ad I) $\displaystyle\int \frac{1+2x}{1+\sqrt{x}}\,dx = \int \frac{1+2x}{1+\sqrt{x}}\cdot 2\sqrt{x}\cdot\frac{1}{2\sqrt{x}}\,dx$

Použijeme 1. větu o substituci: $\varphi(x)=\sqrt{x}$; $\varphi'(x)=\frac{1}{2\sqrt{x}}$, $f(t) = \ldots$

$$f(t) = \frac{1+2t^2}{1+t}\cdot 2t$$

Spočteme $\displaystyle\int f(t)\,dt = \int \frac{4t^3+4t^2-4t^2-4t+6t+6-6}{t+1}\,dt =$

$$= \int 4t^2-4t+6-\frac{6}{t+1}\,dt \overset{c}{=} \frac{4}{3}t^3-2t^2+6t-6\lg|t+1| \text{ na } (-\infty,-1) \text{ a } (-1,+\infty)$$

Hledaná p. f. je $\frac{4}{3}x^{3/2}-2x+6\sqrt{x}-6\lg(\sqrt{x}+1)$ na $(0,+\infty)$, protože $\sqrt{x}>0$ na $(0,+\infty)$.

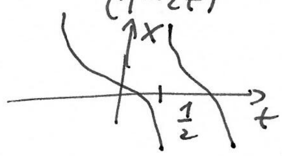
ad II) $\displaystyle\int \frac{1}{x^2}\lg x\,dx = -\frac{\lg x}{x}+\int\frac{1}{x^2}\,dx = -\frac{\lg x}{x}-\frac{1}{x}$ na $(0,+\infty)$

$\downarrow i$ $\quad -\frac{1}{x}$ $\qquad \downarrow d$ $\quad \frac{1}{x}$

ad III) Eulerovou substitucí

$$\sqrt{x^2+x+1} = x+t$$
$$\cancel{x^2}+x+1 = \cancel{x^2}+2tx+t^2$$
$$x(1-2t) = t^2-1$$
$$x = \frac{t^2-1}{1-2t} = \varphi(t)$$

$$\varphi'(t) = \frac{2t(1-2t)+2(t^2-1)}{(1-2t)^2} = \frac{-2t^2+2t-2}{(1-2t)^2} < 0$$



Volíme $\varphi: (\frac{1}{2},+\infty)\to\mathbb{R}$. Pak $\varphi_{-1}(x) = \sqrt{x^2+x+1}-x$ protože $\varphi(1)=0$.

Pokusíme se tedy nalézt $\displaystyle\int \frac{1}{2\frac{t^2-1}{1-2t}+t}\cdot\frac{-2t^2+2t-2}{(1-2t)^2}\,dt$

$$= \int \frac{-2t^2+2t-2}{(-2+t)(1-2t)}\,dt$$

Je potřeba při dosazování za t dát pozor na bod $t=2$ tj. $x=-1$.