Lemma: Let X be a TVS, $A \subset X$ a convex set
$x \in \overline{A}$, $y \in \operatorname{int} A \Rightarrow \{tx + (1-t)y,\ t \in [0,1)\} \subset \operatorname{int} A$

Proof: • $t=0$ ... $0x + (1-0)y = y \in \operatorname{int} A$ by assumption

• $t \in (0,1)$: Fix an open balanced nbhd U of 0
s.t. $y + U \subset A$
Then $x + \frac{1-t}{t}U$ is a nbhd of x and $x \in \overline{A}$, hence
we can find $z \in (x + \frac{1-t}{t}U) \cap A$

Fix $u \in U$ s.t. $z = x + \frac{1-t}{t}u$

Set $v := y - u \in y + U \subset \operatorname{int} A$ (U open and balanced)
Fix V, an open balanced nbhd of 0 with
$v + V \subset \operatorname{int} A$

Since A is convex, we have $tz + (1-t)(v+V) \subset A$
Moreover, $tz + (1-t)(v+V)$ is an open set
(V is open, $1-t \neq 0$) and contains
the element
$$tz + (1-t)v = tx + (1-t)u + (1-t)y - (1-t)u$$
$$= tx + (1-t)y$$
Hence, indeed, $tx + (1-t)y \in \operatorname{int} A$.