Theorem 5.17, implication (iii)⇒(i)

$\mathbb{A}$ not invertible

$$\mathbb{A}$$

$$\begin{pmatrix} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{pmatrix}$$

Theorem 5.17, implication (iii)⇒(i)

$\mathbb{A}$ not invertible $\Rightarrow$ $rk(A) < n$

$$\mathbb{A}$$

$$\begin{pmatrix} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{pmatrix}$$

Theorem 5.17, implication (iii)⇒(i)

$\mathbb{A}$ not invertible $\Rightarrow rk(A) < n$

$$\mathbb{A} \overset{T_1}{\rightsquigarrow} \mathbb{S}$$

$$\begin{pmatrix} \bullet & \bullet & \bullet & \bullet \\ 0 & \bullet & \bullet & \bullet \\ 0 & 0 & \bullet & \bullet \\ 0 & 0 & 0 & \bullet \end{pmatrix}$$

Theorem 5.17, implication (iii)⇒(i)

$\mathbb{A}$ not invertible $\Rightarrow$ $rk(A) < n$

$$\mathbb{A} \overset{T_1}{\rightsquigarrow} \mathbb{S}$$

$$\begin{pmatrix} \bullet & \bullet & \bullet & \bullet \\ 0 & \bullet & \bullet & \bullet \\ 0 & 0 & \bullet & \bullet \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Theorem 5.17, implication (iii)⇒(i)

$\mathbb{A}$ not invertible $\Rightarrow rk(A) < n$

$$\mathbb{A} \overset{T_1}{\rightsquigarrow} \mathbb{S} \qquad \mathbf{b}'$$

$$\begin{pmatrix} \bullet & \bullet & \bullet & \bullet \\ 0 & \bullet & \bullet & \bullet \\ 0 & 0 & \bullet & \bullet \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Theorem 5.17, implication (iii)⇒(i)

$\mathbb{A}$ not invertible $\Rightarrow rk(A) < n$

$$\mathbb{A} \overset{T_1}{\rightsquigarrow} \mathbb{S} \overset{T_2}{\rightsquigarrow} \mathbb{A} \qquad \mathbf{b}' \overset{T_2}{\rightsquigarrow} \mathbf{b}$$

$$\begin{pmatrix} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{pmatrix} \qquad \begin{pmatrix} \bullet \\ \bullet \\ \bullet \\ \bullet \end{pmatrix}$$

Theorem 5.17, implication (iii)$\Rightarrow$(i)

$\mathbb{A}$ not invertible $\Rightarrow rk(A) < n$

$$\mathbb{A} \overset{T_1}{\rightsquigarrow} \mathbb{S} \overset{T_2}{\rightsquigarrow} \mathbb{A} \qquad\qquad \mathbf{b}' \overset{T_2}{\rightsquigarrow} \mathbf{b}$$

$$\begin{pmatrix} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{pmatrix} \qquad \begin{pmatrix} \bullet \\ \bullet \\ \bullet \\ \bullet \end{pmatrix}$$

$\mathbb{A} \cdot \mathbf{x} = \mathbf{b}$

Theorem 5.17, implication (iii)$\Rightarrow$(i)

$\mathbb{A}$ not invertible $\Rightarrow rk(A) < n$

$$\mathbb{A} \overset{T_1}{\rightsquigarrow} \mathbb{S} \overset{T_2}{\rightsquigarrow} \mathbb{A} \overset{T_1}{\rightsquigarrow} \mathbb{S} \qquad \mathbf{b}' \overset{T_2}{\rightsquigarrow} \mathbf{b} \overset{T_1}{\rightsquigarrow} \mathbf{b}'$$

$$\begin{pmatrix} \bullet & \bullet & \bullet & \bullet \\ 0 & \bullet & \bullet & \bullet \\ 0 & 0 & \bullet & \bullet \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$\mathbb{A} \cdot \mathbf{x} = \mathbf{b}$

Theorem 5.17, implication (iii)⇒(i)

$\mathbb{A}$ not invertible $\Rightarrow rk(A) < n$

$$\mathbb{A} \overset{T_1}{\rightsquigarrow} \mathbb{S} \overset{T_2}{\rightsquigarrow} \mathbb{A} \overset{T_1}{\rightsquigarrow} \mathbb{S} \qquad \mathbf{b}' \overset{T_2}{\rightsquigarrow} \mathbf{b} \overset{T_1}{\rightsquigarrow} \mathbf{b}'$$

$$\begin{pmatrix} \bullet & \bullet & \bullet & \bullet \\ 0 & \bullet & \bullet & \bullet \\ 0 & 0 & \bullet & \bullet \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$\mathbb{A} \cdot \mathbf{x} = \mathbf{b} \Rightarrow \mathbb{S} \cdot \mathbf{x} = \mathbf{b}'$

Theorem 5.17, implication (iii)⇒(i)

$\mathbb{A}$ not invertible $\Rightarrow rk(A) < n$

$$\mathbb{A} \overset{T_1}{\rightsquigarrow} \mathbb{S} \overset{T_2}{\rightsquigarrow} \mathbb{A} \overset{T_1}{\rightsquigarrow} \mathbb{S} \qquad \mathbf{b}' \overset{T_2}{\rightsquigarrow} \mathbf{b} \overset{T_1}{\rightsquigarrow} \mathbf{b}'$$

$$\begin{pmatrix} \bullet & \bullet & \bullet & \bullet \\ 0 & \bullet & \bullet & \bullet \\ 0 & 0 & \bullet & \bullet \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$\mathbb{A} \cdot \mathbf{x} = \mathbf{b} \Rightarrow \mathbb{S} \cdot \mathbf{x} = \mathbf{b}'$

$rk(\mathbb{A}) < rk(\mathbb{A}|\mathbf{b}) \Rightarrow \mathbb{A} \cdot \mathbf{x} = \mathbf{b}$ has no solution

$rk(\mathbb{A}) < rk(\mathbb{A}|\mathbf{b})$ and $\mathbb{A} \cdot \mathbf{x} = \mathbf{b}$

$$(\mathbb{A}|\mathbf{b})$$

$$\left(\begin{array}{cccccc|c}
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet
\end{array}\right)$$

$rk(\mathbb{A}) < rk(\mathbb{A}|\mathbf{b}) \Rightarrow \mathbb{A} \cdot \mathbf{x} = \mathbf{b}$ has no solution

$rk(\mathbb{A}) < rk(\mathbb{A}|\mathbf{b})$ and $\mathbb{A} \cdot \mathbf{x} = \mathbf{b}$

$$(\mathbb{A}|\mathbf{b}) \rightsquigarrow (\mathbb{S}|\mathbf{b}')$$

$$\left(\begin{array}{cccccc|c} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & 0 & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & 0 & 0 & 0 & 0 & \bullet \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$rk(\mathbb{A}) < rk(\mathbb{A}|\mathbf{b}) \Rightarrow \mathbb{A}\cdot\mathbf{x} = \mathbf{b}$ has no solution

$rk(\mathbb{A}) < rk(\mathbb{A}|\mathbf{b})$ and $\mathbb{A}\cdot\mathbf{x} = \mathbf{b}$

$$(\mathbb{A}|\mathbf{b}) \rightsquigarrow (\mathbb{S}|\mathbf{b}')$$

$$\left(\begin{array}{cccccc|c}
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
0 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
0 & 0 & 0 & \bullet & \bullet & \bullet & \bullet \\
0 & 0 & 0 & 0 & 0 & 0 & \bullet \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right)$$

$rk(\mathbb{A}) < rk(\mathbb{A}|\mathbf{b}) \Rightarrow \mathbb{A} \cdot \mathbf{x} = \mathbf{b}$ has no solution

$rk(\mathbb{A}) < rk(\mathbb{A}|\mathbf{b})$ and $\mathbb{A} \cdot \mathbf{x} = \mathbf{b} \Rightarrow \mathbb{S} \cdot \mathbf{x} = \mathbf{b}'$

$$(\mathbb{A}|\mathbf{b}) \rightsquigarrow (\mathbb{S}|\mathbf{b}')$$

$$\left(\begin{array}{cccccc|c}
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
0 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
0 & 0 & 0 & \bullet & \bullet & \bullet & \bullet \\
0 & 0 & 0 & 0 & 0 & 0 & \bullet \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right)$$

$rk(\mathbb{A}) = rk(\mathbb{A}|\mathbf{b}) \Rightarrow \mathbb{A} \cdot \mathbf{x} = \mathbf{b}$ has a solution

$rk(\mathbb{A}) = rk(\mathbb{A}|\mathbf{b})$

$$(\mathbb{A}|\mathbf{b})$$

$$\left(\begin{array}{cccccc|c}
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet
\end{array}\right)$$

$rk(\mathbb{A}) = rk(\mathbb{A}|\mathbf{b}) \Rightarrow \mathbb{A} \cdot \mathbf{x} = \mathbf{b}$ has a solution

$rk(\mathbb{A}) = rk(\mathbb{A}|\mathbf{b})$

$$(\mathbb{A}|\mathbf{b}) \rightsquigarrow (\mathbb{S}|\mathbf{b}')$$

$$\left(\begin{array}{cccccc|c} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & 0 & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & 0 & 0 & 0 & \bullet & \bullet \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$rk(\mathbb{A}) = rk(\mathbb{A}|\mathbf{b}) \Rightarrow \mathbb{A} \cdot \mathbf{x} = \mathbf{b}$ has a solution

$rk(\mathbb{A}) = rk(\mathbb{A}|\mathbf{b})$

$$(\mathbb{A}|\mathbf{b}) \rightsquigarrow (\mathbb{S}|\mathbf{b}')$$

$$\left(\begin{array}{cccccc|c}
\color{green}{\bullet} & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
0 & \color{green}{\bullet} & \bullet & \bullet & \bullet & \bullet & \bullet \\
0 & 0 & 0 & \color{green}{\bullet} & \bullet & \bullet & \bullet \\
0 & 0 & 0 & 0 & 0 & \color{green}{\bullet} & \bullet \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right)$$

$rk(\mathbb{A}) = rk(\mathbb{A}|\mathbf{b}) \Rightarrow \mathbb{A} \cdot \mathbf{x} = \mathbf{b}$ has a solution

$rk(\mathbb{A}) = rk(\mathbb{A}|\mathbf{b})$

$\mathbb{A} \cdot \mathbf{x} = \mathbf{b} \Leftrightarrow \mathbb{S} \cdot \mathbf{x} = \mathbf{b}'$

$(\mathbb{A}|\mathbf{b}) \rightsquigarrow (\mathbb{S}|\mathbf{b}')$

$$\left(\begin{array}{cccccc|c} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & 0 & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & 0 & 0 & 0 & \bullet & \bullet \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$