Let $\varphi : [a,b] \to \mathbb{C}$ be a cts piecewise C^1-curve.

i.e. • φ is cts

• $\exists \quad a = t_0 < t_1 < \dots < t_k = b$ s.t.

$\forall j \in \{1, \dots, k\}$ φ' is cts on (t_{j-1}, t_j)

and $\lim_{t \to t_{j-1}+} \varphi'(t)$, $\lim_{t \to t_j -} \varphi'(t)$ exist (finite)

Let $f : \langle \varphi \rangle \to X$ be continuous ($\langle \varphi \rangle = \varphi([a,b])$, X is a Banach space)

Then $\int_\varphi f = \int_a^b f(\varphi(t))\varphi'(t)\,dt$ exists in the Bochner sense

Indeed, let $g(t) := f(\varphi(t))\varphi'(t)$. Then g is cts on $[0,1] \setminus \{t_0, \dots, t_k\}$, so

$g([0,1] \setminus \{t_0, \dots, t_k\})$ is separable (as a cts image of a separable space)

Further, it is cts, so Borel measurable, thus it is strongly measurable by Pettis thm.

Finally, $f(\varphi([0,1]))$ is compact, hence bdd subset of X

φ' is bdd on each (t_{j-1}, t_j), so it is bdd

It follows that g is bdd, hence $\int_a^b \|g\| < \infty$.

Hence, g is Bochner-integrable.