INTRODUCTION

This doctoral thesis consists of the following five research papers: *Remark on the Point of Continuity Property II* (joint paper with P.Holický, Bull. Acad. Polon. Sci., **50** (1995), 105–111), *Note on Connections of Point of Continuity Property and Kuratowski Problem on Function Having the Baire Property* (to appear in Acta Univ. Carolinae, Math. et Phys., **36**(1), 1997), *New Examples of Hereditarily t-Baire Spaces* (submitted for publication to Bull. Acad. Polon. Sci.), *Stegall Compact Spaces Which Are Not Fragmentable* (submitted for publication to Topol. Appl.), *Few remarks on structure of certain spaces of measures* (preprint). These papers deal with several continuity-like properties of mappings between topological spaces. The first three papers are concerned with maps of topological spaces into metric spaces and investigate connections between measurability in some sense and some kind of almost continuity, while in the last two papers sets of continuity points of upper semi-continuous compact-valued maps are studied.

In the first three papers we investigate some possibilities of generalization of the classical Baire theorem on functions of the first class. This theorem says that a function of a complete metric space into a separable metric space is of the first Borel class (i.e., the inverse image of every open set is $\mathcal{F}_\sigma$) if and only if it has the point of continuity property (i.e., its restriction to any nonempty closed set has a point of continuity). Since the 'if' part need not hold for nonmetrizable domains one introduces the notion of 'extended Borel class one'. Then all maps with the point of continuity property with values in a metric space are of this class. So the inverse implication is the interesting one. We study this question for maps of a hereditarily Baire space into a (not necessarily separable) metric space. The 'extended Borel class one' maps are called in various ways - G.Koumoullis calls them 'functions of the first H-class', we call them '$(\mathcal{F} \wedge \mathcal{G})_{\sigma\text{-scattered}}$-measurable' which suggests they are defined via a kind of measurability.

In the paper *Remark on the Point of Continuity Property II,* which is a joint work with P.Holický, we provide some positive results in this direction. We show that whenever X is a hereditarily Baire space such that every 'extended Borel class one' map of X into a metric space of weight at most equal to the tightness of X has the point of continuity property then the same holds for all metric spaces, without any restriction on weight (for the case of regular X this is due to P.Holický). This yields also a slight improvement of a result of R.W.Hansell, namely that the generalized Baire theorem holds for mappings of a hereditarily Baire space with countable tightness into any metric space. Further we prove the generalized Baire theorem for maps of hereditarily ccc hereditarily Baire spaces into metric spaces of weight less than the least weakly inaccessible cardinal and give the obvious combination with the previous result.

The work *Note on Connections of Point of Continuity Property and Kuratowski Problem on Function Having the Baire Property,* studies relationship of the generalized Baire theorem and the question of Kuratowski, whether a function of a

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Baire space into a metric space which has the Baire property is continuous apart from a meager set. Using ideal topologies we establish the equivalence of these two questions. Also some examples of functions for which the generalized Baire theorem fails are given here. These functions are even of the ordinary Borel class one (i.e., $\mathcal{F}_\sigma$-measurable). One of these functions has range of weight $\aleph_1$ which shows that the assumption of separability of the range cannot be weakened if we let the domain be arbitrary hereditarily Baire space.

The third paper, which is entitled as *New Examples of Hereditarily t-Baire Spaces,* deals with a class of spaces which was defined by G.Koumoullis who proved that an 'extended Borel class one' function of a hereditarily t-Baire space into a metric space of nonmeasurable cardinality has the point of continuity property. We introduce a new subclass of Koumoullis' class and using this we show that the cardinality restriction on the range in the mentioned result is necessary. The class of hereditarily t-Baire spaces contains all compact Hausdorff spaces and even all 'hereditarily almost Čech complete' spaces. Our examples also show that there are other hereditarily t-Baire spaces. The function we have constructed is not of the ordinary Borel class one, so it seems to be open whether the cardinality restriction on the range can be dropped if we restrict ourselves to the case of $\mathcal{F}_\sigma$-measurable maps. Another interesting open question is whether the generalized Baire theorem holds without restrictions at least for compact Hausdorff spaces.

In the fourth work (*Stegall Compact Spaces Which Are Not Fragmentable*) we study minimal usco mappings of a Baire space into a compact space. This is related to some questions concerning Gâteaux differentiability of convex continuous functions on a Banach space. Namely, we deal with the class of spaces introduced by C.Stegall. This is the class of all spaces X such that every minimal usco mapping of a Baire space into X is singlevalued (i.e. 'continuous') at points of a residual set. It was proved by C.Stegall that whenever the dual unit ball of a Banach space X, endowed with the $w^\star$ topology, is in the Stegall class then X is a weak Asplund space (i.e., a space such that every (real-valued) continuous convex function on X is Gâteaux differentiable at points of a residual set). It is standard that the Stegall class contains all fragmentable spaces. Using also some theorems on first class functions we give consistent examples of nonfragmentable compact spaces which belong to the Stegall class. However it remains open whether an analogue holds for dual unit balls.

The last paper (*Few remarks on structure of certain spaces of measures*) studies the structure of the spaces of measures on the compact spaces introduced in the previous work. We establish some partial results which may perhaps lead to the proof that the corresponding dual unit ball under some assumptions belongs to the Stegall class. In proving the mentioned result we prove and use two other theorems – one on Baire-property-additive families in compact spaces and the second one on descriptive properties of sets of measures having an atom in a given set.