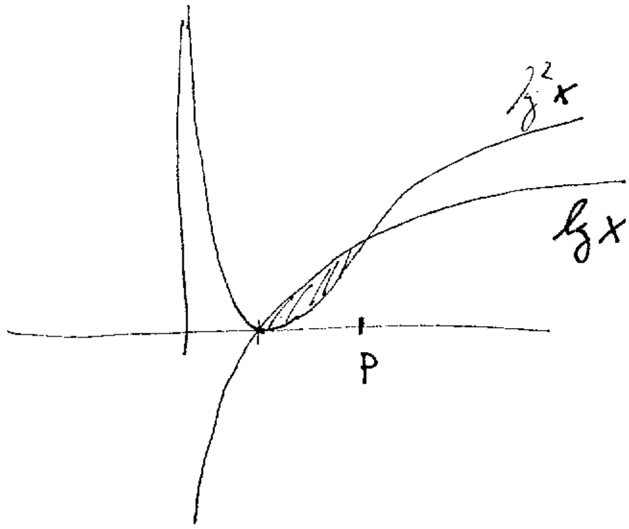
1/2

1. DÚ



$$lg^2 P = lg\,P \Rightarrow lg\,P = 0 \longrightarrow P = 1$$
$$lg\,P = 1 \longrightarrow P = e$$

$$\int_1^e lg\,x - lg^2 x\,dx = I$$

$$\int lg^2 x\,dx = x \cdot lg\,x(lg\,x - 1) - \int lg\,x - 1\,dx = x(lg\,x-1)(lg\,x-1) + x = x((lg\,x-1)^2+1)$$

$\downarrow d$ $\frac{1}{x}$, $\downarrow i$ $x \cdot (lg\,x - 1)$

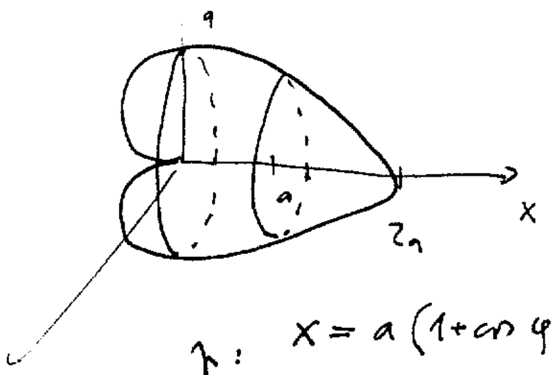
$$\int 1 \cdot lg\,x = x\,lg\,x - 1 = x(lg\,x - 1)$$

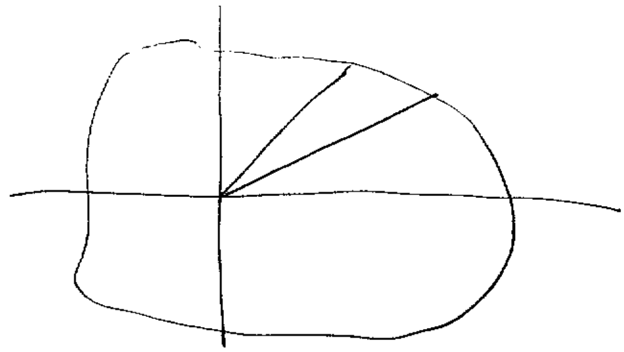
$\downarrow i$ x, $\downarrow d$ $\frac{1}{x}$

$$I = \Big[x(lg\,x-1) - x[(lg\,x-1)^2+1]\Big]_1^e = -e - (-1-2) =$$
$$= 3 - e$$

1/8

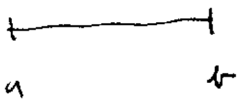
$r = a(1+\cos\varphi)$

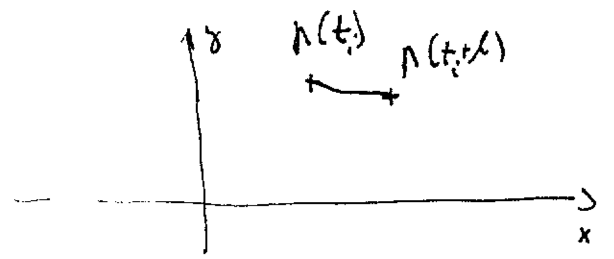




$p: \quad x = a(1+\cos\varphi)\cos\varphi$

$\quad\;\; y = a(1+\cos\varphi)\sin\varphi$





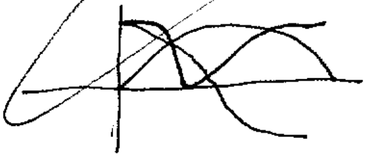
$$\sum \pi |p_2(t)|^2 \cdot \left|\frac{p_1(t+l) - p_1(t)}{l}\right| \cdot l$$

$$\to \pi \int_a^b |p_2(t)|^2 \cdot p_1'(t)\,dt$$

$$V = 2\pi \int_0^{\pi} a(1+\cos\varphi)\sin\varphi \cdot a\left[(1+\cos\varphi)(-\sin\varphi) + \cos\varphi(-\sin\varphi)\right] d\varphi$$

$$= 2\pi a^2 \int_0^{\pi} (1+\cos\varphi)\sin^2\varphi\,(1-2\cos^2\varphi)\,d\varphi = 2\pi a^2 \int_0^{\pi} (1+\cos\varphi)\sin^2\varphi\,d\varphi$$

$$= 2\pi a^2 \left(\frac{\pi}{2} - \left[\frac{\sin^3\varphi}{3}\right]_0^{\pi}\right) = (\pi a)^2$$



Kule: $p = (R\cos\varphi, R\sin\varphi)$ $\quad -\int_0^{\pi} \pi R^2 \sin^2\varphi \cdot R\sin\varphi = \pi R^3 \cdot \int_0^{\pi} \sin^3\varphi\,d\varphi$

$$= \pi R^3 \int_0^{\pi} \sin\varphi(1-\cos^2\varphi)\,d\varphi = -\pi R^3 \left[\cos\varphi - \frac{\cos^3\varphi}{3}\right]_0^{\pi} = -2\pi R^3\left(1-\frac{1}{3}\right) = -\frac{4}{3}\pi R^3$$

$$\pi \int_0^{\pi} a^2(1+\cos\varphi)^2 \sin^2\varphi \; a\Big[\underbrace{(1+\cos\varphi)(-\sin\varphi) + (-\sin\varphi)\cos\varphi}_{-\sin\varphi\,(1+2\cos\varphi)}\Big]\,d\varphi$$

$$(1+\cos\varphi)^2 = 1+2\cos\varphi+\cos^2\varphi$$

$$= -\pi \int_0^{\pi} a^3\left(1+\cos^2\varphi+2\cos\varphi\right)\sin^3\varphi\,(1+2\cos\varphi)\,d\varphi =$$

$$= -\pi a^3 \int_0^{\pi} (1+\cos^2\varphi)\cdot\sin^3\varphi\,(1+2\cancel{\cos\varphi})\,d\varphi$$

$$-\pi a^3 \int_0^{\pi} 2\cos\varphi\,\sin^3\varphi\,(\cancel{1}+2\cos\varphi)\,d\varphi$$

$$= -\pi a^3 \int_0^{\pi} \sin^3\varphi\,d\varphi - \pi a^3 \int_0^{\pi} \sin^3\varphi\cdot 5\cos^2\varphi\,d\varphi =$$

$$-\pi a^3 \int_0^{\pi} \sin\varphi\left(1-\cos^2\varphi + (1-\cos^2\varphi)\cdot 5\cos^2\varphi\right)d\varphi =$$

$$-\pi a^3 \int_0^{\pi} \sin\varphi\left(1+4\cos^2\varphi-5\cos^4\varphi\right)d\varphi =$$

$$+\pi a^3\left[\cos\varphi + 4\frac{\cos^3\varphi}{3} - \cos^5\varphi\right]_0^{\pi} = -2\pi a^3\left(1+\frac{4}{3}-1\right) =$$

$$= -\frac{8}{3}\pi a^3$$

1/12

$$\gamma(t) = \begin{pmatrix} a(\cos t + t \sin t) \\ a(\sin t - t \cos t) \end{pmatrix}$$

$$\gamma'(t) = \left(a(-\sin t + \sin t + t\cos t),\ a(\cos t - \cos t + t \sin t)\right)$$

$$= (a t \cos t,\ a t \sin t)$$

$$|\gamma'(t)| = a t$$

$$\ell = \int_0^{2\pi} a t \, dt = \left[\frac{a t^2}{2} \right]_0^{2\pi} = \frac{4\pi^2 a}{2} = 2\pi^2 a$$