Comments on the retake credit test 1 – solutions and evaluation

General comments: I followed the same rules as in the previous tests – to give a partial credit for a reasonable part of the solution and to require at least one half of the points (i.e., at least 1.5 points). Then seven students passed the test and two students failed. (With the original strict rules, only one student would pass.) Two students did not appear although they still need to succesfully pass one more test.

Overall statistics: Five students have passed two tests, nine students passed one test. Each student have passed at least one test, so up to now nobody have failed the credit.

Problem 1

Solution: We need to find all $x \in \mathbb{R}$ such that

$$\operatorname{arctg} |x-1| \leq \frac{\pi}{4}.$$

The domain of arctg is whole real line, arctg is strictly increasing on $\mathbf{R}$ and $\frac{\pi}{4} = \operatorname{arctg} 1$. Therefore the above inequality is equivalent to

$$|x-1| \leq 1,$$

i.e., $-1 \leq x-1 \leq 1$, in other words $0 \leq x \leq 2$.

So, the solution set is $\langle 0, 2 \rangle$.

Evaluation:

Four students provied a complete correct solution. They obtained 1 point.

One student followed the correct way but made a small mistake. This one obtained 0.8 points.

Two students followed the correct way but supposed that arctg is decreasing. They obtained 0.6 points.

One student made the correct computation but claimed that the domain of arctg is $(-\frac{\pi}{2}, \frac{\pi}{2})$. This one obtained 0.6 points.

One student made the correct computation but failed to put the results together. This one obtained 0.6 points.

Problem 2:

Solution: The integer part $[x]$ satisfies the inequality $x-1<[x]\le x$. Since the denominator is positive, we have

$$\frac{(\sqrt[3]{n}-1)(\sqrt[6]{n}-1)}{\sqrt{n}+10}<\frac{[\sqrt[3]{n}]\cdot[\sqrt[6]{n}])}{\sqrt{n}+10}\le\frac{\sqrt[3]{n}\cdot\sqrt[6]{n}}{\sqrt{n}+10}$$

Further,

$$\lim_{n\to\infty}\frac{\sqrt[3]{n}\cdot\sqrt[6]{n}}{\sqrt{n}+10}=\lim_{n\to\infty}\frac{1}{1+\frac{10}{\sqrt{n}}}=1$$

and

$$\lim_{n\to\infty}\frac{(\sqrt[3]{n}-1)\cdot(\sqrt[6]{n}-1)}{\sqrt{n}+10}=\lim_{n\to\infty}\frac{\sqrt{n}-\sqrt[3]{n}-\sqrt[6]{n}+1}{\sqrt{n}+10}=\lim_{n\to\infty}\frac{1-\frac{1}{\sqrt[6]{n}}-\frac{1}{\sqrt[3]{n}}+\frac{1}{\sqrt{n}}}{1+\frac{10}{\sqrt{n}}}=1.$$

Hence, by the sandwich theorem the original limit equals 1.

Evaluation: Three students got 0 points. One of them made no comuputation at all but claimed that the limit is 0 as it is of type $+\infty/+\infty$ (which is a complete nonsense). One of them did not treat the integer part and, moreover, was writing in a way illegible for theirselves, which resulted in misprints. The third one tried to use sandwich theorem but using a completely nonsense estimates.

The other six students tried to use correct estimates. One of them correctly computed the first limit, the second one was computed with mistakes, some more limits were computed instead. The result was 0.7 points. One student provided a correct way, but some steps of computation were skipped, obtained 0.6 points. Two students computed correctly the first limit but the second one was not comuputed at all or computed in a strange way. These two students got 0.5 points. One student in fact computed the first limit (with skipping a step), but the second one is not really computed. The result was 0.4 points. Finally, one student wrote the estimates but instead of using sandwich theorem just the integer part was omitted. The evaluation was 0.2 points.

Problem 3:

Solution: The solution consists in the following parts - determining the domain of g, drawing a picture of the domain, computing the formulas for partial derivatives and observing that those formulas are valid on the whole domain of g.

The function g can be expressed by

$$g(x,y) = \exp(\cos(xy^2) \cdot \log(\log x - y)),$$

the domain of g is described by the conditions $x > 0$ (in order $\log x$ is defined) and $\log x - y > 0$. (in order $\log(\log x - y)$ is defined. Hence the domain of g is equal to $\{[x,y] \in \mathbb{R}^2 : x > 0 \ \& \ y < \log x\}$, which is the area below the graph of log.

Partial derivatives can be computed in the standard way:

$$\frac{\partial g}{\partial x}(x,y) = \exp(\cos(xy^2) \cdot \log(\log x - y)) \cdot \left(-\sin(xy^2) \cdot y^2 \cdot \log(\log x - y) + \cos(xy^2) \cdot \frac{1}{\log x - y} \cdot \frac{1}{x} \right),$$
$$\frac{\partial g}{\partial y}(x,y) = \exp(\cos(xy^2) \cdot \log(\log x - y)) \cdot \left(-\sin(xy^2) \cdot 2xy \cdot \log(\log x - y) + \cos(xy^2) \cdot \frac{1}{\log x - y} \cdot (-1) \right).$$

It is also clear that these formulas are valid on the whole domain of g. The formulas can be simplified a bit.

Evaluation:

One students provided a complete correct solution and obtained 1 point. Another two provided an almost correct solution – just the picture of domain was in one case reversed, in the other case strange – and obtained 0.9 points. Another one provided an almost correct solution with a mistake in one of the derivatives and obtained 0.7 points.

One student provided a correct domain including the picture, but there were mistakes in derivatives (obtained 0.3 points), another one had incomplete description of the domain and some mistakes in the derivatives (obtained 0.2 points), another one had correct domain but without picture and one of the derivatives almost correct (got 0.2 points).

One student computed the domain assuming $\log x - y$ is substituted by $\log(x-y)$, but the derivatives were computed without this assumption and, moreover, with mistakes. This one obtained 0.3 points.

Finally, one student got 0 points as both domain and formulas for derivatives were a complete nonsense.