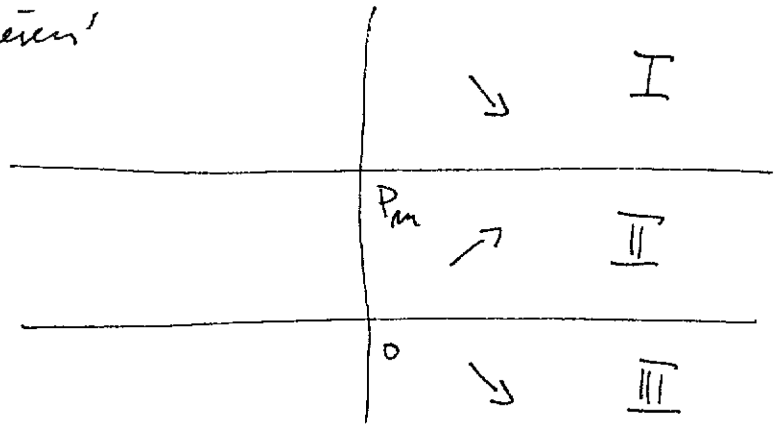
MAF041
2. DÚ

$$y' = \alpha y (P_m - y) = f(x, y) \quad ; \; \alpha > 0, \; P_m > 0$$

1) f spoj. v $\mathbb{R}^2$; $\frac{\partial f}{\partial y}$ spoj. v $\mathbb{R}^2$ $\Rightarrow$ ex. & jedn.

2) monotonie řešení



3) separace & integrace

$$\frac{y'}{\alpha y (P_m - y)} = 1$$

$$\int \frac{1}{y(P_m - y)} \, dy = \frac{1}{P_m} \int \frac{1}{y} + \frac{1}{P_m - y} \, dy = \frac{1}{P_m} \left(\lg|y| - \lg|P_m - y| \right)$$

$$= \frac{1}{P_m} \lg \left| \frac{y}{P_m - y} \right|$$

$$\Rightarrow \lg \left| \frac{y}{P_m - y} \right| = P_m \alpha x - C; \qquad C \in \mathbb{R}$$

4) inverze

$$\left| \frac{y}{P_m - y} \right| = e^{P_m \alpha x} \cdot c \qquad c \in (0, +\infty)$$

v I & $\underline{\text{III}}$: $\frac{-y}{P_m - y} = e^{P_m \alpha x} \cdot c \rightarrow -y = e^{P_m \alpha x} c P_m - c e^{P_m \alpha x} y$

$$y = \frac{c P_m e^{P_m \alpha x}}{-1 + c e^{P_m \alpha x}} \quad ; \; c > 0 \quad \text{řešení na}$$

$\left(-\infty, \frac{1}{\alpha P_m} \lg \frac{1}{c}\right)$ nebo

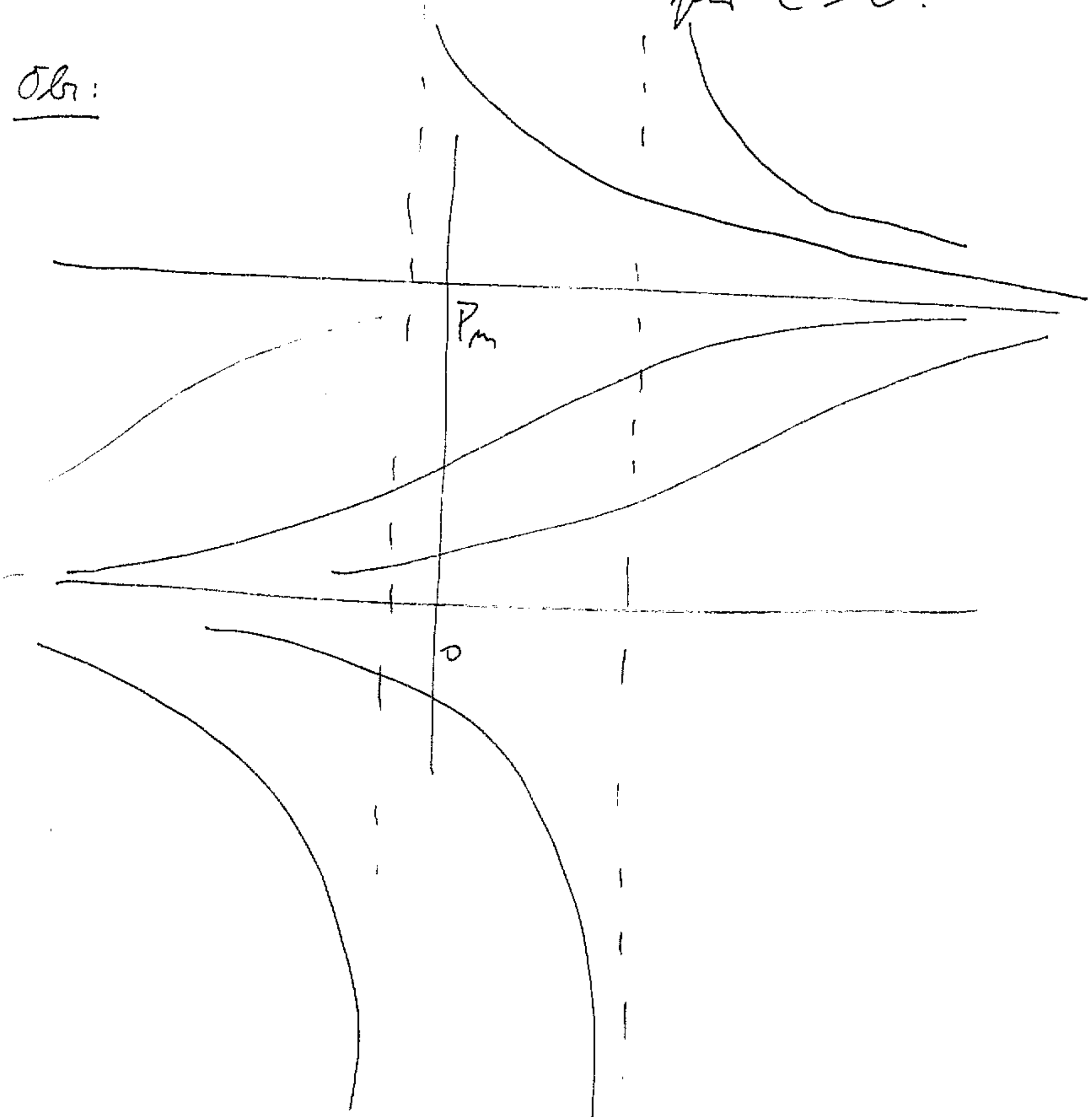
$1 = c e^{P_m \alpha x} \rightarrow \frac{1}{c} = e^{P_m \alpha x}$

na $\left(\frac{1}{P_m \alpha} \lg \frac{1}{c}, +\infty\right)$.

$\rightarrow P_m \alpha x = \lg \frac{1}{c}$; $x = \frac{1}{\alpha P_m} \lg \frac{1}{c}$

II: $y = \dfrac{c P_m e^{\alpha P_m x}}{1 + c e^{\alpha P_m x}}$ je řešení na $x \in \mathbb{R}$ pro $c > 0$.

<u>Obr:</u>



Tedy pokud $y(0) = y_0 \in (0, P_m)$ máme

$$y_0 = y(0) = \frac{c P_m}{1 + c} \Rightarrow y_0 + c y_0 = c P_m$$

$$\longrightarrow \quad c = \frac{y_0}{P_m - y_0}$$ a hledané řešení je

$$y(x) = \frac{y_0}{P_m - y_0} \cdot \frac{P_m e^{\alpha P_m x}}{1 + \frac{y_0}{P_m - y_0} e^{\alpha P_m x}} \quad \text{na } \mathbb{R}$$