the AML of GAMS.
- SLP-IOR (Borland Delphi 6), solvers (Compaq Visual Fortran 6.1).
- FREE OF CHARGE for academic purposes.

Types of SLP problems included in SLP-IOR

- two-stage recourse problems with the subclasses fixed recourse, complete recourse and simple recourse,
- two-stage simple integer recourse problems,
- two-stage multiple simple recourse problems (? integer),
- jointly chance-constrained problems (only RHS stochatic & nondegenerate multivariate normal distribution),
- separate chance constraints problems (only RHS stochatic & $h_i(\xi)$ independent $\rightarrow$ LP),
- multistage recourse problems with scenarios, scenario generation.
- NEW:
  - Integrated probability constraint (joint and separate),
  - CVaR constraint.

ω is a random vector on (Ω, F, P), random parts - affine sums:

$$\begin{aligned} q(\omega) &= q^0 + \sum_{i=1}^{n} q^i \omega_i,\ h(\omega) = h^0 + \sum_{i=1}^{n} h^i \omega_i, \\ T(\omega) &= T^0 + \sum_{i=1}^{n} T^i \omega_i,\ W(\omega) = W^0 + \sum_{i=1}^{n} W^i \omega_i. \end{aligned}$$

Available probability distributions:

- **Univariate discrete distributions:** empirical, uniform, binomial, hypergeometric, geometric, negative binomial, Poisson.
- **Univariate continuous distributions:** uniform, normal, exponencial, gamma, beta, Cauchy, Weibull, chi-quared, Fisher's F, Student's t, extreme value, logistic, lognormal, Pareto, power function, triangular distributions.
- **Multivariate discrete distributions:** empirical and uniform distribution.
- **Multivariate continuous distribution:** uniform and normal distribution.

Setting up the model instance

- Menu-driven fashion enables us to set up
  1. the type of the model,
  2. the dimensions,
  3. the stochastic parts,
  4. the probability distribution of the random vector ω,
  5. the affine stochastic relations.
- Matrix editor - direct edit or import (export):
  - Via the Windows Clipboard.
  - Internal data format.
- SMPS format (CORE, TIME, STOCH),
- AML (Algebraic Modeling Language), GAMS.
- Transformation between models (stochastic vs. deterministic).
- A facility for discretizing probability distributions (recourse models).

Diskrétní Gaussovo rozdělení

Informační Bulletin České statistické společnosti 1/1992, J. Anděl, "Diskrétní Gaussovo rozdělení".

$$
\begin{aligned}
\varphi(x) &= \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \\
\sum_{k=-\infty}^{\infty} \varphi(k) &\overset{?}{=} 1 \\
&\overset{Maple 9.5}{=} 1.000000005.
\end{aligned}
$$

Diskrétní Gaussovo rozdělení

$$\sum_{k=-\infty}^{\infty} \varphi(k) = 1 + 2\sum_{k=1}^{\infty} e^{-2k^2\pi^2} > 1.$$

(Jarník, *Integrální počet II.*)

Two-stage recourse problems: EVPI, VSS, RFS

Facilities for analyzing the model instance and the solution (ONLY for two-stage recourse problems).

- $EV \leq WS \leq HN \leq EEV$, $\mathbb{EVPI} = HN - WS$, $\mathbb{VSS} = EEV - HN$.
  - For continuous distributions SLP–IOR offers sampling for estimating EEV, WS.
- **The reliability of the first-stage solution** (RFS) for fixed $x =$ probability that no recourse will be needed

$$RFS = \mathbb{P}\{\omega : T(\omega)x \geq h(\omega)\}.$$

  (inequalities in the second stage).
- Checking the complete recourse property and the simple recourse structure.
- Computing the recourse objective function for a given first stage decision x.

Test problem batteries

- Import in SMPS format.
- Randomly generated problem batteries:
  - *deterministic LP problems*,
  - *recourse problems* with guaranteed existence of an optiomal solution,
  - *jointly chance-constrained problems* with a known solution.
- Operations that can be performed on each element of the battery
  - discretizing the probability distribution,
  - endowing the test problems with a normal distribution,
  - injecting a fixed distribution,
  - selection of a set of solvers (summary as LaTeX tableaus).

General purpose LP solvers

- **HiPlex** - simplex method (1994).
- **HOPDM** - primal-dual interior-point method (1996).
- **Minos** - simplex method (1995).
- **OB1** - several interior point methods (1989).
- **XMP** - simplex method (1986).

Special purpose LP solvers

- **BPMPD** - augmented interior-point method (1996).
- **MSLiP** - nested Benders decomposition method (1992).
    - Gassmann (1990), *MSLiP: A computer code for the multistage stochastic linear programming problem*, Mathematical Programming 47, 707-423.
- **QDECOM** - regularized decomposition algorithm (1985).
- **SHOR2** - decomposition scheme based on r-algorithm (1998).

Solvers for recourse problems

- **DAPPROX**
- **SDECOM** - stochastic decomposition method
  - Popela: SDECOMP - random sampling withim the L-shaped algorithm.
- Simple recourse:
- **SHOR1**
- **SRAPPROX** - recommended, only RHS stochastic.
  - Popela: LSRAPPROX - discrete approximations, L-shaped algorithm, lower and upper bounds.

Solvers for jointly chance-constrained problems

Only RHS stochatic & nondegenerate multivariate normal distribution!

- **PCSPIOR**
- **PROBALL** - recommended
    - Uses Minos 5.4 for solving LP subproblems
- **PROCON**

- For computing the normal distribution function and its gradient:
    - PCSPNOR3 of Szantai,
    - NORSUBS of Deak.

Solver for simple integer recourse

- **SIRD2SCR** - convex hull method implemented by Mayer and van der Vlerk (1994).
  - K. Haneveland, L. Stougie, M.H. van der Vlerk (1996). *An algorithm for the construction of convex hulls in simple integer recourse programming*. Annals of Operations Research, 64, pp. 67-81.
  - Uses SRAPPROX for solving simple recourse subproblems.
  - Finite discrete distribution.

Solver for multiple simple recourse

- **MScr2Scr** - transformation method (to simple continuous recourse) of van der Vlerk, implemented by Mayer and van der Vlerk (2001).
    - M.H. van der Vlerk (2005). **On multiple simple recourse models**. Mathematical Methods of Operations Research, 62, pp. 225-242.
    - Uses SRAPPROX for solving simple recourse subproblems.

Nested Benders decomposition, MSLiP

- Gassmann (1990), *MSLiP: A computer code for the multistage stochastic linear programming problem*. Mathematical Programming 47, 707-423.
- J. Dupacova, J. Hurt, J. Stepan (2002), *Stochastic modeling in economics and finance*, Kluwer Academic Publishers (P. Popela)
  - L-shaped alg. for two stage models, some modifications for recourse problems.
  - MSLiP (MSLSHAP).
- P. Kall, J. Mayer (2005), *Stochastic linear programming*, International Series in Operations Research & Management Science , Vol. 80, Springer. (Pages 29-53)

MSLiP

=Multistage Stochastic Linear Programming - "nested Benders decomposition with added algorithmic features".

- Support of an arbitrary number of time periods and finite discrete distributions with Markovian structure.

Scenario TREE = a set of nodes $\mathcal{K} = \{1, \ldots, K_T\}$ with stages $\mathcal{K}_t = \{K_{t-1} + 1, \ldots, K_t\}$ and probabilities $p_1, \ldots, p_T > 0$, $\sum_{n \in \mathcal{K}_t} p_n = 1$,
a_n the ancestor of the node n,
$\mathcal{D}(n)$ the set of descentants of the node n,
$t(n)$ the time stage of the node n.

Nested formulation of the discrete MSLP

For starting node ($n = 1$)

$$\begin{aligned} F_1 &= \min_{x_1, \vartheta_1} \left\{ c_1^T x_1 + \vartheta_1 \ \ s.t. \ Ax_1 = b, \ l_1 \le x_1 \le u_1, \ \vartheta_1 \ge Q_1(x_1) \right\}, \\ Q_1(x_1) &= \sum_{m \in \mathcal{D}(1)} \frac{p_m}{p_n} F_m(x_1). \end{aligned}$$

For nested stages $n = 2, \ldots, K_{T-1}$

$$\begin{aligned} F_n(x_{a_n}) &= \min_{x_n, \vartheta_n} \left\{ c_n^T x_n + \vartheta_n \ \ s.t. \ W_n x_n = h_n - T_n x_{a_n}, \right. \\ & \qquad \left. l_n \le x_n \le u_n, \ \vartheta_n \ge Q_n(x_n) \right\}, \\ Q_n(x_n) &= \sum_{m \in \mathcal{D}(n)} \frac{p_m}{p_n} F_m(x_n). \end{aligned}$$

For final stage $n = K_{T-1} + 1, \ldots, K_T$

$$F_n(x_{a_n}) = \min_{x_n} \left\{ c_n^T x_n \ s.t. \ Wx_n = h_n - T_n x_{a_n}, \ l_m \le x_n \le u_n \right\}.$$

(M)(n) Master program = n-th nested two-stage problem:

$$
\begin{aligned}
F_n(x_{a_n}) \quad = \quad \min_{x_n, \vartheta_n} c_n^T x_n + \vartheta_n & \\
s.t. & \\
W_n x_n \quad & = \quad h_n - T_n x_{a_n}, \\
\mathbb{I} x_n \quad & \leq \quad u_n, \\
\mathbb{I} x_n \quad & \geq \quad l_n, \\
\vartheta_n \quad & \geq \quad Q_n(x_n), \text{ convex constraint}, \\
Q_n(x_n) \quad & = \quad \sum_{m \in \mathcal{D}(n)} \frac{p_m}{p_n} F_m(x_n).
\end{aligned}
$$

$F_1 = F_1(x_{a_1})$, where we set $x_{a_1} = 0$, $W_1 = A$ and $h_1 = b$.
We set $\vartheta_n = 0$ for $n = K_{T-1} + 1, \ldots, K_T$.

(RM)(n) Relaxed Master program, $n = 1, \ldots, K_T$:

$$
\begin{aligned}
\widetilde{F}_n(x_{a_n}) \quad = \quad \min_{x_n, \vartheta_n} c_n^T x_n + \vartheta_n & & & \\
s.t. & & & \\
W_n x_n &= h_n - T_n x_{a_n}, & & \\
F_n x_n &\geq f_n, & & \text{feasibility cuts} \\
D_n x_n + \mathbf{1}\vartheta_n &\geq d_n, & & \text{optimality cuts} \\
\mathbb{I} x_n &\leq u_n, & & \\
\mathbb{I} x_n &\geq l_n. & &
\end{aligned}
$$

$\widetilde{F}_1 = \widetilde{F}_1(x_{a_1})$, where we set $x_{a_1} = 0$, $W_1 = A$ and $h_1 = b$.
(RM)(n), $n = K_{T-1} + 1, \ldots, K_T$, compensatory bounds ϑ_n and cuts are not involved.

(RD)(n) Dual problem to the relaxed master problem (RM)(n), $n = 2, \ldots, K_T$:

$$\max_{\pi_n,\alpha_n,\beta_n,\lambda_n,\mu_n} \pi_n^T(h_n - T_n x_{a_n}) + \alpha_n^T f_n + \beta_n^T d_n + \lambda_n^T l_n - \mu_n^T u_n$$

$$\begin{array}{rcl} & s.t. & \\ \pi_n^T W_n + \alpha_n^T F_n + \beta_n^T D_n + \lambda_n - \mu_n & = & c_n, \\ \mathbf{1}^T \beta_n & = & 1, \\ \alpha_n, \beta_n, \lambda_n, \mu_n & \geq & 0, \\ \pi_n & & \text{unrestricted.} \end{array}$$

We set $\alpha_n, \beta_n = 0$ for $n = K_{T-1} + 1, \ldots, K_T$

Algorithm MSLiP

(0)

- Set $\vartheta_n^{(0)} = 0$ for all $n = 1, \dots, K_{T-1}$,
- Solve

$$x_1^{(0)} = \arg\min_{x_1} \left\{ c_1^T x_1 \quad s.t. \; Ax_1 = b, \; l_1 \leq x_1 \leq u_1, \right\}.$$

Algorithm MSLiP

(1)

- Solve the dual problem (RD)(m) to the (RM)(m), $\forall m \in \mathcal{D}(n)$. We get
  - dual optimal solution $(\pi^*_m, \alpha^*_m, \beta^*_m, \lambda^*_m, \mu^*_m), \forall m \in \mathcal{D}(n)$,
  - or feasible direction $(\pi^j_{m(j)}, \alpha^j_{m(j)}, \beta^j_{m(j)}, \lambda^j_{m(j)}, \mu^j_{m(j)})$ in which the dual problem to the subproblem $m(j) \in \mathcal{D}(n)$ is unbounded, i.e.

$$\pi^j_{m(j)}(b_{m(j)} - W_m x_n) + \alpha^j_{m(j)} f_m + \lambda^j_{m(j)} l_m - \mu^j_{m(j)} u_m > 0.$$

$\rightarrow$ **feasibility cut** of the feasible set of (MR)(n):

$$\underbrace{\pi^j_{m(j)} W_m}_{(F_n)_{j\cdot}} x_n \geq \underbrace{\pi^j_{m(j)} b_{m(j)} + \alpha^j_{m(j)} f_m + \lambda^j_{m(j)} l_m - \mu^j_{m(j)} u_m}_{(f_n)_j}.$$

Algorithm MSLiP

(2)

- If $\vartheta_n < Q_n(x_n) \to$ **optimality cut** of the feasible set of (MR)(n)

$$\overbrace{\sum_{m \in \mathcal{D}(n)} p_m \pi_m^i T_m}^{(D_n)_{i\cdot}} x_n + \vartheta_n \geq$$

$$\geq \underbrace{\sum_{m \in \mathcal{D}(n)} p_m \left[\pi_m^i h_m + \alpha_m^i f_m + \beta_m^i d_m + \lambda_m^i l_{t(m)} - \mu_m^i u_{t(m)}\right]}_{(d_n)_i}.$$

- Else if $\vartheta_n \geq Q_n(x_n)$ then we have optimal solution x_n of (MR)(n).

Fast-forward-fast-back (FFFB)

- FORWARD pass ($t = 1, \dots, T$, $n = K_t - 1, \dots, K_t$) terminates by:
  - infeasibility of the relaxed master program (RM)(n) $\longrightarrow$ add feasibility cut to (RM)(a_n) & BACKTRACKING,
  - obtaining optimal solutions $\hat{x}_n$ for all $n = 1, \dots, K_T$ $\longrightarrow$ BACKWARD pass.
- BACKTRACKING ($n \longrightarrow a_n$) terminates by:
  - feasibility of the relaxed master program (RM)(a_n) $\longrightarrow$ FORWARD pass,
  - reaching the root node with an infeasible (RM)(1) $\longrightarrow$ MSLP is infeasible.
- BACKWARD pass always goes through all nodes (adding optimality cuts if necessary).
  - No optimality cuts have been added $\longrightarrow$ optimal solution,
  - else $\longrightarrow$ FORWARD pass.

MSLiP

- The algorithm (FFFB) terminates in a **finite number of iterations**.
- If termination occurs after BACKWARD pass then the current solution is optimal.
- **Validity of**
  - feasibility cuts $\sim$ feasible solutions of (M)(n) are not cut off.
  - optimality cuts $\sim$ objective function of (RM)(n) yields a lower bound to the objective function (M)(n).
- Cuts generated by the algorithm are valid.
- $$„\widetilde{F}_1^{(BACKWARD)} \le F_1 \le \widetilde{F}_1^{(FORWARD)}“$$

QDECOM

= Quadratic DECOMposizion, regularizing quadratic term in the objective (two-stage).
(RMQ) Relaxed Master program

$$
\begin{array}{rcl}
\widetilde{F} & = & \min_{x,\vartheta^m} c^T x_n + \sum_{m\in\mathcal{D}} p_m \vartheta^m \quad + \quad \frac{1}{2}\left\|x - x^{(i-1)}\right\|^2 \\
& & s.t. \\
& & Ax = b, \\
& & Fx \geq f, \\
& & D^m x + \mathbf{1}\vartheta^m \geq d^m, \forall m \in \mathcal{D}, \\
& & \mathbb{I}x \leq u, \\
& & \mathbb{I}x \geq l.
\end{array}
$$

Thank you for your attention.