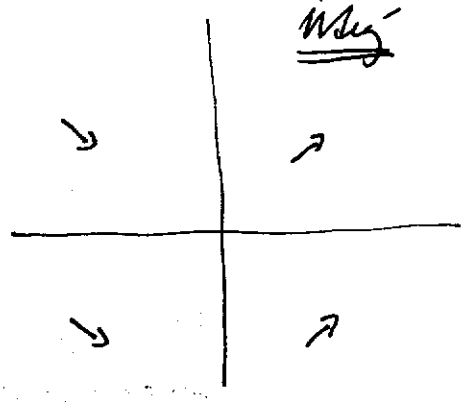
$x\,y' = \sqrt{|y|}$ — pozor na $x=0$!

$y' = \frac{\sqrt{|y|}}{x}$

hor. řešení $y=0$ na $\mathbb{R}$

ex. + jedn. $x \neq 0$; $y \neq 0$



Separace ($y>0$)

$\frac{y'}{\sqrt{|y|}} = \frac{1}{x} \rightsquigarrow (2\sqrt{y})' = (\ln(|x|\cdot c))'$ $\quad c>0$ 1/2

$2\sqrt{y} = \frac{1}{2}\ln(c\cdot|x|) \longrightarrow c\cdot|x|>1 \quad |x|>\frac{1}{c}$

$y_c = \left(\frac{1}{2}\ln(c\cdot|x|)\right)^2$ na $(-\infty, -\frac{1}{c})$ nebo na $(\frac{1}{c}, +\infty)$ 1/2

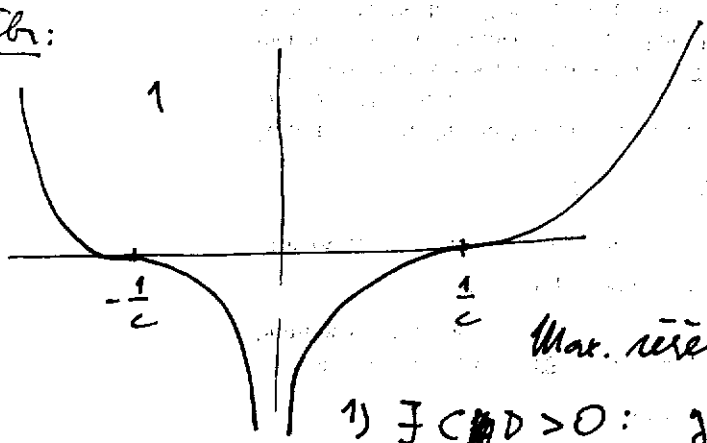
($y<0$): $\frac{y'}{\sqrt{-y}} = (-2\sqrt{-y})' = (\ln(|x|\cdot c))'$ 1/2

$\sqrt{-y} = -\frac{1}{2}\ln(c\cdot|x|) \longrightarrow c\cdot|x|<1$

$-y = \left(\frac{1}{2}\ln(c\cdot|x|)\right)^2$

$y_c(x) = -\left(\frac{1}{2}\ln(c\cdot|x|)\right)^2$ na $(-\frac{1}{c}, 0)$ nebo na $(0, \frac{1}{c})$ 1/2

Obr:



Diskuse: v $\frac{1}{c}$ lze napojit hor. řešení s řešením y_c 1

pokud $x=0 \Rightarrow y \neq 0$

Max. řešení na $\mathbb{R}$:

1) $\exists\, c \neq D > 0$: $y = \begin{cases} \left(\frac{1}{2}\ln(c|x|)\right)^2 & \text{na } (-\infty, -\frac{1}{c}) \\ 0 & \text{na } [-\frac{1}{c}, \frac{1}{D}] \\ \left(\frac{1}{2}\ln(D|x|)\right)^2 & \text{na } (\frac{1}{D}, +\infty) \end{cases}$ 1

$y' + y \operatorname{tg} x = \frac{1}{\cos x}$ na $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

ij: $\exp\left(\int \operatorname{tg} x\right) = \exp(-\ln|\cos x|) = +\frac{1}{|\cos x|}$ 1

$\left(\frac{y}{\cos x}\right)' = \frac{y'}{\cos x} + \left(-\frac{y}{\cos^2 x}\right)(-\sin x) = \frac{1}{\cos^2 x} = (\operatorname{tg} x + C)'$ 2 1

$y(x) = \sin x + c \cdot \cos x$ je řešení $\forall c \in \mathbb{R}$ na

$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) + k\pi$, $k \in \mathbb{Z}$ 1

zk: $\cos x + \frac{\sin^2 x}{\cos x} = \frac{1}{\cos x}$

$-\sin x + \sin x = 0$ ✓

$y^{(3)} + y' = \sin x + x \cos x$

$\lambda^3 + \lambda = 0$ 1

$(\lambda^2 + 1)\lambda = 0 \Rightarrow \lambda = 0, \pm i \Rightarrow$ f.s: $1, \sin x, \cos x$ 1

řešení hledáme ve tvaru:

$(Ax^2 + Bx)(\alpha \sin x + \beta \cos x)$ 1