Let A be a Banach algebra

(a) A unital $\Rightarrow$ $\Delta(A)$ is a weak*-compact subset of S_{A^*}.

$\ulcorner$ By Prop. 21 we have

- $\Delta(A) \subset S_{A^*}$

- $\Delta(A) = \{ h \in A^* ;\ \forall x,y \in A : h(xy) = h(x) \cdot h(y) \ \& \ h(e) = 1 \}$,
  so $\Delta(A)$ is weak*-closed.

So, by Banach-Alaoglu B_{A^*} is w^*-compact,
hence $\Delta(A)$ is w^*-compact. $\lrcorner$

(b) $\Delta(A^+) = \{ \tilde{h} ;\ h \in \Delta(A) \} \cup \{ h_\infty \}$, where
$\tilde{h}(x,\lambda) = h(x) + \lambda$, $(x,\lambda) \in A^+$
$h_\infty(x,\lambda) = \lambda$, $(x,\lambda) \in A^+$

$\ulcorner$ „$\supset$": By Prop. 21 $\tilde{h} \in \Delta(A^+)$ whenever $h \in \Delta(A)$
Moreover, clearly $h_\infty \in \Delta(A^+)$

„$\subset$" Let $g \in \Delta(A^+)$. Define $h(x) = g(x,0)$, $x \in A$
Then $h \in A^*$, h is multiplicative
$\Rightarrow$ either $h \in \Delta(A)$ (then $g = \tilde{h}$)
or $h = 0$ (then $g = h_\infty$). $\lrcorner$

(c) A has no unit $\Rightarrow$ $\Delta(A) \subset B_{A^*}$, $\Delta(A) \cup \{0\}$ is w^*-compact

$\ulcorner$ $\Delta(A) \subset B_{A^*}$ by Prop. 21

$\Delta(A) \cup \{0\} = \{ h \in A^* ;\ \forall x,y \in A \ \ h(xy) = h(x) h(y) \}$
w^* closed, so w^*-compact $\lrcorner$