Written Exam on Mathematics II for IES FSV UK (B)

Summer Semester 2012-2013

Problem 1: Compute determinants of the matrices $\mathbb{A}$ and $\mathbb{B}$, where $\mathbb{A}$ is given below and $\mathbb{B}$ is made from $\mathbb{A}$ by mutliplying all the entries by $-\frac{1}{2}$.

$$\mathbb{A} = \begin{pmatrix} 1 & 2 & 4 & 8 & 16 \\ 2 & 3 & 5 & 9 & 17 \\ 3 & 5 & 8 & 13 & 20 \\ 2 & 4 & 8 & 8 & 16 \\ 16 & 8 & 4 & 2 & 1 \end{pmatrix} \qquad \text{(10 points)}$$

Problem 2: Determine and draw the domain of the function

$$f(x, y) = (x^2 + y^2 - 4)^{\sqrt{xy}},$$

compute its partial derivatives with respect to all the variables at all points where they exist.

(10 points)

Problem 3: Let us consider the equation

$$e^{x+y^2} - \cos(x + y) = 0$$

and the point $[-1, 1]$. Show that this equation defines a C^∞ function $y = f(x)$ defined on a neighborhood of -1, which satisfies $f(-1) = 1$. Compute $f'(-1)$, $f''(-1)$ and determine the equation of the tangent line to the graph of f at the point $[-1, f(-1)]$. (10 points)

Problem 4: Determine sup and inf of the function f on the set M and decide whether these values are attained, if

$$f(x, y, z) = xy \text{ and } M = \{[x, y, z] \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 23, y + z \geq 4\} \qquad \text{(15 points)}$$

Problem 5: Compute the following antiderivative on maximal possible intervals:

$$\int \frac{2x^5 + 54}{x^5 - 2x^4 + 3x^3} \,\mathrm{d}x \qquad \text{(15 points)}$$

Answers to the Written Exam on Mathematics II for IES FSV UK (B)
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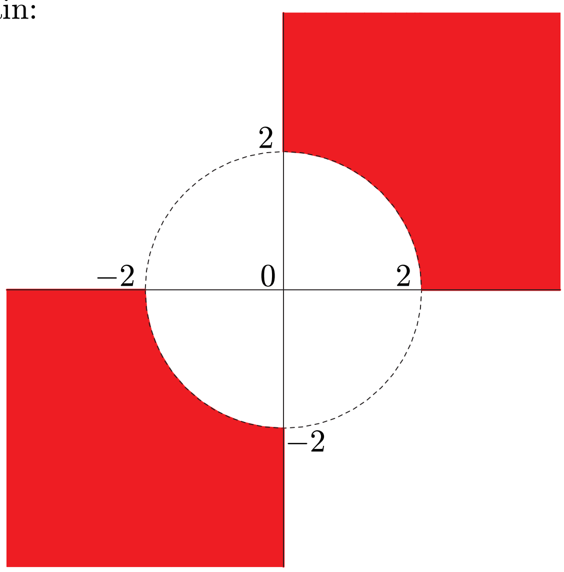
Problem 1: $\det \mathbb{A} = 312$, $\det B = -39/4$.

Problem 2: $D_f = \{[x,y] \in \mathbf{R}^2 : x^2+y^2 > 4 \ \&(x \geq 0, y \geq 0 \text{ or } x \leq 0, y \leq 0)\}$.

Picture of the domain:



$\frac{\partial f}{\partial x}(x,y) = (x^2+y^2-4)^{\sqrt{xy}}\left(\frac{1}{2\sqrt{xy}} \cdot y \cdot \log(x^2+y^2-4) + \sqrt{xy} \cdot \frac{1}{x^2+y^2-4} \cdot 2x\right)$ and $\frac{\partial f}{\partial y}(x,y) = (x^2+y^2-4)^{\sqrt{xy}}\left(\frac{1}{2\sqrt{xy}} \cdot x \cdot \log(x^2+y^2-4) + \sqrt{xy} \cdot \frac{1}{x^2+y^2-4} \cdot 2y\right)$; both partial derivatives on the set $\{[x,y] \in \mathbf{R}^2 : x^2+y^2 > 4 \ \&(x > 0, y > 0 \text{ or } x < 0, y < 0)\}$. At points $[0,y]$, $y \in (-\infty,-2) \cup (2,+\infty)$, the partial derivative with respect to x has no sense (as no horizontal segment centered at $[0,y]$ belongs to D_f) and the partial derivative with respect to y is zero. At points $[x,0]$, $x \in (-\infty,-2) \cup (2,+\infty)$ the partial derivative with respect to y has no sense (as no vertical segment centered at $[x,0]$ is contained in D_f and the partial derivative with respect to x is zero.

Problem 3: $f'(-1) = -\frac{1}{2}$, $f''(-1) = -\frac{3}{8}$, tangent line $y = 1 - \frac{1}{2}(x+1)$.

Problem 4: Maximum $\frac{7\sqrt{21}}{2\sqrt{2}}$ at the point $[\sqrt{\frac{21}{2}}, \frac{7}{2}, \frac{1}{2}]$; minimum $-\frac{7\sqrt{21}}{2\sqrt{2}}$ at the point $[-\sqrt{\frac{21}{2}}, \frac{7}{2}, \frac{1}{2}]$.

Problem 5: $\int \frac{2x^5+54}{x^5-2x^4+3x^3}\,\mathrm{d}x \stackrel{c}{=} 2x - \frac{9}{x^2} - \frac{12}{x} + 2\log|x| + \log(x^2-2x+3) - 6\sqrt{2}\,\mathrm{arctg}\,\frac{x-1}{\sqrt{2}}$ on each of the two intervals $(-\infty,0)$ and $(0,+\infty)$.