• $H_1(f) - H_1(f_\infty)$

• $H_1(f_\infty) = \int_{\mathbb{R}^d} f \lg f_\infty$

$$H_1(f_\infty) = \int_{\mathbb{R}^d} f_\infty \lg f_\infty = \int_{\mathbb{R}^d} \frac{1}{(2\pi)^{d/2}} e^{-\frac{|v|^2}{2}} \left(\lg (2\pi)^{-\frac{d}{2}} \cancel{+} - \frac{|v|^2}{2} \right)$$

$$= \int_{\mathbb{R}^d} \frac{1}{(2\pi)^{d/2}} \left(e^{-\frac{|v|^2}{2}} \cdot \lg\left((2\pi)^{-d/2}\right) - e^{-\frac{|v|^2}{2}} \cdot \frac{|v|^2}{2} \right)$$

$$= \cancel{\int} \lg\left((2\pi)^{-d/2}\right) - \underbrace{\int_{\mathbb{R}^d} \frac{1}{(2\pi)^{d/2}} e^{-\frac{|v|^2}{2}} |v|^2}_{=1} \cdot \frac{1}{2} \, dv$$

$$H_1(f_\infty) = \lg\left((2\pi)^{-1/2}\right) - \frac{1}{2}$$

$$\parallel$$

$$\int_{\mathbb{R}^d} f \lg f_\infty = \int_{\mathbb{R}^d} f(v) \left(\lg\left((2\pi)^{-d/2} - \frac{|v|^2}{2}\right) \right) = \lg\left((2\pi)^{-d/2}\right) - \frac{1}{2}$$

$$H_1(f_\infty) = \int_{\mathbb{R}^d} f \lg f_\infty$$

$$H_1(f) - H_1(f_\infty) = \int_{\mathbb{R}^d} f \lg f - f \lg f_\infty = \int_{\mathbb{R}^d} \frac{f}{f_\infty} \lg\left(\frac{f}{f_\infty}\right) \cdot f_\infty$$

$$\doteq \left| \begin{array}{l} \phi(s) := s \cdot \lg s \\ \phi \text{ je konvexní} \\ \quad \text{na } (0,+\infty) \end{array} \right| = \int_{\mathbb{R}^d} \phi\left(\frac{f}{f_\infty}\right) \cdot f_\infty \, d\nu = \left(= H_\phi(f) \right)$$

$$\phi'(s) = \lg s + 1$$

$$\phi''(s) = \frac{1}{s}$$

$$= \left| \int_{\mathbb{R}^d} f_\infty \, d\nu = 1 \right| \overset{\text{Jensen}}{\geq} \phi\Bigg(\underbrace{\int_{\mathbb{R}^d} f \, d\nu}_{1} \Bigg) = 0$$

$f_\infty \, d\nu$ .... pravděpodobnostní

$$\Rightarrow H_1(f) - H_1(f_\infty) \geq 0$$

slabá formulace:

$$\frac{d}{dt}\int_{\mathbb{R}^d} f\phi = -\frac{1}{4}\int_{\mathbb{R}^d}\int_{\mathbb{R}^d}\int_{S^{d-1}} B(v-w,n)\left[f(v^*) f(w^*) - f(v) f(w)\right] \times$$

$$\times\left[\phi(v^*) + \phi(w^*) - \phi(v) - \phi(w)\right] dn\, dv\, dw$$

$\phi := \lg f$:

$$\frac{d}{dt}\left(H_1(f)\right) = -\frac{1}{4}\iiint B(v-w,n)\left[f(v^*) f(w^*) - f(v) f(w)\right] \times$$

$$\left[\lg\big(\underbrace{f(v^*) f(w^*)}_{A}\big) - \lg\big(\underbrace{f(v)\cdot f(w)}_{B}\big)\right] \geq 0$$

$$(A-B)\cdot\left(\lg(A) - \lg(B)\right) \geq 0$$