TEST 1, PŘÍKLAD A1 | ŘEŠENÍ 4

$$\lim_{x\to 0}\frac{(1-x)^{\frac{1}{x}}-\frac{1}{e}}{x} \overset{1\text{ bod}}{=} \lim_{x\to 0}\frac{\exp\left(\frac{\ln(1-x)}{x}\right)^{*}-\frac{1}{e}}{x} =$$

*) dobře definováno, neboť $1-x>0$ na $(-1,1)$

$$\underset{\text{"}\frac{0}{0}\text{"}}{\overset{l'H\ 2\text{ body}}{=}} \lim_{x\to 0}\frac{\exp\left(\frac{\ln(1-x)}{x}\right)\cdot\left(\frac{\frac{1}{1-x}\cdot(-1)\cdot x-\ln(1-x)}{x^2}\right)-0}{1} =$$

$$= \lim_{x\to 0}\underbrace{\exp\left(\frac{\ln(1-x)}{x}\right)}_{\to\frac{1}{e}}\cdot\frac{-x-(1-x)\ln(1-x)}{x^2\cdot\underbrace{(1-x)}_{\to 1}} =$$

$$\underset{1\text{ bod}}{\overset{AL}{=}} \frac{1}{e}\cdot 1\cdot\lim_{x\to 0}\frac{-x-(1-x)\ln(1-x)}{x^2} = (*)$$

Použijeme Taylorův polynom řádu 2:

2 body:
$$\ln(1-x) = -x-\frac{x^2}{2}+o(x^2) \quad \text{pro } x\to 0$$
$$(1-x)\ln(1-x) = -x-\frac{x^2}{2}+x^2+o(x^2) = -x+\frac{1}{2}x^2+o(x^2)$$

$$(*) = \frac{1}{e}\cdot\lim_{x\to 0}\frac{-x-\left(-x+\frac{x^2}{2}+o(x^2)\right)}{x^2} = \frac{1}{e}\cdot\lim_{x\to 0}\frac{-\frac{x^2}{2}+o(x^2)}{x^2} =$$

$$= \frac{1}{e}\lim_{x\to 0}\left(-\frac{1}{2}+\frac{o(x^2)}{x^2}\right) = -\frac{1}{2e}$$

2 body

TEST 1, PŘÍKLAD B1 | KRUH 4 | VARIANTA

$$\lim_{x\to 0} \frac{(1+x)^{\frac{2}{x}} - e^2}{x}$$

$$(1+x)^{\frac{2}{x}} \overset{1\,bod}{=} \exp\left(2\cdot \frac{\ln(1+x)}{x}\right) \overset{1\,bod}{=} \exp(2)\cdot \exp\left(2\cdot\frac{\ln(1+x)}{x} - 2\right)$$

($\frac{\ln(1+x)}{x} \to 1$, 1 bod)

2 body:

$$2\left(\frac{\ln(1+x)}{x} - 1\right) = 2\cdot\left(\frac{x - \frac{x^2}{2} + o(x^2)}{x} - 1\right) =$$

$$= 2\left(1 - \frac{x}{2} + o(x) - 1\right) = -x + o(x) \quad \text{pro } x\to 0$$

3 body:

$$\exp(x) = 1 + x + o(x) \quad \text{pro } x \to 0$$

$$\exp\left(2\cdot\frac{\ln(1+x)}{x} - 2\right) = 1 + \left(2\frac{\ln(1+x)}{x} - 2\right) + o(x) =$$

protože $\lim_{x\to 0} \frac{2\frac{\ln(1+x)}{x} - 2}{x} = -1$

$$= 1 - x + o(x)$$

Tedy limita je:

2 body:

$$\lim_{x\to 0} \frac{(1+x)^{\frac{2}{x}} - e^2}{x} = \lim_{x\to 0} e^2\cdot\frac{\left(\exp\left(\frac{2\ln(1+x)}{x} - 2\right) - 1\right)}{x} =$$

$$= \lim_{x\to 0} e^2\cdot\frac{1 - x + o(x) - 1}{x} = \lim_{x\to 0} e^2\left(-1 + \frac{o(x)}{x}\right) = -e^2$$

TEST 1, PRIKLAD B2 | ŘEŠENÍ

$$\lim_{x\to 1}\left(\frac{4}{\pi}\operatorname{arctg} x\right)^{\frac{1}{x-1}} \overset{1\text{ bod}}{=} \lim_{x\to 1}\exp\left(\frac{\ln\left(\frac{4}{\pi}\operatorname{arctg} x\right)}{x-1}\right)$$

→ 1, tedy limita má tvar 1^∞ (1 bod)

<u>lim. v exponentu:</u>

$$\lim_{x\to 1}\frac{\ln\left(\frac{4}{\pi}\operatorname{arctg} x\right)}{x-1} \overset{\text{„}0/0\text{"},\ 2\text{ body}}{\underset{l'H}{=}} \lim_{x\to 1}\frac{\frac{1}{\frac{4}{\pi}\operatorname{arctg} x}\cdot\frac{4}{\pi}\cdot\frac{1}{1+x^2}}{1} =$$

↑ 3 body

$$\overset{3\text{ body}}{=} \lim_{x\to 1}\frac{1}{\operatorname{arctg} x\cdot(1+x^2)} = \frac{1}{\frac{\pi}{4}\cdot 2} = \frac{2}{\pi}$$

Tedy původní limita je $e^{\frac{2}{\pi}}$ 1 bod

PŘÍKLAD A2 | VARIANTA

$$\lim_{x\to 0}\left(\frac{2}{\pi}\arccos x\right)^{\frac{1}{x}} = \lim_{x\to 0}\exp\left(\frac{\ln\left(\frac{2}{\pi}\arccos x\right)}{x}\right) \ 1\text{ bod}$$

($\arccos x \to \frac{\pi}{2}$)

$$\lim_{x\to 0}\frac{\ln\left(\frac{2}{\pi}\arccos x\right)}{x} = \lim_{x\to 0}\frac{\ln\left(\frac{2}{\pi}\arccos x\right)}{\frac{2}{\pi}\arccos x - 1}\cdot\frac{\frac{2}{\pi}\arccos x - 1}{x} =$$

(3 body, → 1)

$$= 1\cdot\lim_{x\to 0}\frac{\frac{2}{\pi}\arccos x - 1}{x} \underset{3\text{ body}}{=} \lim_{x\to 0}\frac{\frac{2}{\pi}\left(\frac{\pi}{2}-\arcsin x\right)-1}{x} =$$

$$= \lim_{x\to 0} -\frac{2}{\pi}\cdot\frac{\arcsin x}{x} \underset{2\text{ body}}{=} -\frac{2}{\pi}$$

limita je $e^{-2/\pi}$ 1 bod

TEST 1, PŘÍKLAD 43 } kruh 4

$$\sum_{n=1}^{\infty} \frac{(-i)^n z^n}{\sqrt{n}}$$

• Poloměr konvergence: $\lim_{n\to\infty} \sqrt[n]{\left|\frac{(-i)^n}{\sqrt{n}}\right|} = \lim_{n\to\infty} \sqrt[n]{\frac{1}{\sqrt{n}}} = 1$ } 2 bod

$\Rightarrow R = 1$

Tedy: pro $|z| < 1$ K.A., pro $|z| > 1$ D. 2 bod

---

na kružnici, tj. pro $|z| = 1$:

absolutní konvergence: $\left|\frac{(-i)^n z^n}{\sqrt{n}}\right| = \frac{1}{\sqrt{n}}$, $\sum \frac{1}{\sqrt{n}}$ D. } 2 b

tedy na kružnici nekonverguje absolutně

---

konvergence (neabsol.): $\frac{(-i)^n z^n}{\sqrt{n}} = \frac{(-iz)^n}{\sqrt{n}}$

Pokud $-iz = 1$, tj. $z = i$, pak řada diverguje 1 bod

3 b { Pokud $|z| = 1$, $z \neq i$, tj. $-iz \neq 1$, pak $\sum (-iz)^n$ má omezené částečné součty a $\frac{1}{\sqrt{n}} \downarrow 0$, tedy řada konverguje dle Dirichletova kr.

Závěr: $|z| < 1$ K.A.

$|z| = 1, z \neq i$ K.N.A.

$z = i$ nebo $|z| > 1$ D.