Téma: K.11 ... operátory s nekonst. koef.

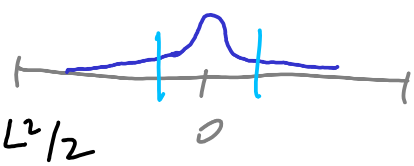
$$A : u(x) \longmapsto \sum_{j=0}^{m} a_j(x)\, u^{(j)}(x)$$

Př. (11.1)

$$u' + \underbrace{x}_{a(x)} u = 0 \qquad x \in (-L, L)$$

$$u = 0 \qquad x = \pm L \ \text{... velké}$$

$$\Longrightarrow \quad u(x) = e^{-\frac{x^2}{2}}$$



$$\tilde{u}(x) = e^{-x^2/2} - e^{-L^2/2}$$

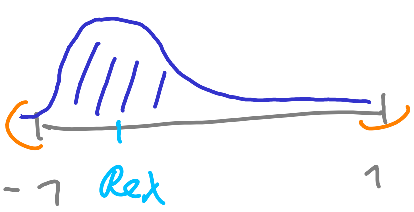
škálování: $h = \frac{1}{L}$ ... malé

$$x \longleftrightarrow x/h$$

lokalizováno v okolí $x^* = \operatorname{Re}\lambda$, $\xi^* = -\operatorname{Im}\lambda$

$$\Longrightarrow \quad hu' + xu = \lambda u \ , \quad x \in (-1, 1)$$

$$u = 0 \qquad x = \pm 1$$



$$u(x) = e^{-\frac{(x-\lambda)^2}{2h}}$$

$$= C \cdot e^{-\frac{(x-\operatorname{Re}\lambda)^2}{2h}} \cdot e^{\frac{i x \operatorname{Im}\lambda}{h}}$$

$\mathcal{F}$

$$\hat{u}(\xi) = C\, e^{-\frac{(\xi+\operatorname{Im}\lambda)^2}{2h}} \ \text{---}$$

wave packet

("vlnové klubko")

Pozn.

$$\hat{u}(\xi) = \int_{\mathbb{R}} e^{\frac{i\xi x}{\cancel{h}}} u(x)\, dx$$

$$\check{u}(x) = \frac{1}{2\pi} \int_{\mathbb{R}} e^{-ix\xi} u(\xi)\, d\xi$$

např.: $$e^{-\frac{x^2}{2}} \xrightarrow{\mathcal{F}} e^{-\frac{\xi^2}{2}}$$

$$\left[u(x)\, e^{ilx}\right]^{\wedge}(\xi) = \hat{u}(\xi + l)$$

$$\left[u(x-a)\right]^{\wedge}(\xi) = \hat{u}(\xi)\cdot e^{-ia\xi}$$

? zobecnění:

$$u' + a(x)u = 0, \quad x \in (-L, L)$$
$$u = 0 \qquad x = \pm L$$

např.: $\boxed{a(x) \simeq x}$ v okolí 0

$$\Rightarrow \quad u(x) = e^{-\frac{x^2}{2}}$$

Obecná situace:

$$D_h = h\frac{d}{dx}, \quad h>0 \text{ malé}$$

$$a_j(x) \in C(a,b), \quad j = 0, 1, \dots, m$$

$$\{A_h\}_{h>0} \text{ --- operátory}$$

$$(A_h u)(x) = \sum_{j=0}^{n} a_j(x)\,(D_h^j u)(x)$$

$x \in (a,b)$

ansatz:

$$v(x) = e^{-\frac{i\xi x}{h}}$$

$$\Rightarrow \quad (A_h v)(x) = f(x,\xi)\,v(x)$$

$$f(x,\xi) = \sum_{j=0}^{n} a_j(x)\,(-i\xi)^j$$

Symbol op.

A_h (v bodě x)

Pozn: A ... konst. koef. (X.10)

$$Au \xrightarrow{\mathcal{F}} f(\xi)\,\hat{u}(\xi)$$

ale: $A_h u \;\cancel{\xrightarrow{\mathcal{F}}}\; f(x,\xi)\,\hat{u}(\xi)$

Def. $I(f,x,\lambda)$ ... index křivky $t \mapsto f(x,t)$ vůči $\lambda \in \mathbb{C}$

Def. Twist condition: Řekneme, že symbol $f = f(x,\xi)$ splňuje (T.C.) v bodě (x^*,ξ^*), jestliže

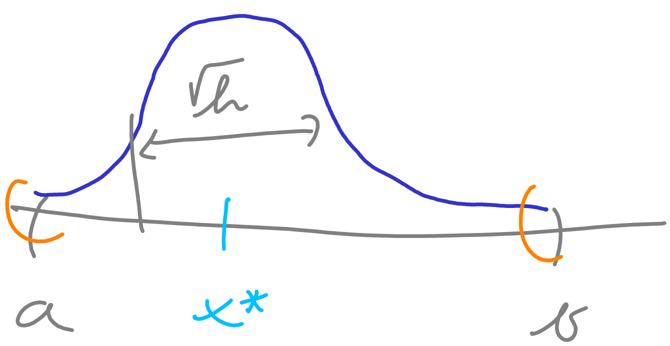
$$\boxed{\operatorname{Im}\left(\frac{\partial f}{\partial x} \Big/ \frac{\partial f}{\partial \xi}\right) > 0} \qquad (11.11)$$

Věty 11.1 a 11.2

$$\mathcal{O}(h^N)$$

$$A_h v - \lambda v = 0 \qquad \mathcal{O}\left(e^{-\frac{c}{h}}\right)$$

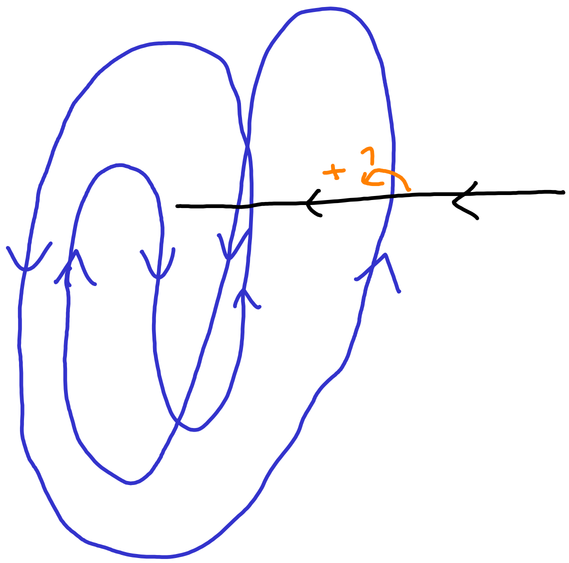
$$\lambda = f(x^*,\xi^*)$$



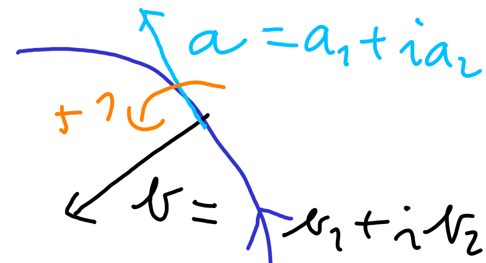
BÚNO: $\|v^{(h)}\| = 1 \qquad (11.12)$

Pozn. ad podmínky (11.11) – heuristika

1) malíkové věse:



obecněji :



a, b .. svírají kladně orientovaný úhel

$\iff +1$

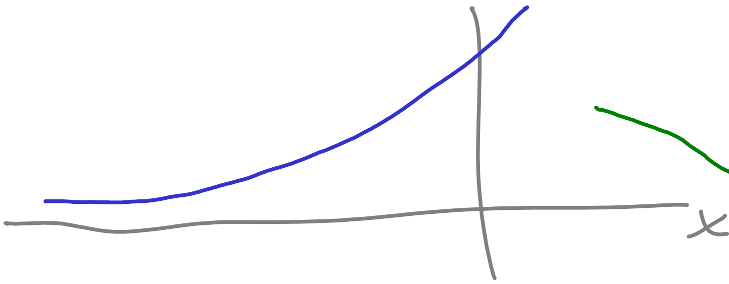
$$\iff \underbrace{\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}}_{\substack{\parallel \\ \operatorname{Im}(\bar{a} b)}} > 0 \quad , \quad \bar{a} = \frac{|a|^2}{a}$$

$$\iff \boxed{\operatorname{Im}\left(b / a \right) > 0}$$

$$a = \frac{\partial f}{\partial z}$$

$$b = \frac{\partial f}{\partial x}$$

Pozn.: význam $I(f, x, \lambda)$:
počet řešení $(f(x, z) - \lambda)$ v $\{\operatorname{Re} z > 0\}$



d – $I(f(\underline{\underline{x}}), \lambda)$ ... počet řešen v $\{\mathrm{Re}\, z < 0\}$