Let A be a C^*-algebra and $B \subset A$ its C^*subalgebra

(a) $\forall x \in B$: $\sigma_B(x) \cup \{0\} = \sigma_A(x) \cup \{0\}$

(b) A unital, $e \in B \Rightarrow \forall x \in B: \sigma_B(x) = \sigma_A(x)$.

In particular $G(B) = B \cap G(A)$

Proof (b) ① $x \in B$, $x^* = x \underset{\text{Prop. 32 (a)}}{\Rightarrow} \sigma_A(x) \subset \mathbb{R} \Rightarrow \mathbb{C} \setminus \sigma_A(x)$ is connected

$\Rightarrow \sigma_A(x) = \sigma_B(x)$ by Prop. 14 (c)

② $x \in B \cap G(A)$ ... $y := x^{-1}$

$\Rightarrow x^* \in B \cap G(A)$, $(x^*)^{-1} = y^*$

Further, $x^* x \in B \cap G(A)$, $(x^* x)^{-1} = y y^*$

Since $x^* x$ is selfadjoint, $\sigma_A(x^* x) = \sigma_B(x^* x)$, in part. $0 \notin \sigma_B(x^* x)$, so $x^* x \in G(B)$, i.e. $y y^* \in B$

Finally, $y = y y^* x^* \in B$, so $x \in G(B)$

③ $x \in B$, $\lambda \in \mathbb{C}$

By ② $\lambda e - x \in G(B) \Leftrightarrow \lambda e - x \in G(A)$

So, $\sigma_A(x) = \sigma_B(x)$

(a) Consider A^+ and let $\tilde{B} = \{(b, \lambda), b \in B\} \subset A^+$

Then for $x \in B$ we have

$$\begin{array}{ccc} \sigma_{A^+}(x,0) & \overset{(b)}{=} & \sigma_{\tilde{B}}(x,0) \\ \| & & \| \\ \sigma_A(x) \cup \{0\} & & \sigma_B(x) \cup \{0\} \end{array}$$