$$f(x,y) = \frac{1}{3}x^3 - xy + \frac{1}{2}y^2$$

$$D_f = \mathbb{R}^2$$

$$f \in C^2(\mathbb{R}^2) \quad (\text{dokonce } C^\infty(\mathbb{R}^2))$$

$$\nabla f(x,y) = \left(x^2 - y, -x + y\right) = o \iff y = x^2 \ \& \ y = x$$

$$x = x^2$$

$$x = 0 \quad x = 1$$

stac. body $[0,0]$
$[1,1]$

$$D^2 f(x,y) = \begin{pmatrix} 2x & -1 \\ -1 & 1 \end{pmatrix}$$

$$D^2 f(0,0) = \begin{pmatrix} 0 & -1 \\ -1 & 1 \end{pmatrix}$$

Sylv. $\begin{vmatrix} 0 & -1 \\ -1 & 1 \end{vmatrix} = -1$

ID

$$\sim \begin{pmatrix} 1 & -1 \\ -1 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 \\ 0 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$\Rightarrow [0,0]$ sedl. bod

$$D^2 f(1,1) = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

PD

$\Rightarrow [1,1]$ je ostré lok. minimum

$$f(x,y,z) = -x^3 + 3xz + 2y - y^2 - 3z^2$$

$D_f = \mathbb{R}^3$, $f \in C^2(\mathbb{R}^3)$

$$Df(x,y,z) = \left(-3x^2 + 3z,\ 2-2y,\ 3x - 6z\right) = o \iff$$

$x^2 = z$ & $y = 1$ & $x = 2z$

$4z^2 = z$

$z = 0 \quad 4z = 1$

$z = \frac{1}{4}$

stac. body : $[0,1,0]$

$\left[\frac{1}{2}, 1, \frac{1}{4}\right]$

$$\nabla^2 f(x,y,z) = \begin{pmatrix} -6x & 0 & 3 \\ 0 & -2 & 0 \\ 3 & 0 & -6 \end{pmatrix}$$

$$\nabla^2 f(0,1,0) = \begin{pmatrix} 0 & 0 & 3 \\ 0 & -2 & 0 \\ 3 & 0 & -6 \end{pmatrix} \sim \begin{pmatrix} 0 & -2 & 0 \\ 0 & 0 & 3 \\ 3 & 0 & -6 \end{pmatrix} \sim \begin{pmatrix} -2 & 0 & 0 \\ 0 & 0 & 3 \\ 0 & 3 & -6 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} -2 & 0 & 0 \\ 0 & -6 & 3 \\ 0 & 3 & 0 \end{pmatrix} \sim \begin{pmatrix} -2 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & \frac{3}{2} \end{pmatrix} \quad \text{ID}$$

$[0,1,0]$ sedlový bod

$$\nabla^2 f\left(\frac{1}{2}, 1, \frac{1}{4}\right) = \begin{pmatrix} -3 & 0 & 3 \\ 0 & -2 & 0 \end{pmatrix} \sim \begin{pmatrix} -3 & 0 & 0 \\ 0 & -2 & 0 \end{pmatrix}$$

$\begin{pmatrix} \dots \\ 3 & 0 & -6 \end{pmatrix} \quad \begin{pmatrix} \dots \\ 0 & 0 & -3 \end{pmatrix}$ ND

$\left[\frac{1}{2}, 1, \frac{1}{4}\right]$ ostré lok. maximum

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$f(x,y) = x^4 + y^4 - x^2 - 2xy - y^2$

$D_f = \mathbb{R}^2, \ f \in C^2(\mathbb{R}^2)$

$\nabla f(x,y) = (4x^3 - 2x - 2y, \ 4y^3 - 2x - 2y) = o$

$$\Leftrightarrow \left.\begin{array}{l} 2x^3 - x - y = 0 \\ \& \ 2y^3 - x - y = 0 \end{array}\right\} \begin{array}{l} 2x^3 = 2y^3 \\ x = y \end{array}$$

$$2x^3 = 2x$$

$$x^3 = x \ \begin{cases} x = 0 \\ x^2 = 1 \ \begin{cases} x = 1 \\ x = -1 \end{cases} \end{cases}$$

stac. body $[0,0]$

$[-1,-1]$

$[1,1]$

$$D^2 f(x,y) = \begin{pmatrix} 12x^2-2 & -2 \\ -2 & 12y^2-2 \end{pmatrix}$$

$$D^2 f(-1,-1) = D^2 f(1,1) = \begin{pmatrix} 10 & -2 \\ -2 & 10 \end{pmatrix} \sim \begin{pmatrix} 10 & 0 \\ 0 & 10-\frac{2}{5} \end{pmatrix} \quad \text{PD}$$

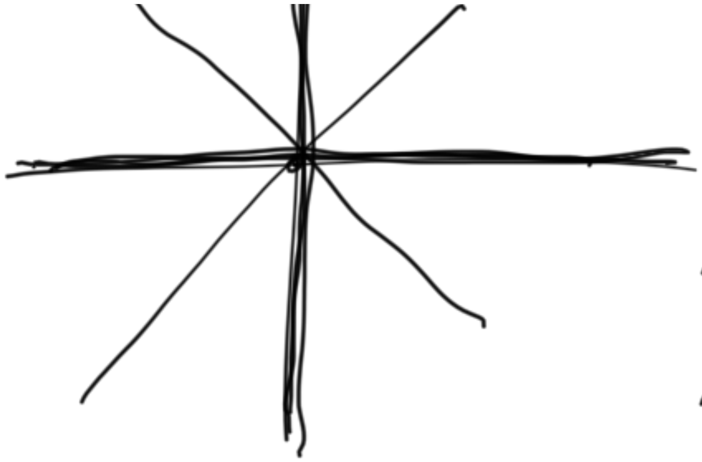
$[-1,1]$ a $[1,1]$ jsou ostrá lokální minima

$$D^2 f(0,0) = \begin{pmatrix} -2 & -2 \\ -2 & -2 \end{pmatrix} \sim \begin{pmatrix} -2 & -2 \\ 0 & 0 \end{pmatrix} \sim \underbrace{\begin{pmatrix} -2 & 0 \\ 0 & 0 \end{pmatrix}}_{\text{nenulová} \Rightarrow \text{není PSD}} \quad \text{NSD}$$

$\Rightarrow$ v $[0,0]$ není lok. min.

VG7

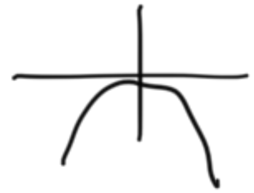
může tam být lok. max. nebo sedlový bod



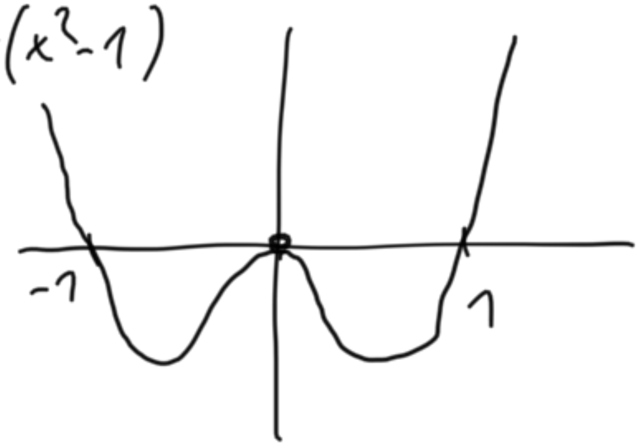
$f(x,y) = x^2 - y^2$

$x \mapsto f(x,0) = x^2$

$y \mapsto f(0,y) = -y^2$



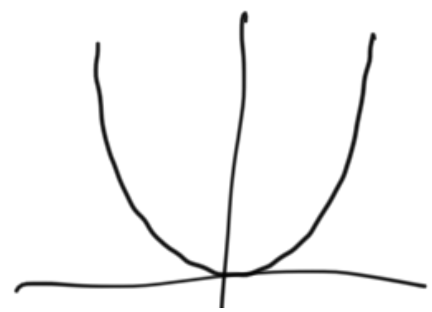
$f(x,0) = x^4 - x^2 = x^2(x^2-1)$



$f(0,y) = y^4 - y^2$

$\underline{x=y}$ $\quad f(x,x) = 2x^4 - x^2 - 2x^2 - x^2 = 2(x^4 - 2x^2) = 2x^2(x^2-2)$

$y = -x$ $\quad f(x,-x) = 2x^4 - x^2 + 2x^2 - x^2 = 2x^4$



$\Rightarrow$ $[0,0]$ sedlový bod