DÚ 4 MAFOBA 041
3,4/ca

uzávěr grafu $f(x) = \begin{cases} \sin 1/x & x \neq 0 \\ 0 & x = 0 \end{cases}$

$G = \{(x,y) \in \mathbb{R}^2 :$ kde $y = \sin \frac{1}{x}$ pokud $x \neq 0$ nebo $y = 0$ pokud $x = 0\}$

$\overline{G} = G \cup \{(0,y); y \in [-1,1]\} \overset{\text{ozn.}}{=} H$

Dk: $x \in H$ ukážu, že $x \in \overline{G}$

stačí $\forall U_\varepsilon(x)$ ukázat $U_\varepsilon(x) \cap G \neq \emptyset$

Vol $\varepsilon > 0$ $\quad \bar{x} \in G$ jasné

$\bar{x} \in \{(0,y); y \in [-1,1]\}$

$\bar{x} = (0, y')$; $y' \in [-1,1]$

Hledám bod $\bar{z} \in G \cap U_\varepsilon(\bar{x})$. Bod $\bar{z}$ hledám na úsečce $\{(x, y'), x \in (0,\varepsilon)\} \subset U_\varepsilon(\bar{x})$

Najdu $k \in \mathbb{N}$ tak, aby $\left(\frac{1}{2\pi k + \pi}, \frac{1}{2\pi k - \pi}\right) \subset (0, \varepsilon)$.

$\sin \frac{1}{\frac{1}{2\pi k + \pi}} = 1$ a $\sin \frac{1}{\frac{1}{2\pi k - \pi}} = -1$ Protože je sin spojitá

má Darbouxovu vlastnost $\forall y' \in [-1,1] \Rightarrow \exists x' \in \left(\frac{1}{2\pi k + \frac{\pi}{2}}, \frac{1}{2\pi k - \frac{\pi}{2}}\right)$:

: $\sin \frac{1}{x'} = y' \Rightarrow (x', y') \in G \cap U_\varepsilon(\bar{x}) \Rightarrow H \subset \overline{G}$

? $\bar{x} \notin H \Rightarrow \bar{x} \notin \overline{G}$ $\quad$ Je potřeba nalézt $U_\varepsilon(\bar{x})$ tak aby $U_\varepsilon(\bar{x}) \cap G = \emptyset$

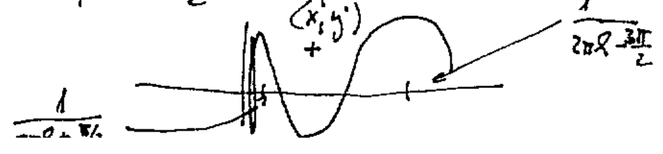
$\bar{x} = (x', y')$ $\quad$ snadné pokud $|y'| > 1$.

Pokud $|y'| \leq 1$. Bůno $x' > 0$. $\exists k \in \mathbb{N}$: $x' \in \left(\frac{1}{2k\pi + \frac{\pi}{2}}, \frac{1}{2k\pi - \frac{3\pi}{2}}\right)$

$\sin \frac{1}{x'} < y'$

značí

$\varepsilon = \operatorname{dist}((x', y'), G \cap \{(x,y) \in \mathbb{R}; x \in \left(\frac{1}{2\pi k + \frac{\pi}{2}}, \frac{1}{2\pi k - \frac{3\pi}{2}}\right)\}) > 0$. Pak $U_\varepsilon(\bar{x}) \cap G = \emptyset$



Teď:

$\bar{x} \notin H \Rightarrow$ buď $\bar{x} = (x', y')$, $|y'| > 1$ snadné

nebo $|y'| < 1 \wedge |x'| > 0$

Pak ovšem $K := G \cap \{(x, y) \in \mathbb{R}^2, |x| \geq \frac{|x'|}{2}\}$ je uz.

protože $\sin \frac{1}{x}$ je spojitá na $[\frac{|x'|}{2}, +\infty) \cup (-\infty, -\frac{|x'|}{2}]$

$\Rightarrow \exists\, U_\varepsilon(\bar{x}) : U_\varepsilon(\bar{x}) \cap K = \emptyset \quad (\bar{x} \notin K)$.

Volíme $U(\bar{x}) = U_\varepsilon(\bar{x}) \cap U_{\frac{|x'|}{2}}(\bar{x})$ je hledané okolí!

3,4/cb

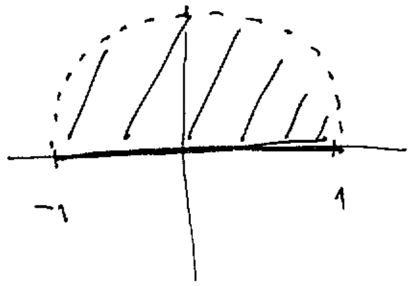
$G := \{(x,y) \in \mathbb{R}^2 :\ y = 1 \text{ pokud } x \in \mathbb{R} \setminus \mathbb{Q},\ y = 0 \text{ pokud } x \in \mathbb{Q}\}$

$\overline{G} = \{(x,y) \in \mathbb{R}^2 : \ \dots\} = \{(x,y) \in \mathbb{R}^2 ; y = 1\} \cup \{(x,y) \in \mathbb{R}^2 : y = 0\}$

3,4/da)

$\overline{\mathbb{Q}} = \mathbb{R}\ ;\ \mathbb{Q}^\circ = \emptyset\ ;\ \partial\mathbb{Q} = \overline{\mathbb{Q}} \setminus \mathbb{Q}^\circ = \mathbb{R}$

db) $G := \{(x,y) \in \mathbb{R}^2 ;\ x^2 + y^2 < 1,\ y \geq 0\}$



$\overline{G} = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1,\ y \geq 0\}$

(je uzavřený vzor uz. mn. při spoj. zobrazení!)

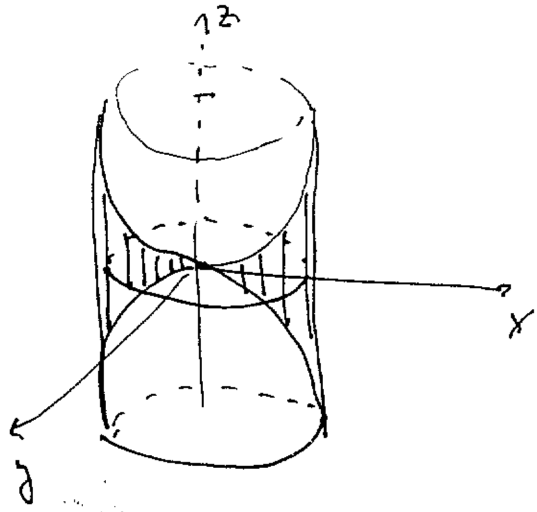
$G^\circ = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 < 1\ ;\ y > 0\}$

$\partial G = \overline{G} \setminus G^\circ = \left(\{(x,y) \in \mathbb{R}^2 : x^2 + y^2 = 1\} \cap \{y \geq 0\}\right) \cup$

$\left(\{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1,\ y = 0\}\right)$

c) $G := \{(x,y,z) \in \mathbb{R}^3 :\ |z| < x^2 + y^2 \leq 1\}$

$\overline{G} = \{|z| \leq x^2 + y^2 \leq 1\}\ ;\ G^\circ = \{|z| < x^2 + y^2 < 1\}$



d) $G := \mathbb{R} \setminus \{ \frac{1}{n} ; n \in \mathbb{N} \}$

$\overline{G} = G \cup \mathbb{R}$

$G^\circ = G \setminus \{0\}$

$\partial G = \{ \frac{1}{n}, n \in \mathbb{N} \} \cup \{0\}$

e) $G = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 < 1\} \setminus \{(x,0) : x \in [-\frac{1}{2}, \frac{1}{2}]\}$

$\overline{G} = \{ x^2 + y^2 \leq 1 \}$

$G = G^\circ$

$\partial G = \{ x^2 + y^2 = 1 \} \cup \{ (x,0) : x \in [-\frac{1}{2}, \frac{1}{2}] \}$