Proof of Lemma XI. 25

Let $\mathbb{A}$ be an angelic space

(1) $A \subset T$ compact $\Rightarrow$ A sequentially compact

⌈ Let $(t_n) \subset A$ be a sequence. Then $B := \{t_n, n \in \mathbb{N}\}$ is relatively countably compact. It follows that there is $x \in A$, a cluster point of (t_n). If $x \in B$, then $x = t_n$ for infinitely many $n \in \mathbb{N}$, thus (t_n) has a constant subsequence. Suppose $x \notin B$. Since $x \in \overline{B}$, by the property (ii) there is a sequence (n_k) with $t_{n_k} \to x$. Up to passing to a subsequence we may assume that (n_k) is increasing ⌋

(2) So, for $A \subset T$ we have

A rel. ctbly compact $\Leftrightarrow$ A rel. compact $\Leftrightarrow$ rel. sequentially compact

$\Rightarrow$ by the property (i)

$\Leftarrow$ trivial

$\Rightarrow$ by (1)

$\Leftarrow$ by the property (i)

(3) $A \subset T$ ctbly compact $\Rightarrow$ A compact

⌈ by the property (i) we deduce that $\overline{A}$ is compact. We will show that $A = \overline{A}$. Fix $x \in \overline{A}$. By the property (ii) there is a sequence $(t_n) \subset A$ with $t_n \to x$. Since A is ctbly compact, the sequence (t_n) has a cluster point in A. But its only cluster point is x, so $x \in A$ ⌋

(4) Using (1) and (3) it follows that for $A \subset T$

A is ctbly compact $\Leftrightarrow$ A is compact $\Leftrightarrow$ A is sequentially compact