

Limity funkcí II

Základní limity

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \quad \lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1 \quad \lim_{x \rightarrow 0} \frac{\ln(1+x)}{x} = 1$$

Pro výpočet limit typu “ 1^∞ ”:

$$\lim_{x \rightarrow a} (f(x))^{g(x)} = e^{\lim_{x \rightarrow a} g(x) \ln(f(x))}.$$

Příklady

1. $\lim_{x \rightarrow 0^+} \frac{\arccos(1-x)}{\sqrt{x}}$
2. $\lim_{x \rightarrow 0^+} \frac{\left(\frac{\pi}{2} - \arcsin \frac{1}{\sqrt{x^2+1}}\right)}{x}$
3. $\lim_{x \rightarrow 0} \frac{\ln \cos ax}{\ln \cos bx}$, $a, b \in R$
4. $\lim_{x \rightarrow 0} \frac{\ln(a+x) + \ln(a-x) - 2 \ln a}{x^2}$, $a > 0$
5. $\lim_{x \rightarrow 0} \frac{\ln(\operatorname{tg}(\frac{\pi}{4} + ax))}{\sin bx}$, $a, b \in R$
6. $\lim_{x \rightarrow 0^+} \ln(x \ln a) \ln\left(\frac{\ln ax}{\ln \frac{x}{a}}\right)$, $a > 0$
7. $\lim_{x \rightarrow 0} \frac{\ln(1+xe^x)}{\ln(x + \sqrt{1+x^2})}$
8. $\lim_{x \rightarrow 1} (1-x) \log_x 2$
9. $\lim_{x \rightarrow 0^+} (1+x)^{\frac{1}{x}}$
10. $\lim_{x \rightarrow \frac{\pi}{2}} (\sin x)^{\operatorname{tg} x}$

11. $\lim_{x \rightarrow 0} \left(\frac{1 + \operatorname{tg} x}{1 + \sin x} \right)^{\frac{1}{\sin^3 x}}$
12. $\lim_{x \rightarrow 1} (1 + \sin \pi x)^{\operatorname{cotg} \pi x}$
13. $\lim_{x \rightarrow 0^+} (\cos \sqrt{x})^{\frac{1}{x}}$
14. $\lim_{x \rightarrow 0} (1 + x^2)^{\operatorname{cotg} \pi x}$
15. $\lim_{x \rightarrow \frac{\pi}{4}} (\operatorname{tg} x)^{\operatorname{tg} 2x}$
16. $\lim_{x \rightarrow 1} \frac{\sin \pi x^\alpha}{\sin \pi x^\beta}, \alpha, \beta \in R$
17. $\lim_{x \rightarrow 0} \frac{e^{\alpha x} - e^{\beta x}}{\sin \alpha x - \sin \beta x}, \alpha, \beta \in R$
18. $\lim_{x \rightarrow a} \frac{a^x - x^a}{x - a}, a \in R^+$
19. $\lim_{x \rightarrow 0} \left(\frac{1 + x2^x}{1 + x3^x} \right)^{\frac{1}{x^2}}$
20. $\lim_{x \rightarrow 0} \left(\frac{a^{x^2} + b^{x^2}}{a^x + b^x} \right)^{\frac{1}{x}}, a, b \in R^+$