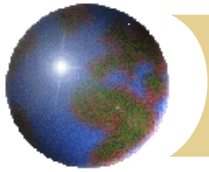# *Introduction to derivatives*
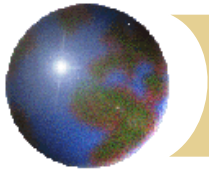
Miloš Kopa

based on

Options, Futures, and Other Derivatives, 8th Edition,
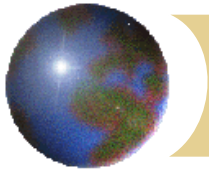
Copyright © John C. Hull 2012

What is a Derivative?

- A derivative is an instrument whose value depends on, or is derived from, the value of another asset.

- Examples: futures, forwards, swaps, options, exotics…

Why Derivatives Are Important

- Derivatives play a key role in transferring risks in the economy

- The underlying assets include stocks, currencies, interest rates, commodities, debt instruments, electricity, insurance payouts, the weather, etc

- Many financial transactions have embedded derivatives

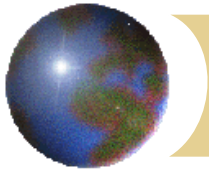- The real options approach to assessing capital investment decisions has become widely accepted

How Derivatives Are Traded

- On exchanges such as the Chicago Board Options Exchange

- In the over-the-counter (OTC) market where traders working for banks, fund managers and corporate treasurers contact each other directly
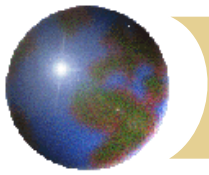
How Derivatives are Used

- To hedge risks
- To speculate (take a view on the future direction of the market)
- To lock in an arbitrage profit
- To change the nature of a liability
- To change the nature of an investment without incurring the costs of selling one portfolio and buying another
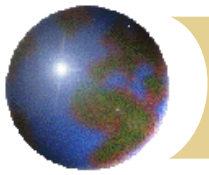
Forward Price

- The forward price for a contract is the delivery price that would be applicable to the contract if were negotiated today (i.e., it is the delivery price that would make the contract worth exactly zero)

- The forward price may be different for contracts of different maturities
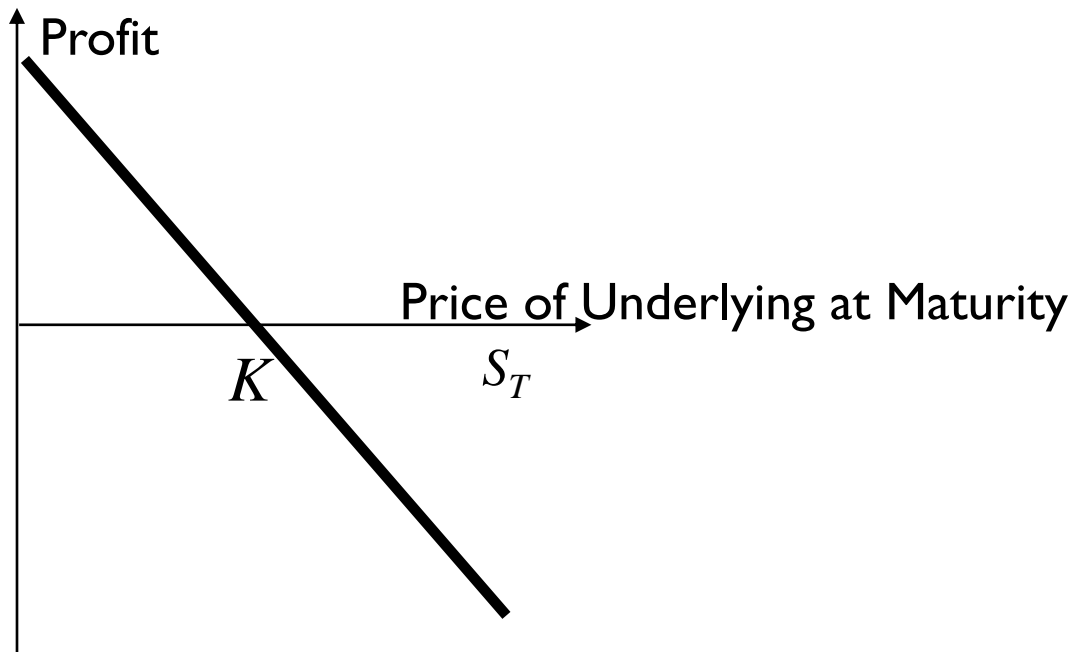
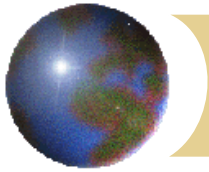# *Profit from a Long Forward Position* *(K= delivery price=forward price at time contract is entered into)*
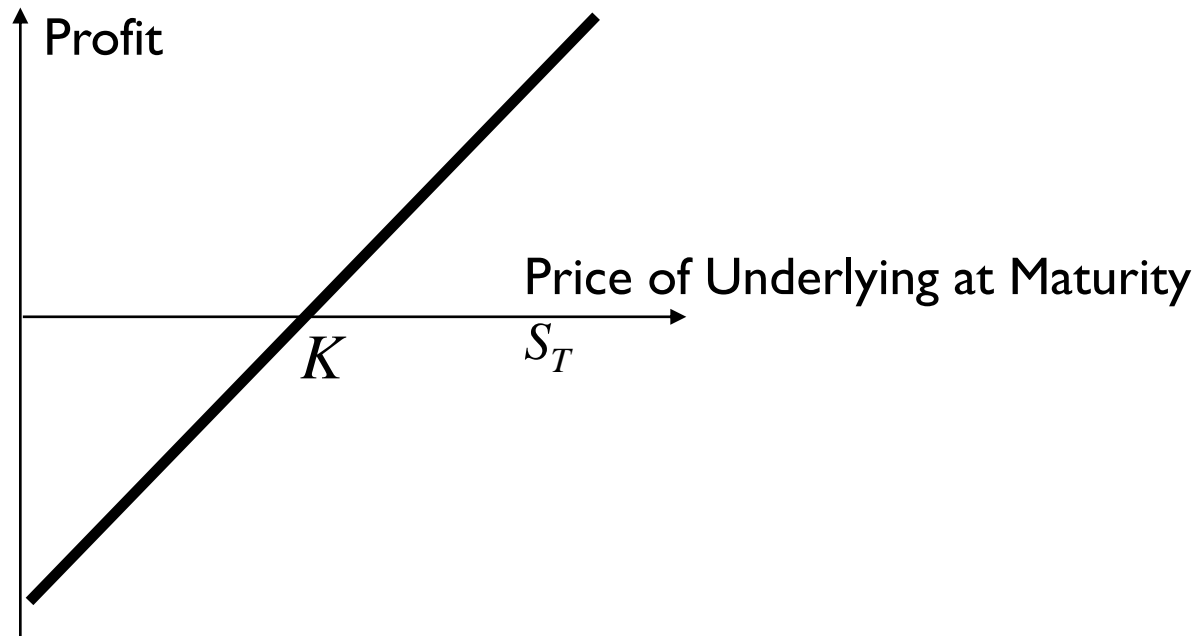
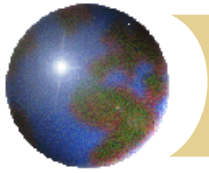# *Profit from a Short Forward Position* *(K= delivery price=forward price at time contract is entered into)*

Futures Contracts

- Agreement to buy or sell an asset for a certain price at a certain time
- Similar to forward contract
- Whereas a forward contract is traded OTC, a futures contract is traded on an exchange
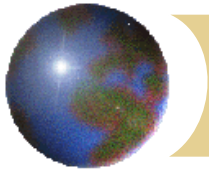
Exchanges Trading Futures

- CME Group (formerly Chicago Mercantile Exchange and Chicago Board of Trade)
- NYSE Euronext
- BM&F (Sao Paulo, Brazil)
- TIFFE (Tokyo)
- and many more (see list at end of book)
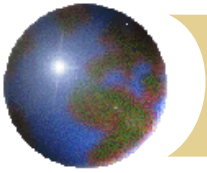
Examples of Futures Contracts

Agreement to:

- Buy 100 oz. of gold @ US$1400/oz. in December
- Sell £62,500 @ 1.4500 US$/£ in March
- Sell 1,000 bbl. of oil @ US$90/bbl. in April

1. Gold: An Arbitrage Opportunity?

Suppose that:
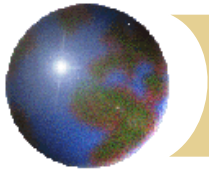
The spot price of gold is US$1,400

The 1-year forward price of gold is US$1,500

The 1-year US$ interest rate is 5% per annum

Is there an arbitrage opportunity?

To give an answer, we need to price the contract.

The Forward Price of Gold
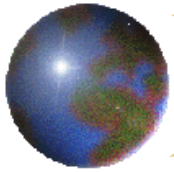
(ignores the gold lease rate)

If the spot price of gold is S and the forward price for a contract deliverable in T years is F, then

$$F = S (1+r)^T$$

where r is the 1-year (domestic currency) risk-free rate of interest.
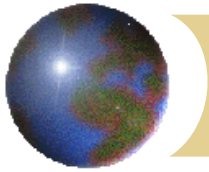
In our examples, S = 1400, T = 1, and r =0.05 so that

$$F = 1400(1+0.05) = 1470$$

Options

- A call option is an option to buy a certain asset by a certain date for a certain price (the strike price)

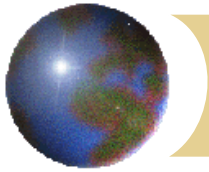- A put option is an option to sell a certain asset by a certain date for a certain price (the strike price)

American vs European Options

- An American option can be exercised at any time during its life
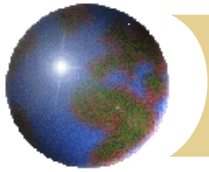
- A European option can be exercised only at maturity

Google Call Option Prices (June 15, 2010; Stock Price is bid 497.07, offer 497.25); *See Table 1.2 page 8; Source: CBOE*

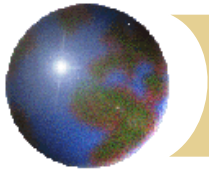| Strike Price	Jul 2010 Bid	Jul 2010 Offer	Sep 2010 Bid	Sep 2010 Offer	Dec 2010 Bid	Dec 2010 Offer
460	43.30	44.00	51.90	53.90	63.40	64.80
480	28.60	29.00	39.70	40.40	50.80	52.30
500	17.00	17.40	28.30	29.30	40.60	41.30
520	9.00	9.30	19.10	19.90	31.40	32.00
540	4.20	4.40	12.70	13.00	23.10	24.00
560	1.75	2.10	7.40	8.40	16.80	17.70

Options vs Futures/Forwards

- A futures/forward contract gives the holder the obligation to buy or sell at a certain price

- An option gives the holder the right to buy or sell at a certain price

Types of Traders

- ⊕ Hedgers
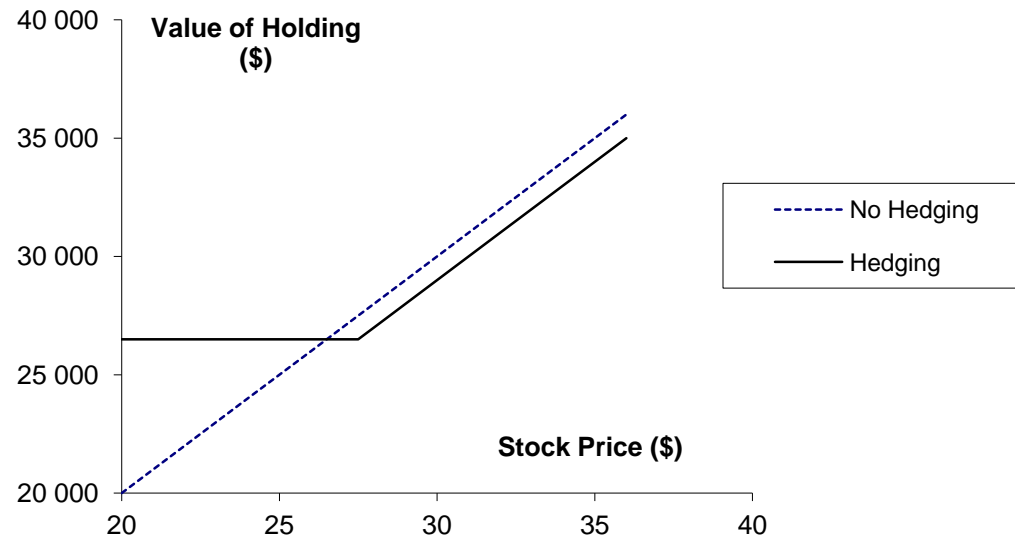- ⊕ Speculators
- ⊕ Arbitrageurs

Hedging Examples *(pages 10-12)*

- A US company will pay £10 million for imports from Britain in 3 months and decides to hedge using a long position in a forward contract

- An investor owns 1,000 Microsoft shares currently worth $28 per share. A two-month put with a strike price of $27.50 costs $1. The investor decides to hedge by buying 10 contracts
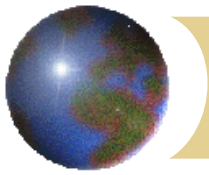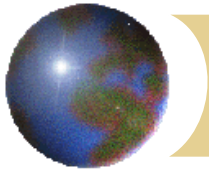
Value of Microsoft Shares with and without Hedging (Fig 1.4, page 12)

# *Speculation Example*

⬥ An investor with $2,000 to invest feels that a stock price will increase over the next 2 months. The current stock price is $20 and the price of a 2-month call option with a strike of 22.50 is $1
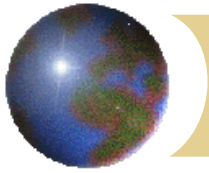
⬥ What are the alternative strategies?

Arbitrage Example

- A stock price is quoted as £100 in London and $140 in New York

- The current exchange rate is 1.4300
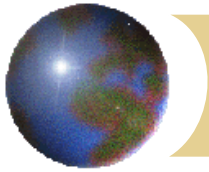
- What is the arbitrage opportunity?
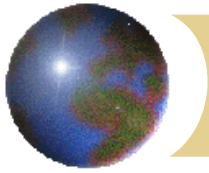
Short Selling *(Page 102-103)*

- Short selling involves selling securities you do not own

- Your broker borrows the securities from another client and sells them in the market in the usual way

Short Selling *(continued)*

- ⊕ At some stage you must buy the securities so they can be replaced in the account of the client

- ⊕ You must pay dividends and other benefits the owner of the securities receives

- ⊕ There may be a small fee for borrowing the securities

Example

- You short 100 shares when the price is $100 and close out the short position three months later when the price is $90

- During the three months a dividend of $3 per share is paid

- What is your profit?

- What would be your loss if you had bought 100 shares?

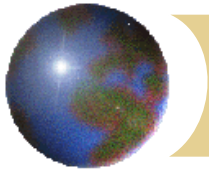# *Notation for Valuing Futures and Forward Contracts*

S_0: Spot price today

F_0: Futures or forward price today

T: Time until delivery date

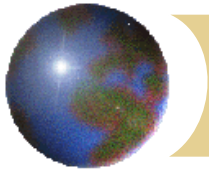r: Risk-free interest rate for maturity T

The Forward Price

If the spot price of an investment asset is S_0 and the futures price for a contract deliverable in T years is F_0, then
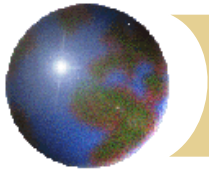
$$F_0 = S_0 e^{rT}$$

where r is the T-year risk-free rate of interest.

When an Investment Asset Provides a Known Income (page 107, equation 5.2)
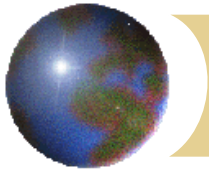
$$F_0 = (S_0 - I)e^{rT}$$

where I is the present value of the income during life of forward contract

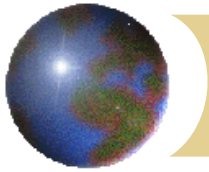# When an Investment Asset Provides a Known Yield *(Page 109, equation 5.3)*

$$F_0 = S_0\, e^{(r-q)T}$$

where q is the average yield during the life of the contract (expressed with continuous compounding)

Valuing a Forward Contract

- A forward contract is worth zero (except for bid-offer spread effects) when it is first negotiated

- Later it may have a positive or negative value

- Suppose that K is the delivery price and F_0 is the forward price for a contract that would be negotiated today
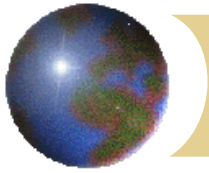
Valuing a Forward Contract
Page 109-11

◆ By considering the difference between a contract with delivery price K and a contract with delivery price F_0 we can deduce that:

  ❖ the value of a long forward contract, f, is
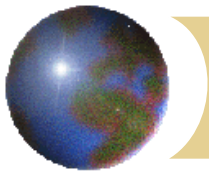$$(F_0 - K)e^{-rT}$$

  ❖ the value of a short forward contract is
$$(K - F_0)e^{-rT}$$

Forward vs Futures Prices

⬥ When the maturity and asset price are the same, forward and futures prices are usually assumed to be equal. (Eurodollar futures are an exception)
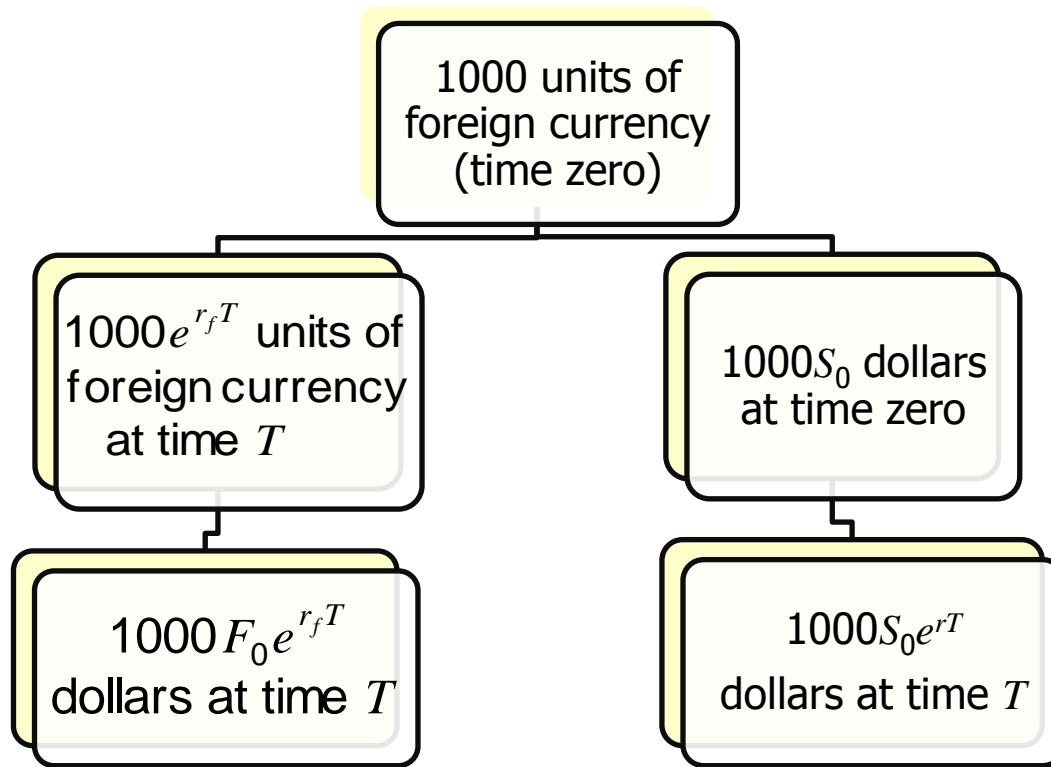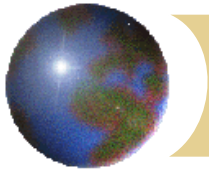
Futures and Forwards on Currencies *(Page 112-115)*

- A foreign currency is analogous to a security providing a yield

- The yield is the foreign risk-free interest rate

- It follows that if r_f is the foreign risk-free interest rate
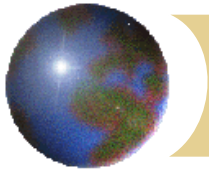
$$F_0 = S_0 e^{(r-r_f)T}$$

Explanation of the Relationship Between Spot and Forward *(Figure 5.1)*

```
        ┌──────────────────┐
        │   1000 units of  │
        │ foreign currency │
        │   (time zero)    │
        └──────────────────┘
         ┌─────────┴──────────┐
┌─────────────────┐    ┌─────────────────┐
│ $1000e^{r_fT}$ units of │    │ $1000S_0$ dollars │
│ foreign currency │    │   at time zero   │
│   at time $T$    │    └─────────────────┘
└─────────────────┘             │
         │              ┌─────────────────┐
┌─────────────────┐    │  $1000S_0e^{rT}$  │
│  $1000F_0e^{r_fT}$  │    │ dollars at time $T$ │
│ dollars at time $T$ │    └─────────────────┘
└─────────────────┘
```

Review of Option Types

- A call is an option to buy

- A put is an option to sell

- A European option can be exercised only at the end of its life

- An American option can be exercised at any time

Option Positions

- Long call
- Long put
- Short call
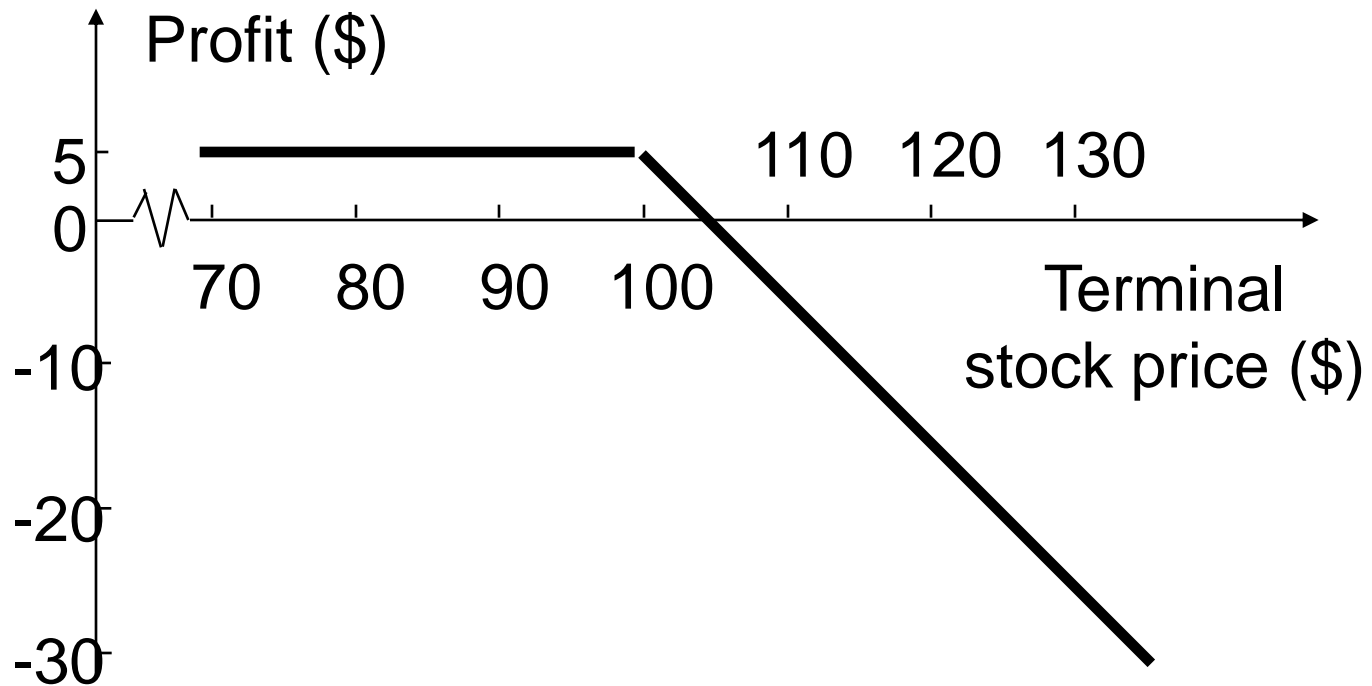- Short put

Long Call *(Figure 9.1, Page 195)*

Profit from buying one European call option: option price = $5, strike price = $100, option life = 2 months
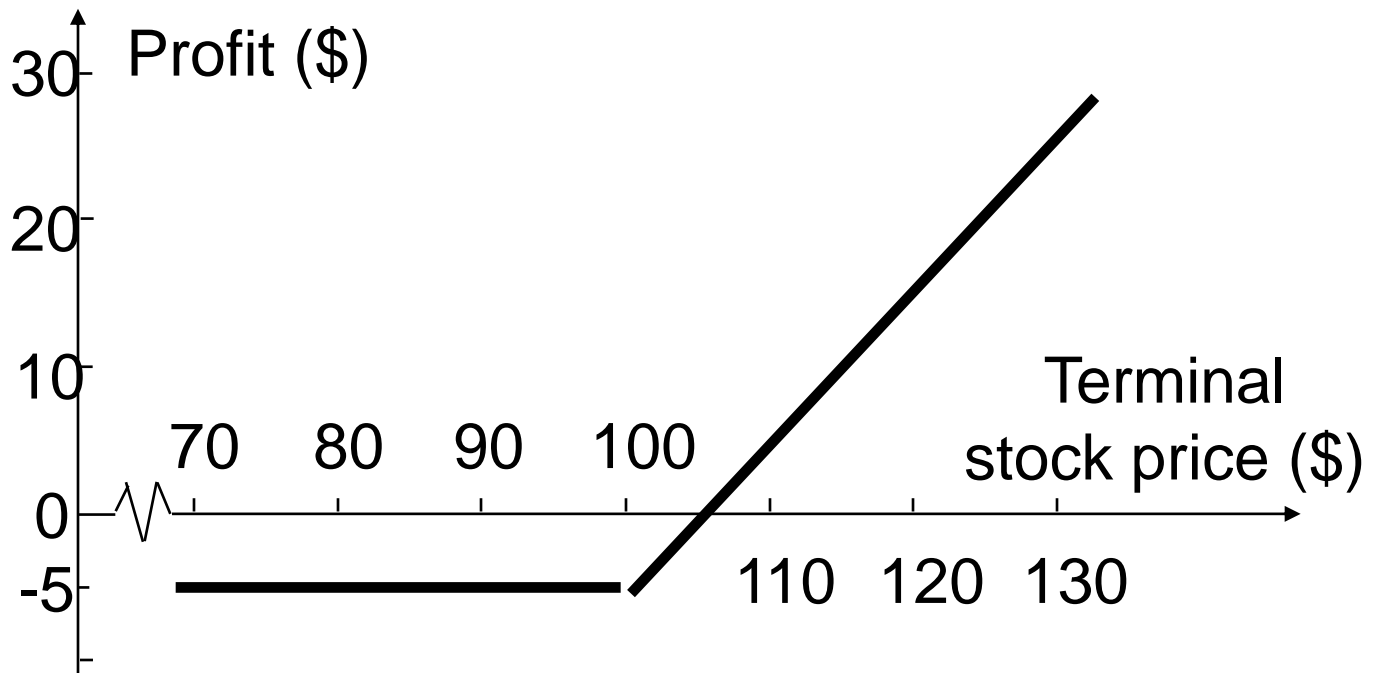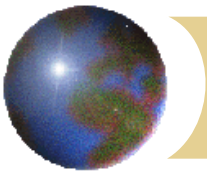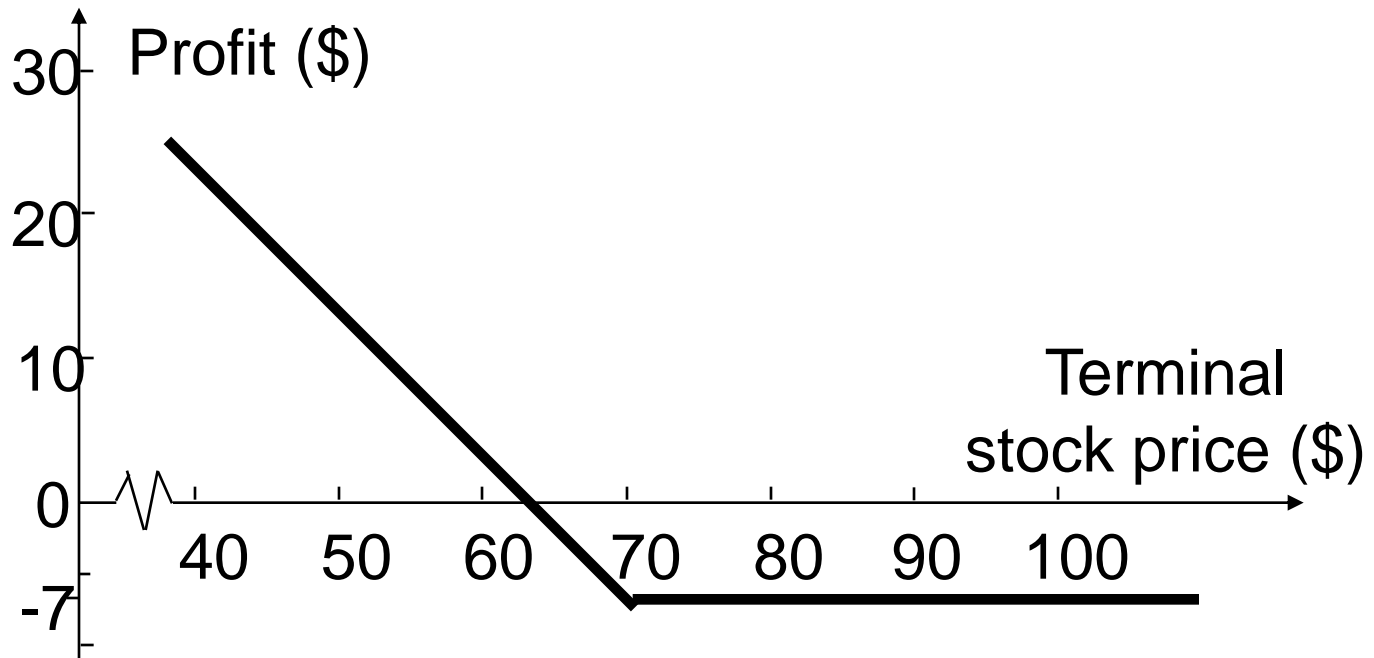
Short Call *(Figure 9.3, page 197)*

Profit from writing one European call option: option price = $5, strike price = $100

Long Put *(Figure 9.2, page 196)*

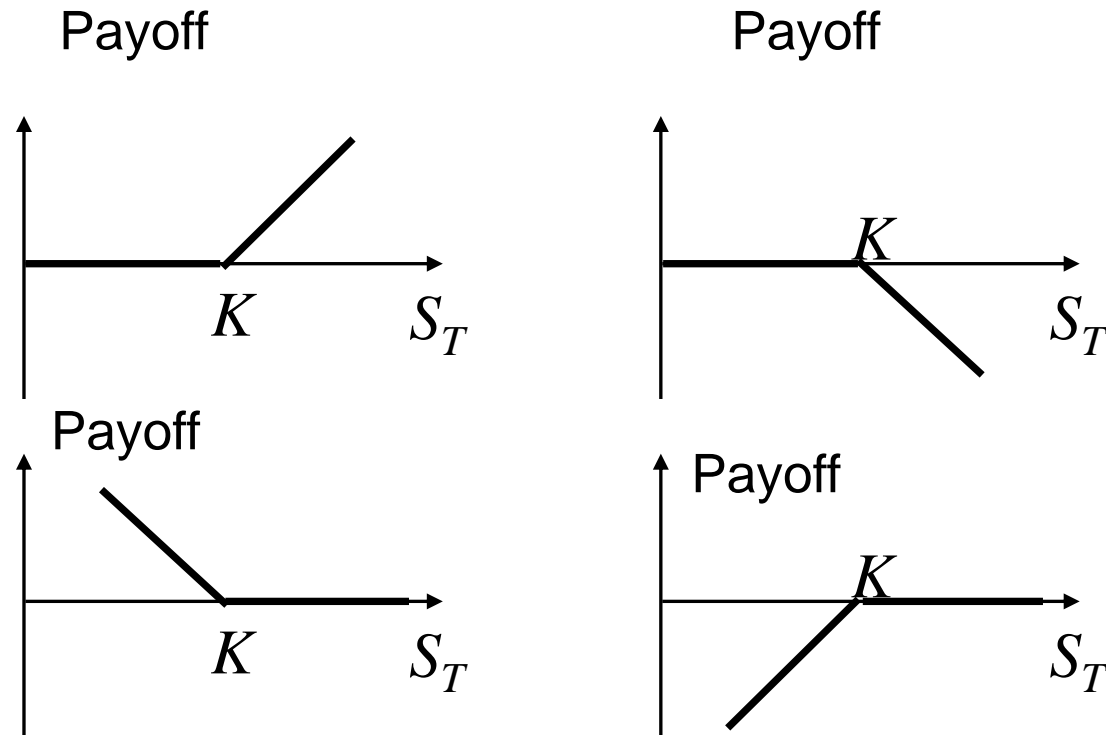Profit from buying a European put option: option price = $7, strike price = $70
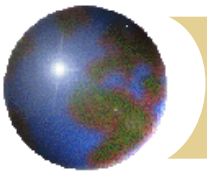
Short Put *(Figure 9.4, page 197)*

Profit from writing a European put option: option price = $7, strike price = $70
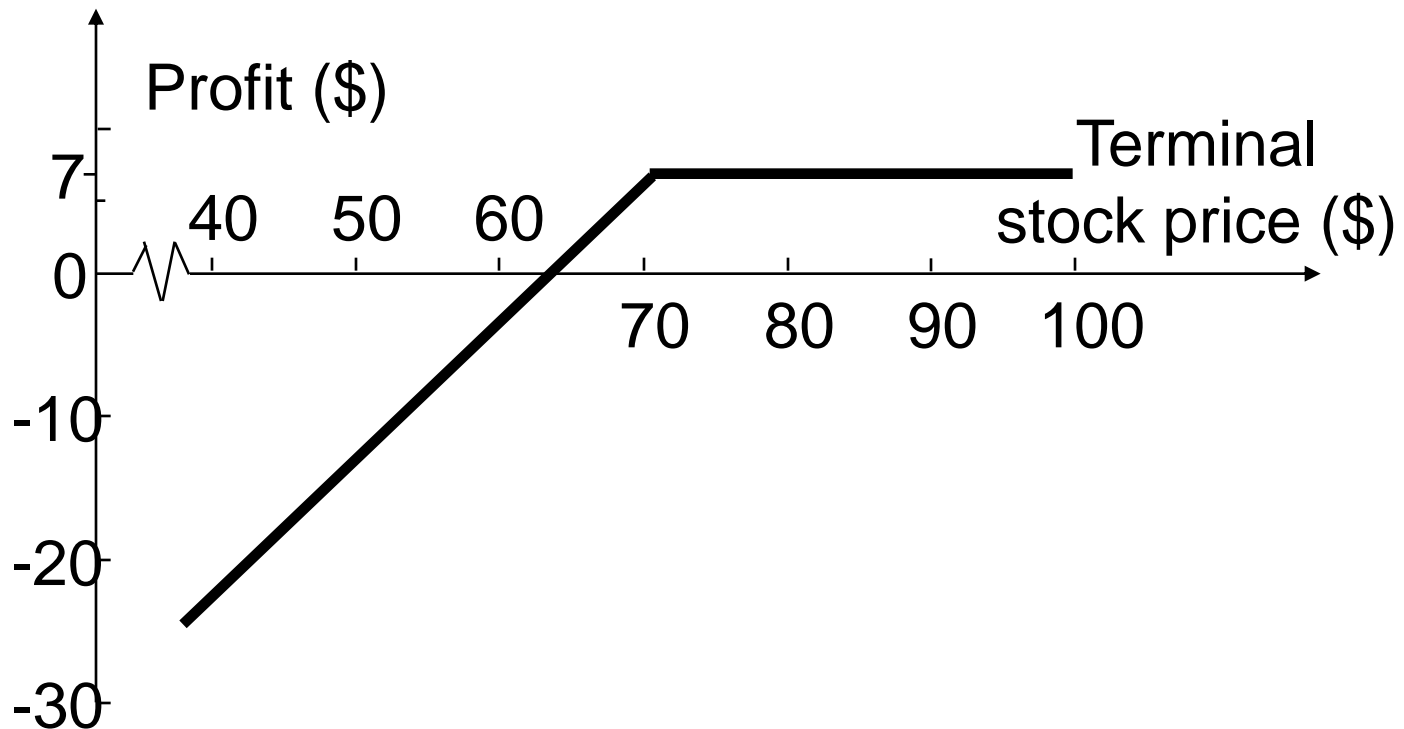
Payoffs from Options
What is the Option Position in Each Case?

K = Strike price, S_T = Price of asset at maturity

Payoff

Payoff

Payoff

Payoff

41

Assets Underlying Exchange-Traded Options

Page 198-199

- Stocks
- Foreign Currency
- Stock Indices
- Futures

Specification of Exchange-Traded Options

- Expiration date
- Strike price
- European or American
- Call or Put (option class)

Warrants

- Warrants are options that are issued by a corporation or a financial institution

- The number of warrants outstanding is determined by the size of the original issue and changes only when they are exercised or when they expire

Notation

c: European call option price

p: European put option price

S_0: Stock price today

K: Strike price

T: Life of option

σ: Volatility of stock price

C: American call option price

P: American put option price

S_T: Stock price at option maturity

D: PV of dividends paid during life of option

r Risk-free rate for maturity T with cont. comp.

Put-Call Parity: No Dividends

- Consider the following 2 portfolios:
  - Portfolio A: European call on a stock + zero-coupon bond that pays K at time T
  - Portfolio C: European put on the stock + the stock
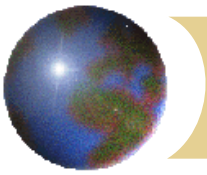
Values of Portfolios

		$S_T > K$	$S_T < K$
Portfolio A	Call option	$S_T - K$	0
	Zero-coupon bond	K	K
	Total	S_T	K
Portfolio C	Put Option	0	$K - S_T$
	Share	S_T	S_T
	Total	S_T	K

The Put-Call Parity Result *(Equation 10.6, page 222)*

- Both are worth $\max(S_T, K)$ at the maturity of the options

- They must therefore be worth the same today. This means that

$$c + Ke^{-rT} = p + S_0$$

(a)

(b)

(c)

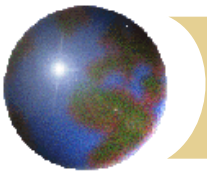(d)
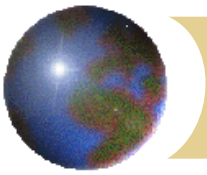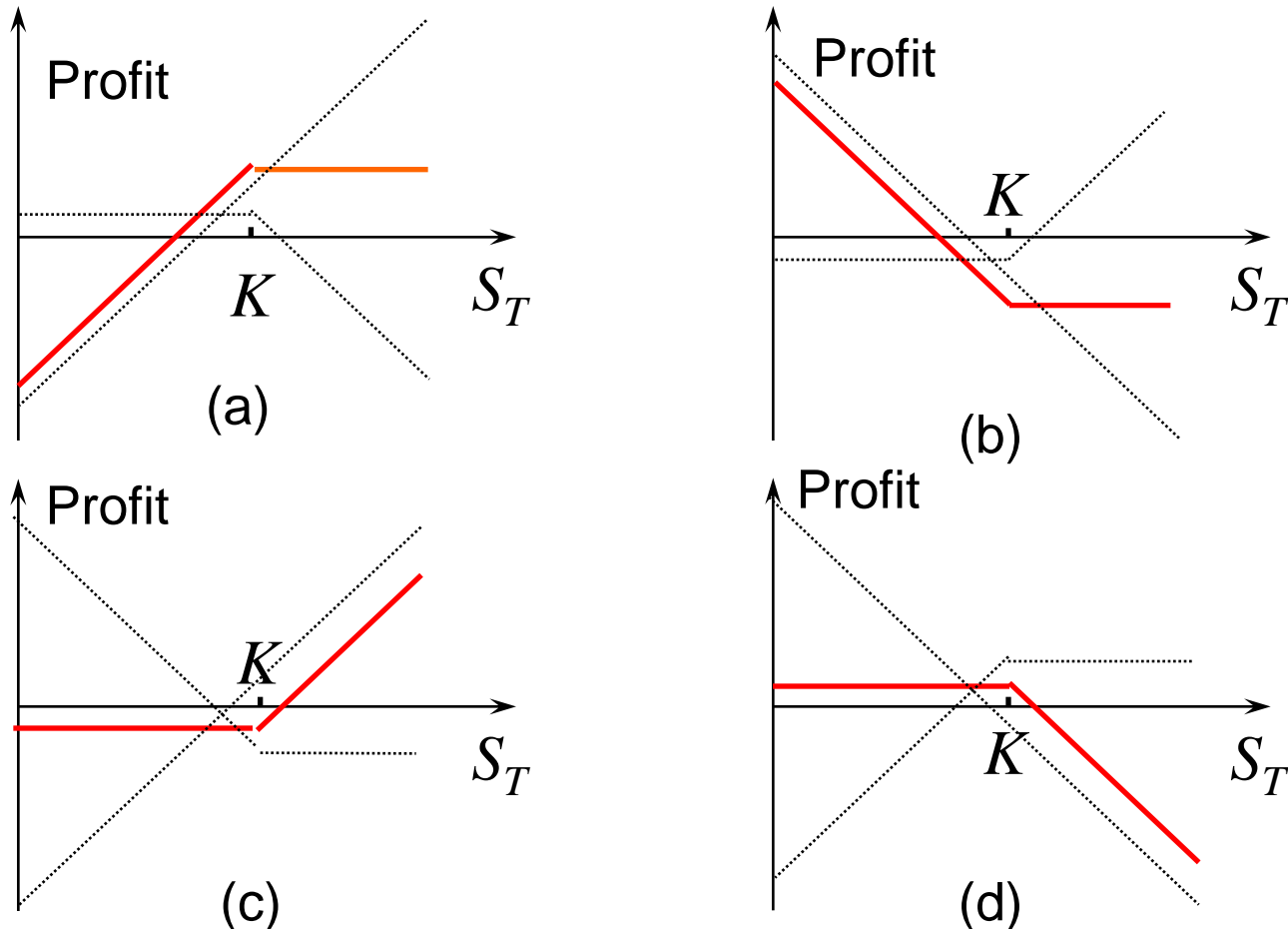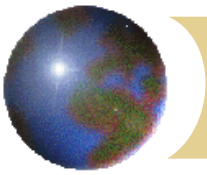
Bull Spread Using Calls
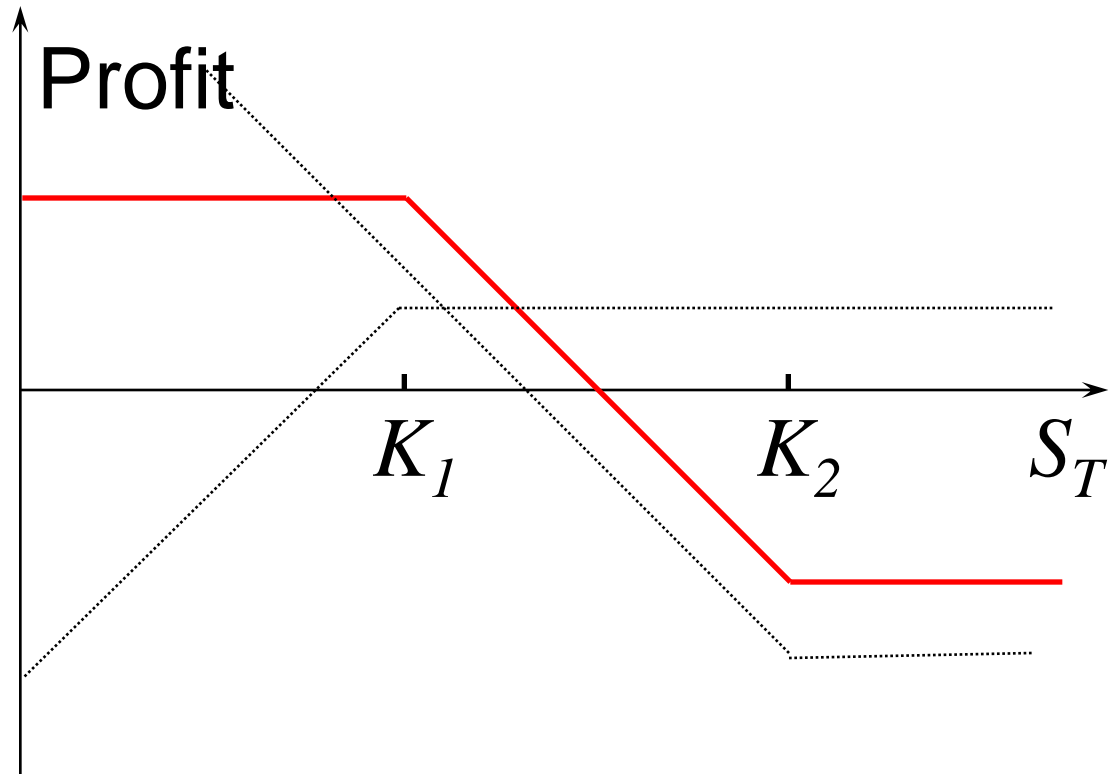## *(Figure 11.2, page 238)*
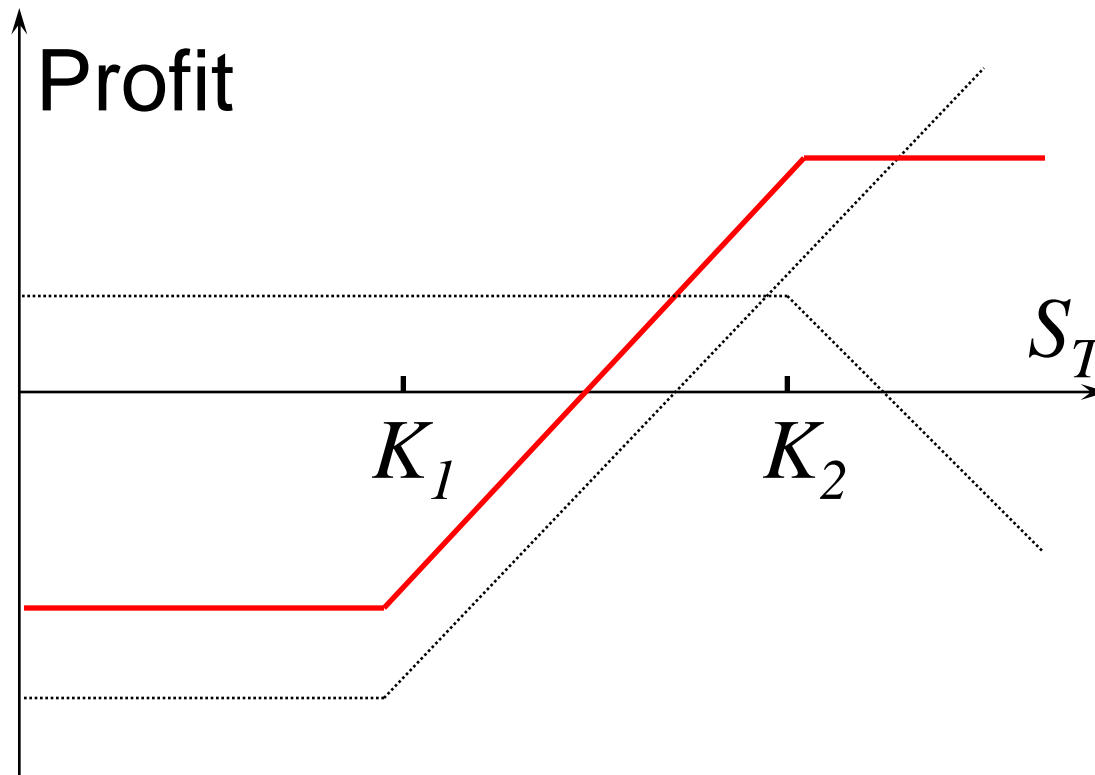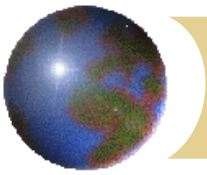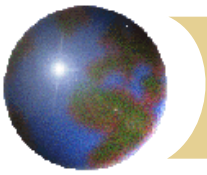
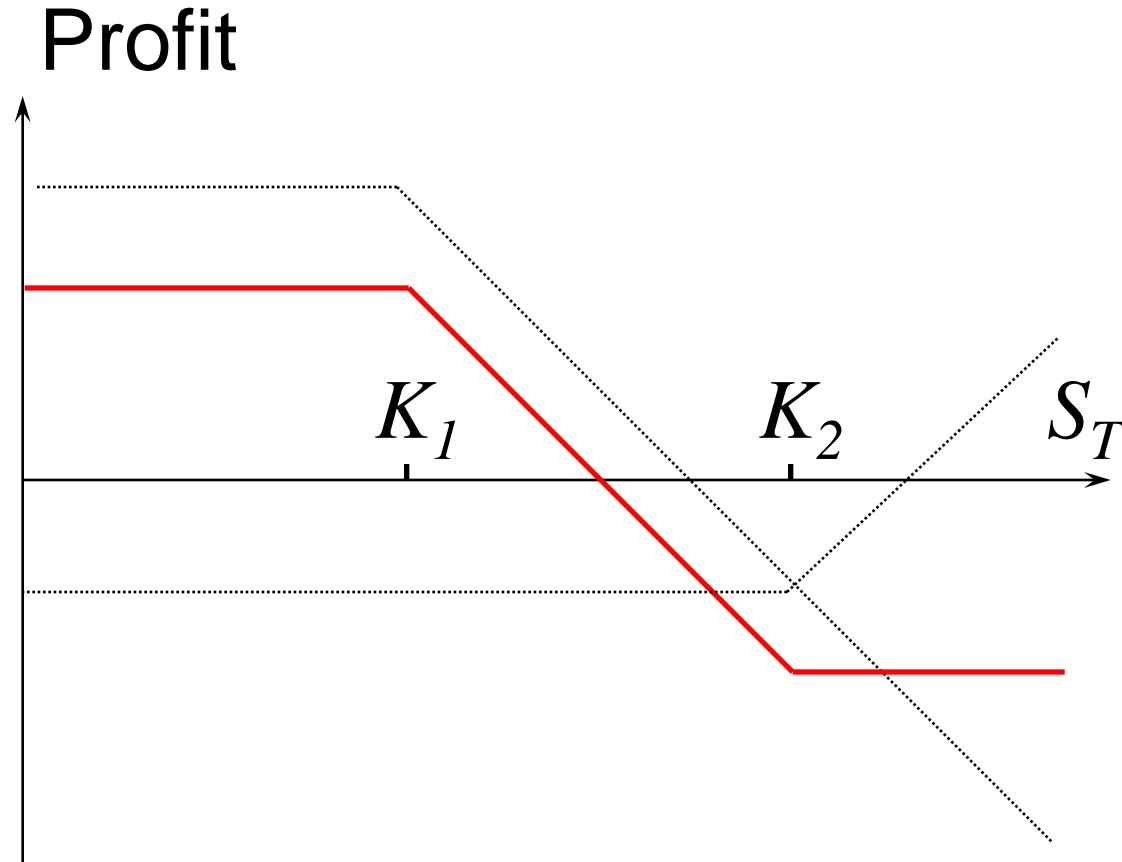# Bull Spread Using Puts

Figure 11.3, page 239
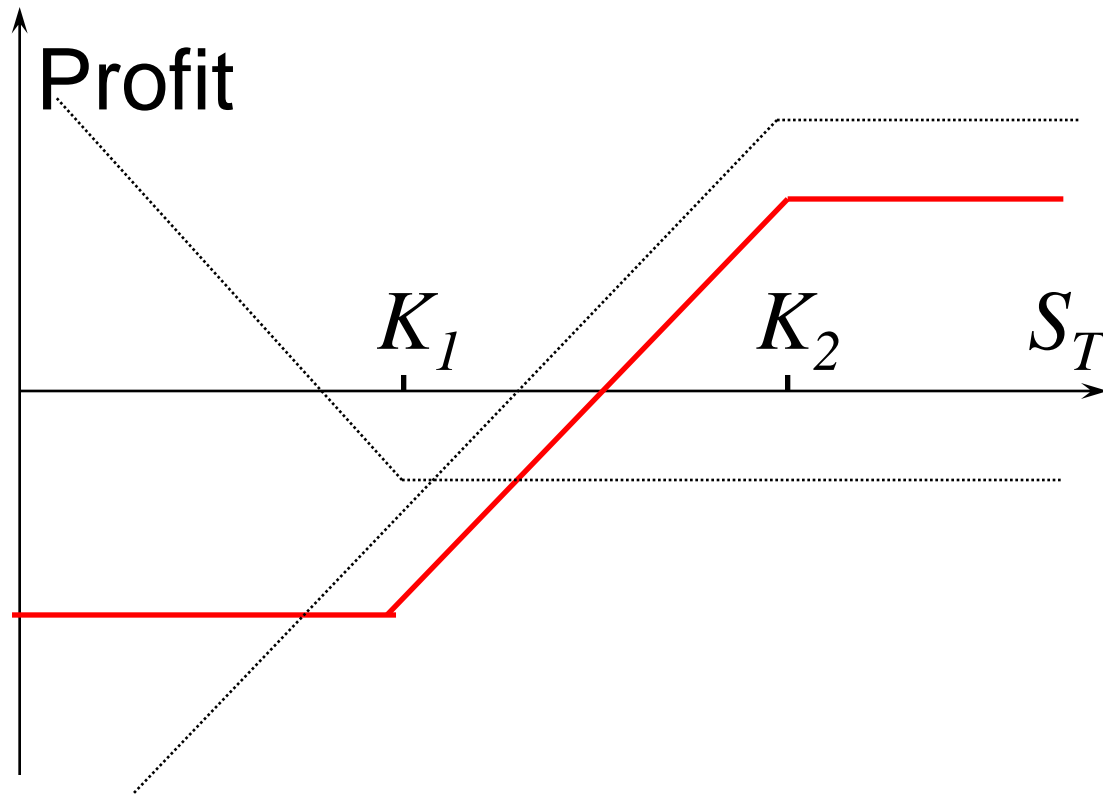
Bear Spread Using Puts

Figure 11.4, page 240
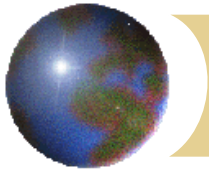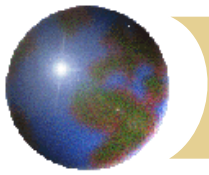
Bear Spread Using Calls

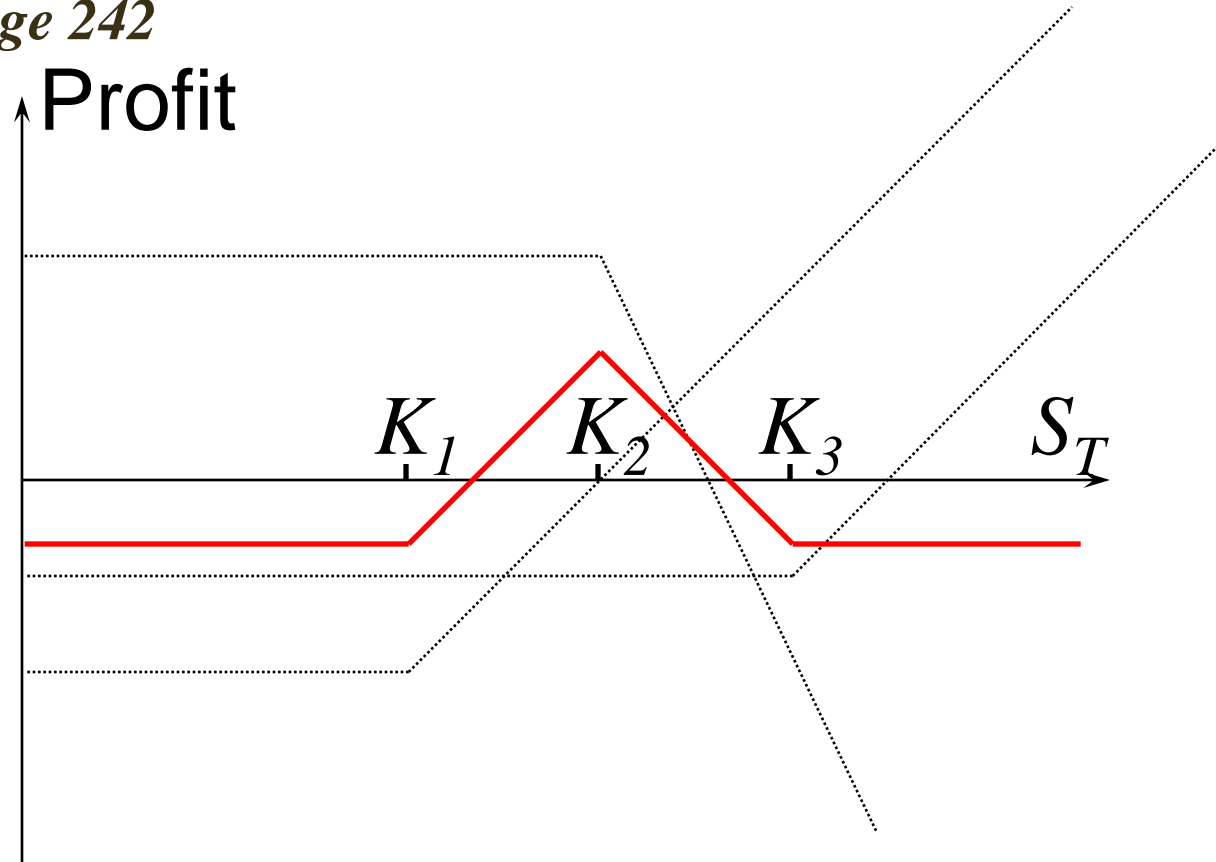*Figure 11.5, page 241*

Profit

K_1 K_2 S_T

Box Spread

- A combination of a bull call spread and a bear put spread

- If all options are European a box spread is worth the present value of the difference between the strike prices

- If they are American this is not necessarily so (see Business Snapshot 11.1)
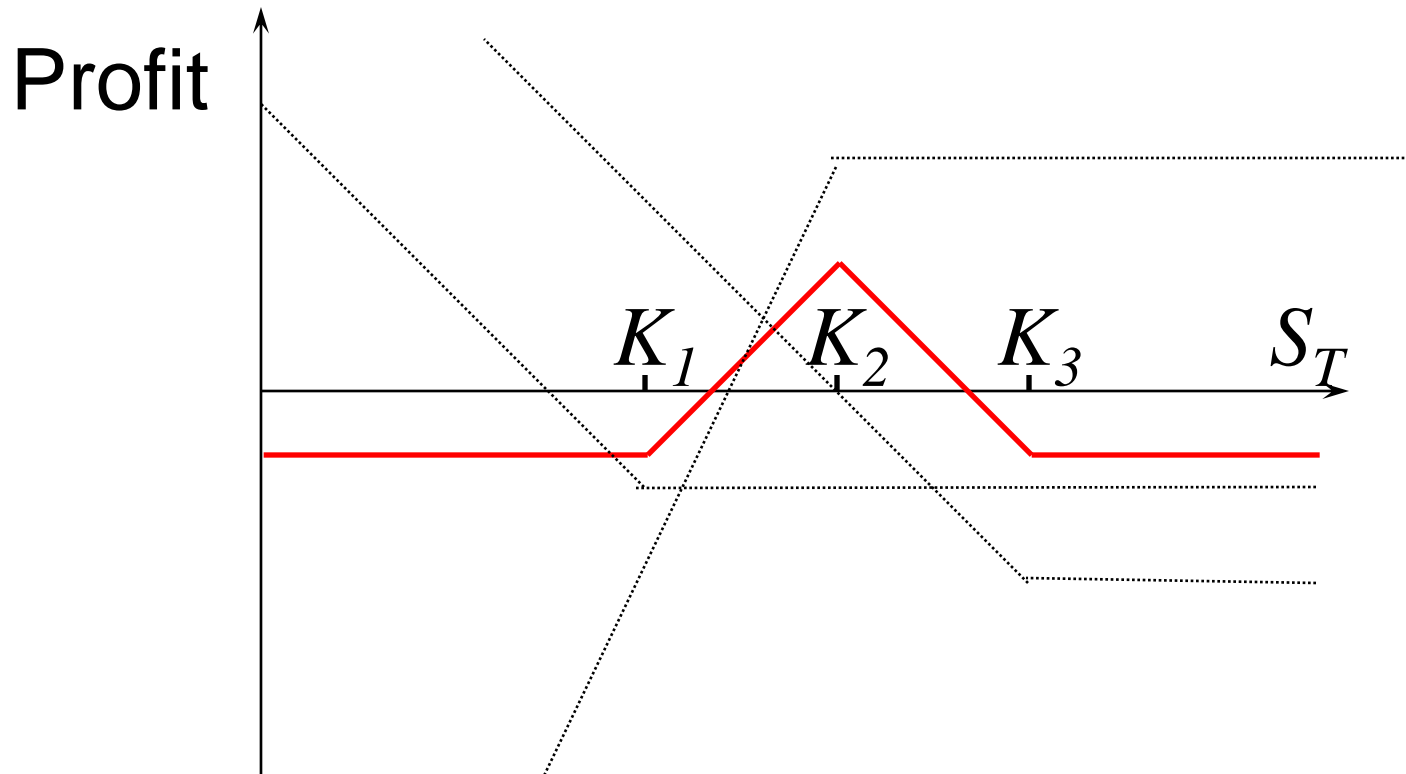
Butterfly Spread Using Calls

Figure 11.6, page 242

Profit

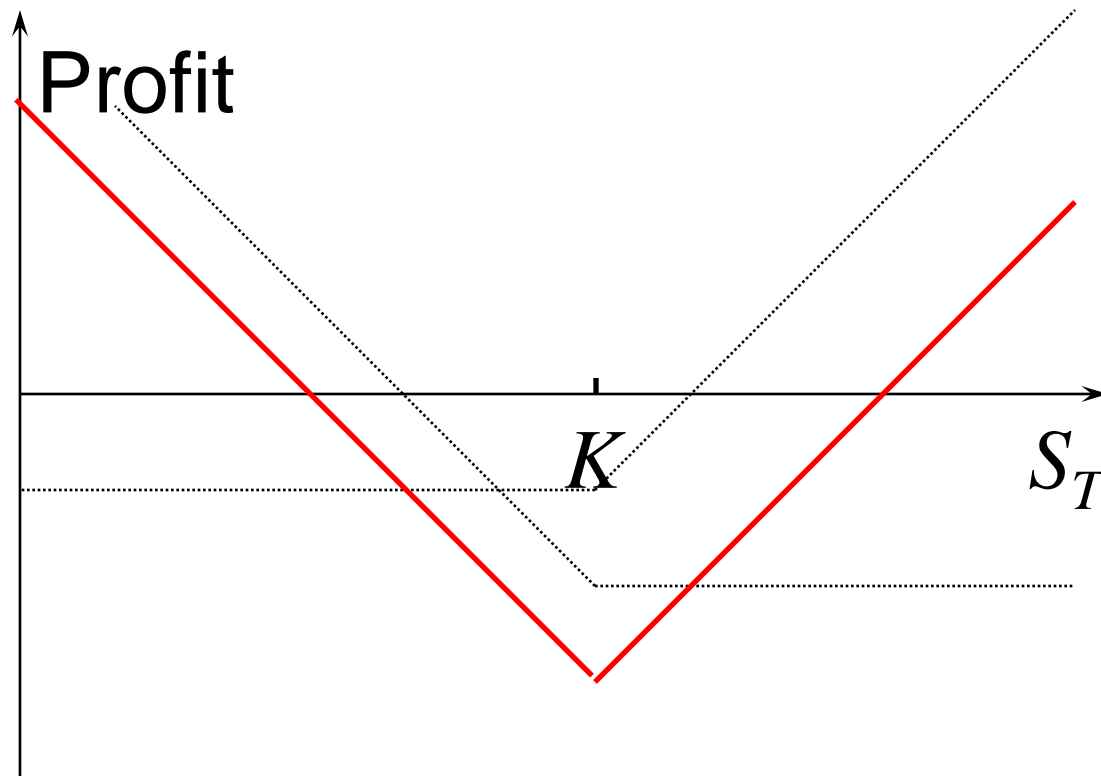K_1 K_2 K_3 S_T

Butterfly Spread Using Puts

Figure 11.7, page 243

A Straddle Combination
Figure 11.10, page 246

A Strangle Combination

Figure 11.12, page 249