V. Spektrum operátoru

Pro následující operátory $T \in L(X)$ určete $\sigma(T)$ a $\sigma_p(T)$.

1. $X = \ell^2$, $T((x_n)) = (0, x_1, x_2, \dots)$. **2.** $X = \ell^2$, $T((x_n)) = (x_2, x_3, x_4, \dots)$.

3. $X = \ell^2$, $T((x_n)) = (0, x_2, 0, x_4, \dots)$. **4.** $X = \ell^2$, $T((x_n)) = (\frac{1+i}{n} x_n)_{n=1}^\infty$.

5. $X = c_0$, $T((x_n)) = (\frac{1}{n} x_n)_{n=1}^\infty$. **6.** $X = c_0$, $T((x_n)) = (x_1, \frac{1}{2}x_2, x_3, \frac{1}{4}x_4, x_5, \frac{1}{6}x_6, \dots)$.

7. $X = \ell^2$, $T((x_n)) = (\frac{n+1}{n} x_n)_{n=1}^\infty$. **8.** $X = \ell^2$, $T((x_n)) = (\frac{n}{n+1} x_n)_{n=1}^\infty$.

9. $X = c_0$, $T((x_n)) = (\frac{x_1 + \dots + x_n}{n})_{n=1}^\infty$.

10. $X = \ell^1$, $T((x_n)) = (q_n x_n)_{n=1}^\infty$, kde $\{q_n; n \in \mathbb{N}\} = \mathbb{Q} \cap (0,1)$.

11. $X = \mathcal{C}([0,1])$, $T(f) = f + f(1) - f(0)$. **12.** $X = \mathcal{C}([0,1])$, $T(f)(t) = \int_0^t f,\ t \in [0,1]$.

13. $X = \mathcal{C}([0,1])$, $T(f)(t) = t f(t)$. **14.** $X = \mathcal{C}([0,1])$, $T(f)(t) = (t - \frac{1}{2})^+ \cdot f(t)$.

15. $X = \mathcal{C}([-1,1])$, $T(f)(t) = f(|t|)$. **16.** $X = L^p([0,1])$, kde $p \in [1,\infty]$, $T(f) = \chi_{[0,\frac{1}{2}]} \cdot f$;

17. $X = L^p([0,1])$, kde $p \in [1,\infty]$, $T(f)(t) = (t - \frac{1}{2})^+ \cdot f(t),\ t \in [0,1]$;

18. $X = L^p([0,1])$, kde $p \in [1,\infty]$, $T(f) = (\chi_{[0,\frac{1}{2}]} - \chi_{(\frac{1}{2},1]}) \cdot f$.

19. $X = L^p([0,1])$, kde $p \in [1,\infty]$, $T(f)(t) = t f(t)$.

20. $X = L^p(\mathbb{R})$, kde $p \in [1,\infty]$, $T(f)(t) = f(-t)$. **21.** $X = \mathcal{C}_0(\mathbb{R})$, $T(f)(t) = f(-t)$.

22. $X = L^p(\mathbb{R})$, kde $p \in [1,\infty]$, $T(f)(t) = f(t-1)$. **23.** $X = \mathcal{C}_0(\mathbb{R})$, $T(f)(t) = f(t-1)$.

24. $X = \mathcal{C}_0(\mathbb{R})$, $T(f)(t) = f(2t)$. **25.** $X = L^p(\mathbb{R})$, kde $p \in [1,\infty]$, $T(f)(t) = f(2t)$.

- -

V. Spektrum operátoru

Pro následující operátory $T \in L(X)$ určete $\sigma(T)$ a $\sigma_p(T)$.

1. $X = \ell^2$, $T((x_n)) = (0, x_1, x_2, \dots)$. **2.** $X = \ell^2$, $T((x_n)) = (x_2, x_3, x_4, \dots)$.

3. $X = \ell^2$, $T((x_n)) = (0, x_2, 0, x_4, \dots)$. **4.** $X = \ell^2$, $T((x_n)) = (\frac{1+i}{n} x_n)_{n=1}^\infty$.

5. $X = c_0$, $T((x_n)) = (\frac{1}{n} x_n)_{n=1}^\infty$. **6.** $X = c_0$, $T((x_n)) = (x_1, \frac{1}{2}x_2, x_3, \frac{1}{4}x_4, x_5, \frac{1}{6}x_6, \dots)$.

7. $X = \ell^2$, $T((x_n)) = (\frac{n+1}{n} x_n)_{n=1}^\infty$. **8.** $X = \ell^2$, $T((x_n)) = (\frac{n}{n+1} x_n)_{n=1}^\infty$.

9. $X = c_0$, $T((x_n)) = (\frac{x_1 + \dots + x_n}{n})_{n=1}^\infty$.

10. $X = \ell^1$, $T((x_n)) = (q_n x_n)_{n=1}^\infty$, kde $\{q_n; n \in \mathbb{N}\} = \mathbb{Q} \cap (0,1)$.

11. $X = \mathcal{C}([0,1])$, $T(f) = f + f(1) - f(0)$. **12.** $X = \mathcal{C}([0,1])$, $T(f)(t) = \int_0^t f,\ t \in [0,1]$.

13. $X = \mathcal{C}([0,1])$, $T(f)(t) = t f(t)$. **14.** $X = \mathcal{C}([0,1])$, $T(f)(t) = (t - \frac{1}{2})^+ \cdot f(t)$.

15. $X = \mathcal{C}([-1,1])$, $T(f)(t) = f(|t|)$. **16.** $X = L^p([0,1])$, kde $p \in [1,\infty]$, $T(f) = \chi_{[0,\frac{1}{2}]} \cdot f$;

17. $X = L^p([0,1])$, kde $p \in [1,\infty]$, $T(f)(t) = (t - \frac{1}{2})^+ \cdot f(t),\ t \in [0,1]$;

18. $X = L^p([0,1])$, kde $p \in [1,\infty]$, $T(f) = (\chi_{[0,\frac{1}{2}]} - \chi_{(\frac{1}{2},1]}) \cdot f$.

19. $X = L^p([0,1])$, kde $p \in [1,\infty]$, $T(f)(t) = t f(t)$.

20. $X = L^p(\mathbb{R})$, kde $p \in [1,\infty]$, $T(f)(t) = f(-t)$. **21.** $X = \mathcal{C}_0(\mathbb{R})$, $T(f)(t) = f(-t)$.

22. $X = L^p(\mathbb{R})$, kde $p \in [1,\infty]$, $T(f)(t) = f(t-1)$. **23.** $X = \mathcal{C}_0(\mathbb{R})$, $T(f)(t) = f(t-1)$.

24. $X = \mathcal{C}_0(\mathbb{R})$, $T(f)(t) = f(2t)$. **25.** $X = L^p(\mathbb{R})$, kde $p \in [1,\infty]$, $T(f)(t) = f(2t)$.