

Transformation of a matrix to the row-echelon form

$$\begin{pmatrix} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \end{pmatrix}$$

Transformation of a matrix to the row-echelon form

$$\begin{pmatrix} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \end{pmatrix}$$

Transformation of a matrix to the row-echelon form

$$\begin{pmatrix} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \end{pmatrix}$$

Transformation of a matrix to the row-echelon form

$$\begin{pmatrix} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \end{pmatrix}$$

Transformation of a matrix to the row-echelon form

$$\begin{pmatrix} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & \bullet & \bullet & \bullet & \bullet & \bullet \end{pmatrix}$$

Transformation of a matrix to the row-echelon form

$$\begin{pmatrix} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & \bullet & \bullet & \bullet & \bullet & \bullet \end{pmatrix}$$

Transformation of a matrix to the row-echelon form

$$\begin{pmatrix} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & \bullet & \bullet & \bullet & \bullet \end{pmatrix}$$

Transformation of a matrix to the row-echelon form

$$\begin{pmatrix} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & \bullet & \bullet & \bullet & \bullet \end{pmatrix}$$

Transformation of a matrix to the row-echelon form

$$\begin{pmatrix} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & \bullet & \bullet & \bullet & \bullet \end{pmatrix}$$

Transformation of a matrix to the row-echelon form

$$\begin{pmatrix} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & 0 & \bullet & \bullet & \bullet \\ 0 & 0 & 0 & \bullet & \bullet & \bullet \\ 0 & 0 & 0 & \bullet & \bullet & \bullet \end{pmatrix}$$

Transformation of a matrix to the row-echelon form

$$\begin{pmatrix} \bullet & & & & & & \\ & 0 & & & & & \\ & 0 & 0 & & & & \\ & 0 & 0 & 0 & & & \\ & 0 & 0 & 0 & \bullet & & \\ & & & & \bullet & & \\ & & & & \bullet & & \\ & & & & \bullet & & \end{pmatrix}$$

Transformation of a matrix to the row-echelon form

$$\begin{pmatrix} \bullet & & & & & & \\ & 0 & & & & & \\ & 0 & & & & & \\ & 0 & & & & & \\ & 0 & & & & & \end{pmatrix}$$

Transformation of a matrix to the row-echelon form

$$\begin{pmatrix} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & 0 & \bullet & \bullet & \bullet \\ 0 & 0 & 0 & 0 & \bullet & \bullet \\ 0 & 0 & 0 & 0 & \bullet & \bullet \end{pmatrix}$$

Transformation of a matrix to the row-echelon form

$$\begin{pmatrix} \bullet & & & & & & \\ & 0 & & & & & \\ & 0 & & & & & \\ & 0 & & & & & \\ & 0 & & & & & \end{pmatrix}$$

Transformation of a matrix to the row-echelon form

$$\begin{pmatrix} \bullet & & & & & & \\ & 0 & & & & & \\ & 0 & 0 & & & & \\ & 0 & 0 & 0 & & & \\ & 0 & 0 & 0 & 0 & 0 & \bullet \\ & 0 & 0 & 0 & 0 & 0 & \bullet \end{pmatrix}$$

Transformation of a matrix to the row-echelon form

$$\begin{pmatrix} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & 0 & \bullet & \bullet & \bullet \\ 0 & 0 & 0 & 0 & 0 & \bullet \\ 0 & 0 & 0 & 0 & 0 & \bullet \end{pmatrix}$$

Transformation of a matrix to the row-echelon form

$$\begin{pmatrix} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & 0 & \bullet & \bullet & \bullet \\ 0 & 0 & 0 & 0 & 0 & \bullet \\ 0 & 0 & 0 & 0 & 0 & \bullet \end{pmatrix}$$

Transformation of a matrix to the row-echelon form

$$\begin{pmatrix} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & 0 & \bullet & \bullet & \bullet \\ 0 & 0 & 0 & 0 & 0 & \bullet \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Transformation of a matrix to the row-echelon form

$$\begin{pmatrix} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & 0 & \bullet & \bullet & \bullet \\ 0 & 0 & 0 & 0 & 0 & \bullet \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Computing the inverse of a matrix

$$rk(\mathbb{A}) = n$$

$$\left(\begin{array}{cccc|cccc} & \mathbb{A} & & & & \mathbb{I} & & & \\ & & & & & & & & \\ \bullet & & & & 1 & 0 & 0 & 0 \\ \bullet & & & & 0 & 1 & 0 & 0 \\ \bullet & & & & 0 & 0 & 1 & 0 \\ \bullet & & & & 0 & 0 & 0 & 1 \end{array} \right)$$

Computing the inverse of a matrix

$$\text{rk}(A) = n$$

$$A \rightsquigarrow S$$

$$I \rightsquigarrow B$$

$$\left(\begin{array}{cccc|cccc} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & 0 & \bullet & \bullet & \bullet & \bullet & \bullet \end{array} \right)$$

Computing the inverse of a matrix

$$\text{rk}(A) = n$$

 $A \rightsquigarrow S$ $I \rightsquigarrow B$

$$\left(\begin{array}{cccc|cccc} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & 0 & \bullet & \bullet & \bullet & \bullet & \bullet \end{array} \right)$$

Computing the inverse of a matrix

$$\text{rk}(A) = n$$

$$A \rightsquigarrow S'$$

$$I \rightsquigarrow B'$$

$$\left(\begin{array}{cccc|cccc} 1 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 1 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & 1 & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & 0 & 1 & \bullet & \bullet & \bullet & \bullet \end{array} \right)$$

Computing the inverse of a matrix

$$\text{rk}(A) = n$$

$$A \rightsquigarrow S''$$

$$I \rightsquigarrow B''$$

$$\left(\begin{array}{cccc|cccc} 1 & \bullet & \bullet & 0 & \bullet & \bullet & \bullet & \bullet \\ 0 & 1 & \bullet & 0 & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & 1 & 0 & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & 0 & 1 & \bullet & \bullet & \bullet & \bullet \end{array} \right)$$

Computing the inverse of a matrix

$$\text{rk}(A) = n$$

$$A \rightsquigarrow S'''$$

$$I \rightsquigarrow B'''$$

$$\left(\begin{array}{cccc|cccc} 1 & \bullet & 0 & 0 & \bullet & \bullet & \bullet & \bullet \\ 0 & 1 & 0 & 0 & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & 1 & 0 & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & 0 & 1 & \bullet & \bullet & \bullet & \bullet \end{array} \right)$$

Computing the inverse of a matrix

$$\text{rk}(A) = n$$

$$A \rightsquigarrow S''' \rightsquigarrow I$$

$$I \rightsquigarrow B''' \rightsquigarrow A^{-1}$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \bullet & \bullet & \bullet & \bullet \\ 0 & 1 & 0 & 0 & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & 1 & 0 & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & 0 & 1 & \bullet & \bullet & \bullet & \bullet \end{array} \right)$$

\mathbb{A} invertible $\Rightarrow rk(\mathbb{A}) = n$

$rk(\mathbb{A}) < n$ and \mathbb{A} is invertible

$$\begin{array}{cccc|cccc} & \mathbb{A} & & & \mathbb{I} & & & & \\ \left(\begin{array}{cccc|cccc} \bullet & \bullet & \bullet & \bullet & 1 & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & 0 & 1 & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & 0 & 0 & 1 & 0 \\ \bullet & \bullet & \bullet & \bullet & 0 & 0 & 0 & 1 \end{array} \right) \end{array}$$

$$\mathbb{A} \cdot \mathbb{A}^{-1} = \mathbb{I}$$

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$rk(\mathbb{A}) < n$ and \mathbb{A} is invertible

$$\begin{array}{ccc} & \mathbb{A} \rightsquigarrow \mathbb{S} & \mathbb{I} \rightsquigarrow \mathbb{B} \\ \left(\begin{array}{cccc|cccc} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & 0 & \bullet & \bullet & \bullet & \bullet & \bullet \end{array} \right) \end{array}$$

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$$\begin{array}{ccc} & \mathbb{A} \rightsquigarrow \mathbb{S} & \mathbb{I} \rightsquigarrow \mathbb{B} \\ \left(\begin{array}{cccc|cccc} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & 0 & \color{red}{0} & \bullet & \bullet & \bullet & \bullet \end{array} \right) \end{array}$$

$$\mathbb{A} \cdot \mathbb{A}^{-1} = \mathbb{I}$$

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$\mathbb{A} \rightsquigarrow \mathbb{S}$

$\mathbb{I} \rightsquigarrow \mathbb{B}$

$$\left(\begin{array}{cccc|cccc} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & 0 & 0 & \bullet & \bullet & \bullet & \bullet \end{array} \right)$$

$$\mathbb{A} \cdot \mathbb{A}^{-1} = \mathbb{I} \Rightarrow \mathbb{S} \cdot \mathbb{A}^{-1} = \mathbb{B}$$

\mathbb{A} invertible $\Rightarrow rk(\mathbb{A}) = n$

$rk(\mathbb{A}) < n$ and \mathbb{A} is invertible

$$\begin{array}{ccc} & \mathbb{A} \rightsquigarrow \mathbb{S} & \mathbb{I} \rightsquigarrow \mathbb{B} \\ \left(\begin{array}{cccc|cccc} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right) \end{array}$$

$$\mathbb{A} \cdot \mathbb{A}^{-1} = \mathbb{I} \Rightarrow \mathbb{S} \cdot \mathbb{A}^{-1} = \mathbb{B}$$

\mathbb{A} invertible $\Rightarrow rk(\mathbb{A}) = n$

$rk(\mathbb{A}) < n$ and \mathbb{A} is invertible

$$\begin{array}{ccc} & \mathbb{A} \rightsquigarrow \mathbb{S} & \mathbb{I} \rightsquigarrow \mathbb{B} \\ \left(\begin{array}{cccc|cccc} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right) \end{array}$$

$$\mathbb{A} \cdot \mathbb{A}^{-1} = \mathbb{I} \Rightarrow \mathbb{S} \cdot \mathbb{A}^{-1} = \mathbb{B} \Rightarrow rk(\mathbb{B}) < n$$

\mathbb{A} invertible $\Rightarrow rk(\mathbb{A}) = n$

$rk(\mathbb{A}) < n$ and \mathbb{A} is invertible

$$\mathbb{A} \rightsquigarrow \mathbb{S}$$

$$\mathbb{I} \rightsquigarrow \mathbb{B} \Rightarrow rk(\mathbb{B}) = n$$

$$\left(\begin{array}{cccc|cccc} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

$$\mathbb{A} \cdot \mathbb{A}^{-1} = \mathbb{I} \Rightarrow \mathbb{S} \cdot \mathbb{A}^{-1} = \mathbb{B} \Rightarrow rk(\mathbb{B}) < n$$