

10. sa'p'itorn' p'isemla

Najdete primitivni funkciu: $\int \frac{\sin x (1 + \cos x)}{1 + \cos^2 x} dx = P$

Resen': Nalemeno najch'ne

$$\int \frac{1+t}{1+t^2} dt = \frac{1}{2} \int \frac{2t}{1+t^2} dt + \int \frac{1}{1+t^2} dt = \frac{1}{2} \ln |1+t^2| + \arctan t \quad \text{na } \mathbb{R}$$

Patle 1. u'et' a substitucij: $P = -\frac{1}{2} \ln |1 + \cos^2 x| + \arctan \cos x \quad \text{na } \mathbb{R}$

$$= -\frac{1}{2} \ln |1 + \cos^2 x| + \arctan(\cos x) \quad \text{na } \mathbb{R}$$

11. sa'p'itorn' p'isemla - mat 059 - 2013

Najdete nejve maximalni intervaly monotonicity pre f, f' .

$$f(x) = \frac{3-x}{x^2+2x+1}$$

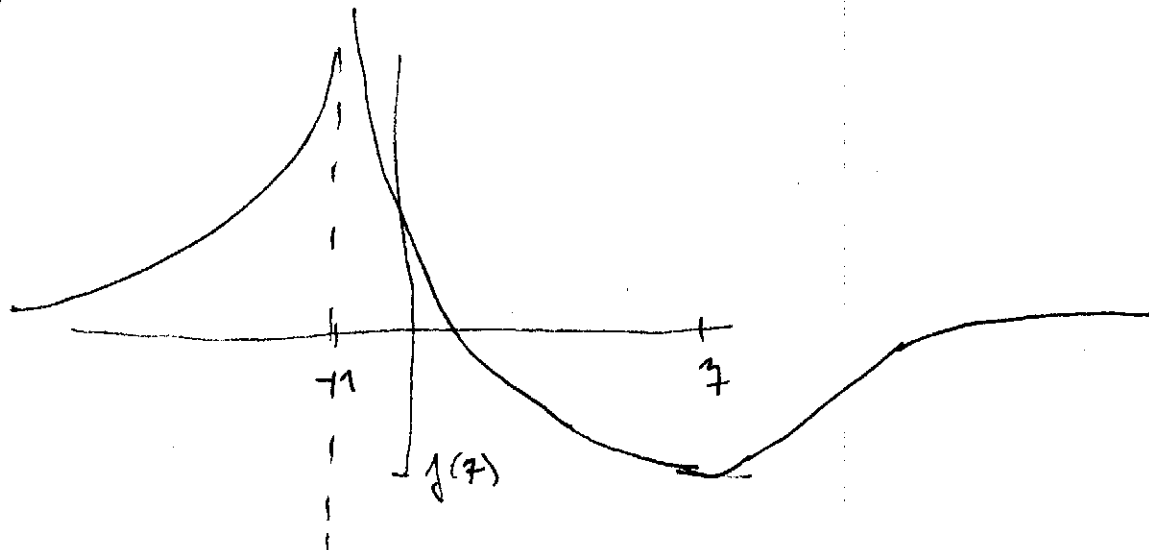
Resen': 1) $D(f) = \mathbb{R} \setminus \{-1\}$ proto $x^2+2x+1 = (x+1)^2$

$$2) \text{ na } D(f): f'(x) = \frac{-(x^2+2x+1) - (3-x)(2x+2)}{(x^2+2x+1)^2} = \frac{-x^2-2x-1-6x-6+2x^2+2x}{(x^2+2x+1)^2}$$

$$= \frac{x^2-6x-7}{(x^2+2x+1)^2} = \frac{(x-7)(x+1)}{(x^2+2x+1)^2}$$

$\Rightarrow f' > 0$ na $(-\infty, -1) \cup (7, +\infty)$ a $f' < 0$ na $(-1, 7)$

$\Rightarrow f$ je rostouci na $(-\infty, -1) \cup (7, +\infty)$ a klesajici na $(-1, 7)$



12. súčtová funkcia - máj 051-2013

$$\text{Najdite } T_{0,5}^{\uparrow}(x), \text{ je-li } f(x) = \frac{x^2 - x^3}{x^2 + 3x + 2}$$

$$\text{Riešenie: } f(x) = x^2 \frac{1-x}{x^2+3x+2} \quad \text{Derivácia } g(x) = \frac{1-x}{x^2+3x+2}$$

$$g(0) = \frac{1}{2}; \quad g'(x) = \frac{-(x^2+3x+2) - (1-x)(2x+3)}{(x^2+3x+2)^2} = \frac{-x^2-3x-2-2x-3+2x^2+3x}{(x^2+3x+2)^2}$$

$$= \frac{x^2 - 2x - 5}{(x^2+3x+2)^2}; \quad g'(0) = -\frac{5}{4}$$

$$g''(x) = \frac{(2x-2)(x^2+3x+2)^2 - (x^2-2x-5) \cdot 2 \cdot (x^2+3x+2)(2x+3)}{(x^2+3x+2)^4}$$

$$g''(0) = \frac{(-2) \cdot 4 - (-5) \cdot 2 \cdot 2 \cdot 3}{2^4} = \frac{-8 + 60}{16} = \frac{52}{16}$$

$$\begin{aligned} \text{Tedy } T_{0,5}^{\uparrow}(x) &= x^2 \left(\frac{1}{2} - \frac{5}{4}x + \frac{52}{16} \cdot \frac{1}{2} \cdot x^2 \right) = \\ &= \frac{1}{2}x^2 - \frac{5}{4}x^3 + \frac{52}{32}x^4 \end{aligned}$$