5. zápočtová písemka – nmaf051 – 14.11.2012

Spočtěte $\lim\limits_{x\to 0} \dfrac{\sqrt{\cos x-\frac{1}{2}}-\sqrt{\frac{1}{2}}}{x^2} = \lim\limits_{x\to 0} \dfrac{\cos x-\frac{1}{2}-\frac{1}{2}}{x^2\left(\sqrt{\cos x-\frac{1}{2}}+\sqrt{\frac{1}{2}}\right)} =$

$\overset{AL}{=} \lim \dfrac{\cos x-1}{x^2}\cdot\dfrac{1}{2\sqrt{\frac{1}{2}}} \underset{\uparrow}{=} -\dfrac{1}{2\sqrt{2}}$

známá limita

1.

6. zápočtová písemka – nmaf051 – 21.11.2012

Buď $f(x) = \begin{cases} \sin x & \text{pro } x \geq 0 \\ x & \text{pro } x < 0 \end{cases}$.

Rozhodněte, zda existuje $f'(0)$. Pokud ano, spočtěte ji.

Řešení: Máme spočítat $\lim\limits_{x\to 0} \frac{f(x)}{x} = L$ pokud existuje.

Protože $\lim\limits_{x\to 0-} \frac{x}{x} = 1$ a $\lim\limits_{x\to 0+} \frac{\sin x}{x} = 1$ a $x|_{x=0} = \sin x|_{x=0} = 0$

existuje L a $f'(0) = 1$.

7. zápočtová písemka – nmaf051 – 28.11.2012

Najděte $D(f)$, $D(f')$, a f', je-li $f(x) = \sqrt[3]{\sin^2 x}$

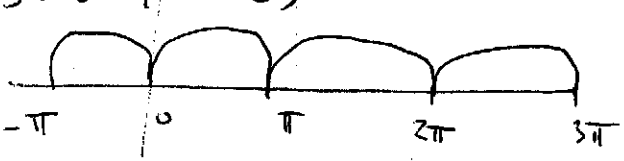
$D(f) = \mathbb{R}$; pro $x \neq 0 + k\pi, k\in\mathbb{Z}$: $f'(x) = \frac{2}{3}\sin^{-\frac{1}{3}} x \cos x$

a tedy $D(f') \subset \mathbb{R}\setminus(\{0\}\cup\{k\pi, k\in\mathbb{Z}\})$

Jak je tomu v 0? $\lim\limits_{x\to 0\pm} \dfrac{\sqrt[3]{\sin^2 x}}{x} = \lim\limits_{x\to 0\pm} \underbrace{\sqrt[3]{\dfrac{\sin^2 x}{x^2}}}_{\to 1}\cdot\underbrace{\dfrac{x^{2/3}}{x}}_{=x^{-1/3}} = \pm\infty$

Tedy f' v 0 neex. a $D(f') = \mathbb{R}\setminus(\{0\}\cup\{k\pi, k\in\mathbb{Z}\})$

a v $k\pi$, $k\in\mathbb{Z}$ taktéž!



8. zápočtová písemka – nmaf051 – 3.12.2012

Spočtěte primitivní funkci na maximální intervalech

$$\int x^2 \sin x\,dx = \left(-x^2\cos x\right) + 2\int x\cos x\,dx =$$

$$\begin{array}{cccc} \downarrow d & \downarrow i & \downarrow d & \downarrow i \\ 2x & -\cos x & 1 & \sin x \end{array}$$

$$-x^2\cos x + 2x\sin x - 2\int \sin x\,dx =$$

$$-x^2\cos x + 2x\sin x + 2\cos x \quad \text{na } \mathbb{R}.$$

9. zápočtová písemka – nmaf051 – 12.12.12

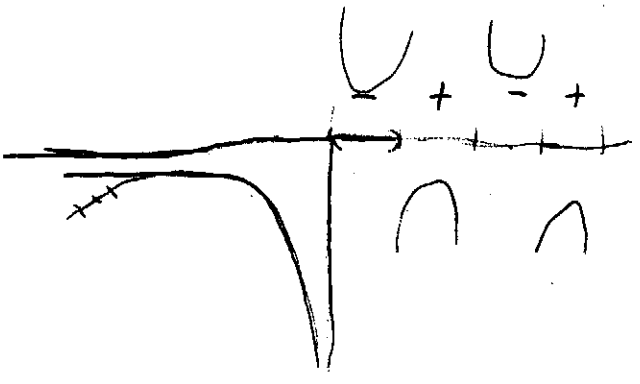
Spočtěte primitivní fci na max. intervalech

$$\int \frac{12x+20}{12x^2+12x+2012}\,dx = \frac{1}{2}\int \frac{24x+12}{12x^2+12x+2012}\,dx + \int \frac{14}{12x^2+12x+2012}\,dx$$

$$= \lg\sqrt{12x^2+12x+2012} + \frac{14}{12}\int \frac{1}{\left(x+\frac{1}{2}\right)^2 + \underbrace{\frac{2012}{12} - \frac{1}{4}}_{\frac{2009}{12}}}\,dx = \lg\sqrt{} +$$

$$\frac{14}{12}\,\frac{1}{\sqrt{\frac{2009}{12}}}\cdot\int \frac{1}{\sqrt{\frac{2009}{12}}}\,\frac{1}{\left(\frac{\sqrt{12}\left(x+\frac{1}{2}\right)}{\sqrt{\frac{2009}{12}}}\right)^2+1}\,dx =$$

$$= \lg\sqrt{12x^2+12x+2012} + \frac{14}{\sqrt{12}\cdot\sqrt{2009}}\cdot \operatorname{arctg}\frac{\sqrt{12}\,x+\sqrt{3}}{\sqrt{2009}} \quad \text{na } \mathbb{R}$$



$$\frac{a}{(b+a)!} = (a)!$$

$$(a)!\cdot a = (b+a)!$$

$$\frac{(b+a+k)!\,k!}{b} \qquad \sum_{k=0}^{a-b} (-1)^k$$