Some results from students' presentation from Complex Analysis 1

Jensen inequality for holomorphic functions. *Let f be a holomorphic function on $U(0,R)$ such that $f(0) \neq 0$. Then for any $r \in (0,R)$ the following inequality holds:*

$$\log|f(0)| \leq \frac{1}{2\pi} \int_0^{2\pi} \log|f(re^{it})| \, \mathrm{d}t$$

This inequality is proved within the presentation on Jensen formula and is used in Section IX.3 (in the proof of Theorem 13).

Theorem. *Let $f \in N$ be a function which is not constant zero. Let (α_n) be all its roots in $U(0,1)$ listed according to theic multiplicities. Then*

$$\sum_{n=1}^{\infty}(1 - |\alpha_n|) < \infty.$$

Blaschke products. *Let (α_n) be a sequence $P(0,1)$ satisfying*

$$\sum_{n=1}^{\infty}(1 - |\alpha_n|) < \infty$$

and $k \in \mathbb{N} \cup \{0\}$. Then the formula

$$B(z) = z^k \prod_{n=1}^{\infty} \frac{\alpha_n - z}{1 - \overline{\alpha_n} z} \frac{|\alpha_n|}{\alpha_n}$$

defines a holomorphic function on $U(0,1)$ which has no roots except for the points α_n and zero (if $k > 0$).

These theorems are proved within the presentation on Blaschke products. They are used in Section IX.3 to prove Lemma 16. The space N is defined in Section IX.3 (and also in the respective presentation).