3rd problem set

A critical pair is given by two rules $l_1 \approx r_1$, $l_2 \approx r_2$ (possibly the same rule taken twice). We rename the variables in both rules so that the variables of $l_1 \approx r_1$ and $l_2 \approx r_2$ are disjoint. Next we take an address a such that $l_1[a]$ is defined and not a variable and there exists a most general unifier θ for $l_1[a]$ and l_2. The pair is θr_1 and $\theta l_1[a\colon \theta l_2 \to \theta r_2]$.

Problem 1. Explain why it is a bad idea to consider the simpler looking pair θr_1, $l_1[a\colon \theta l_2 \to \theta r_2]$.

Problem 2. Show that there was an error in problem 4 of the last set: The system $\mathcal{E}$ of equations $w(x,x,y) \approx w(x,y,x)$, $w(x,y,x) \approx w(y,x,x)$, $w(x,x,x) \approx x$ gives a graph $D(\mathcal{E})$ that is *not* finitely terminating.

Problem 3. Find all critical pairs of the system $x(yz) \approx (xy)z, 1x \approx x$.

Problem 4. Show that the equality $f(f(x)) \approx g(x)$ gives rise to a term rewrite system that does not have confluent critical pairs.

Problem 5 (Problem 4 from set 2 as it should have been)**.** Show that the system $w(x,x,y) \approx u(x,y)$, $w(x,y,x) \approx u(x,y)$, $w(y,x,x) \approx u(x,y)$, $u(x,x) \approx x$ is compatible with some reduction order and has confluent critical pairs.

Problem 6. The term rewrite system given by $f(f(x)) \approx g(x)$ is not convergent. Add one more rule to it to make it convergent.