$$f(x) = \arcsin \underbrace{\frac{4x}{x^2+4}}_{g(x)}$$

$$g(x) \in \langle -1, 1 \rangle: \quad -1 \leq \frac{4x}{x^2+4} \leq 1 \quad \Big/ \cdot (x^2+4) > 0$$

$$-(x^2+4) \leq 4x \leq x^2+4$$

$$\underbrace{x^2+4x+4}_{(x+2)^2} \geq 0 \quad \Big| \quad \underbrace{x^2-4x+4}_{(x-2)^2} \geq 0$$

to platí $\forall x \in \mathbb{R}$

$$D_f = \mathbb{R}$$

g ... rac. fce, spojitá na $D_g = \mathbb{R}$

$g: \mathbb{R} \to \langle -1, 1 \rangle$

arcsin je spojitá na $\langle -1, 1 \rangle$

$\Rightarrow$ skládání spojití

f je spojitá na $D_f = \mathbb{R}$

$h(y) = \arcsin y$, $\quad h'(y) = \dfrac{1}{\sqrt{1-y^2}}$, $y \in (-1, 1)$!

$f = h \circ g$

věta o der. složené fce lze použít jen pro ta $x \in \mathbb{R}$, pro která $g(x) \in (-1, 1)$

$$g(x) = 1: \quad \frac{4x}{x^2+4} = 1 \quad \Big| \quad \Updownarrow$$

$$4x = x^2+4$$
$$(x-2)^2=0$$
$$\underline{x=2}$$

$\underline{g(x)=-1}$:
$$\frac{4x}{x^2+4}=-1$$
$$4x=-x^2-4$$
$$(x+2)^2=0$$
$$\underline{x=-2}$$

$$\underline{x \in \mathbb{R} \setminus \{-2,2\}}$$

$$\underline{f'(x)} = h'(g(x))\cdot g'(x) = \frac{1}{\sqrt{1-\left(\frac{4x}{x^2+4}\right)^2}} \cdot \frac{4\cdot(x^2+4)-4x\cdot 2x}{(x^2+4)^2} =$$

$$= \frac{1}{\sqrt{\frac{(x^2+4)^2-16x^2}{(x^2+4)^2}}} \cdot \frac{4x^2+16-8x^2}{(x^2+4)^2} = \frac{\cancel{x^2+4}}{\sqrt{(x^2+4+4x)(x^2+4-4x)}} \cdot \frac{16-4x^2}{(x^2+4)^{\cancel{2}}} =$$

$$= \frac{4(4-x^2)}{\sqrt{(x+2)^2(x-2)^2}\cdot(x^2+4)} = \frac{4(4-x^2)}{\sqrt{(x^2-4)^2}\cdot(x^2+4)} = \frac{4(4-x^2)}{\underbrace{|x^2-4|}_{|4-x^2|}(x^2+4)} =$$

$$4 \cdot \operatorname{sgn}(4-x^2)$$

$$= \frac{4 \cdot \operatorname{sgn}(\ldots)}{x^2+4}, \quad x \in \mathbb{R} \setminus \{-2, 2\}$$

$\frac{u}{|u|} = \operatorname{sgn} u, \ u \neq 0$

$f'(-2), f'(2)$?

z definice, např. $f'(2) = \lim_{x \to 2} \frac{f(x) - f(2)}{x-2} = \lim_{x \to 2} \frac{\arcsin \frac{4x}{x^2+4} - \frac{\pi}{2}}{x-2} = \ldots\ldots$

pomocí V44:

$$f'_+(2) = \lim_{x \to 2+} f'(x) = \lim_{x \to 2+} \frac{4 \operatorname{sgn}(4-x^2)}{x^2+4} = -\frac{1}{2}$$

$\operatorname{sgn}(4-x^2) = 1 \ \ldots \ -2 < x < 2$

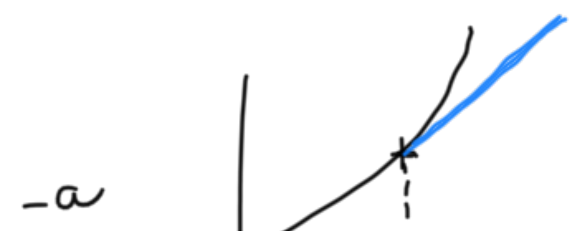
$\operatorname{sgn}(4-x^2) = -1 \ \ldots \ x > 2$

f je spojitá ve 2

$$f'_-(2) = +\frac{1}{2}$$

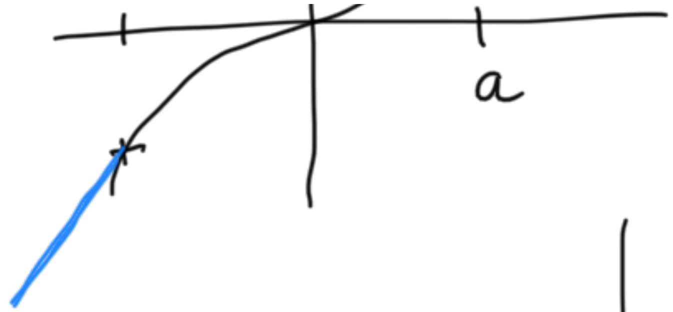
$$f'_+(-2) = \lim_{x \to -2+} f'(x) = \lim_{x \to -2+} \frac{4 \operatorname{sgn}(4-x^2)}{x^2+4} = \frac{1}{2}$$

$$f'_-(-2) = -\frac{1}{2}$$

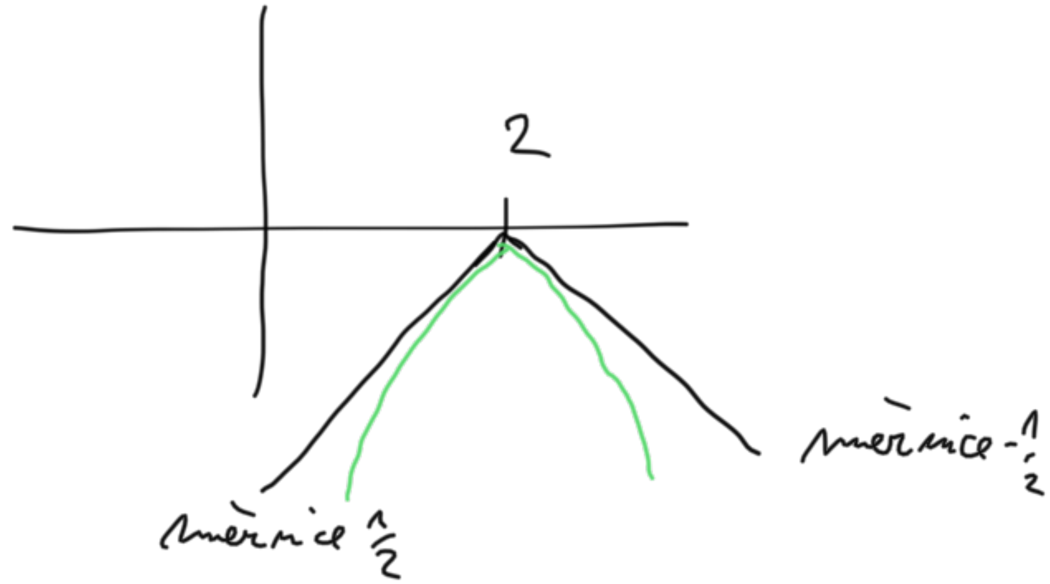


$$f'_+(a) = f'_-(-a)$$

$\searrow$ f je lichá



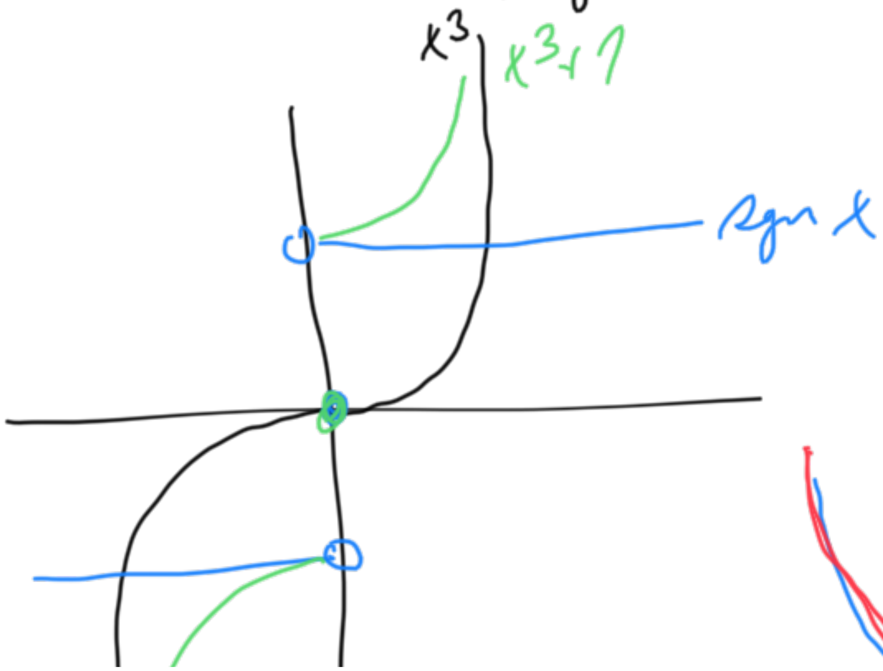
$f'(2)$ a $f'(-2)$ neexistují.



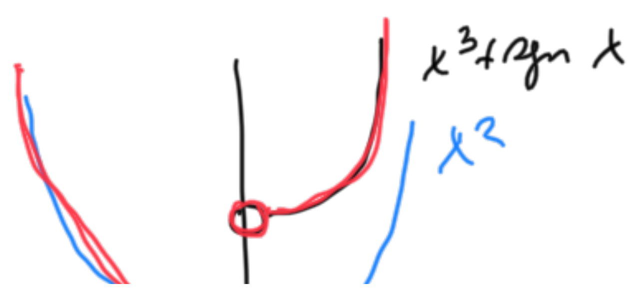
$$f(x) = \max\{x^2, x^3 + \operatorname{sgn} x\}$$

$D_f = \mathbb{R}$

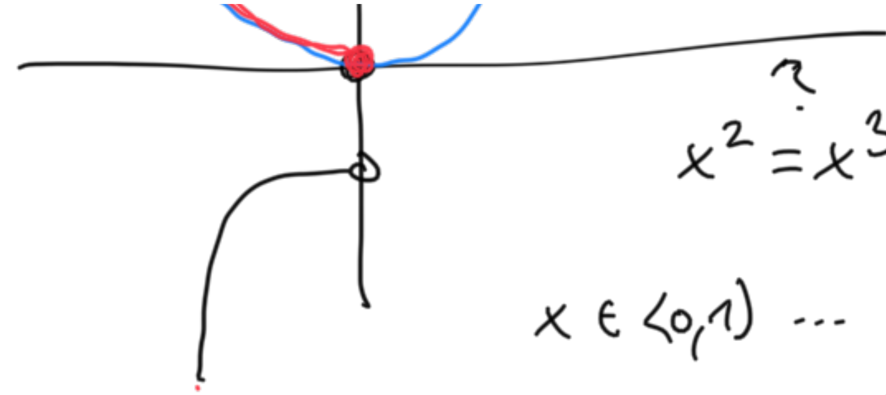
$$\left.\begin{array}{ll} x \mapsto x^2 & \text{spoj. na } \mathbb{R} \\ x \mapsto x^3 & \text{— " —} \\ x \mapsto \operatorname{sgn} x & \text{spoj. na } \mathbb{R} \setminus \{0\} \end{array}\right\} \Rightarrow f \text{ je spojitá na } \mathbb{R} \setminus \{0\}$$



$$x^3 + \operatorname{sgn} x = \begin{cases} x^3 + 1 & \dots\ x > 0 \\ 0 & \dots\ x = 0 \\ x^3 - 1 & \dots\ x < 0 \end{cases}$$



x^3-1



$$x^2 \overset{?}{=} x^3+1$$

$x \in \langle 0,1)$ ... $x^2 < 1$

$x^3+1 \geq 1$

$x > 1$ ... $x^3 > x^2$

$x^3+1 > x^2$

$x = 1$ ... $2 > 1$

$$f(x) = \begin{cases} x^2, & x \in (-\infty, 0\rangle \\ x^3+1, & x \in (0, +\infty) \end{cases}$$

$$\lim_{x \to 0+} f(x) = \lim_{x \to 0+} x^3+1 = 1 \quad \times$$

$$\lim_{x \to 0-} f(x) = \lim_{x \to 0-} x^2 = 0 = f(0)$$

$\Rightarrow$ f je spojitá zleva v 0

f není spojitá zprava v 0

$$f'(x) = \begin{cases} (x^2)' = 2x, & x \in (-\infty, 0) \\ (x^3+1)' = 3x^2, & x \in (0, +\infty) \end{cases}$$

$$f'_-(0) \overset{V44}{=} \lim_{x \to 0-} f'(x) = \lim_{x \to 0-} 2x = 0$$

f je spoj. v 0 zleva

$f'_+(0)$ NELZE počítat pomocí V44

z definice: $$f'_+(0) = \lim_{x \to 0+} \frac{f(x)-f(0)}{x-0} = \lim_{x \to 0+} \frac{x^3+1-0}{x} =$$

$$= \lim_{x \to 0+} \left(x^2 + \frac{1}{x}\right) \overset{AL}{=} +\infty$$

$$f(x) = \sqrt{\underbrace{\operatorname{arctg}(\log^2 x)}_{g(x)}}$$

log ... $(0,+\infty)$

$\sqrt{\ }$ .... $\operatorname{arctg}(\log^2 x) \geq 0$

$\log^2 x \geq 0$

$D_f = (0,+\infty)$

$\sqrt{\ }$, arctg, log spojité na svých def. oborech

aritmetika + skládání spoj. fcí $\Rightarrow$

f je spojitá na $D_f = (0,+\infty)$

$h(y) = \sqrt{y}$

$D_h = (0,+\infty)$

$D_h = (0,+\infty)$

$h'(y)$ existuje vlastní jen pro $y \in (0,+\infty)$

$$g(x) > 0 \iff \log^2 x > 0 \iff x \in (0,+\infty) \setminus \{1\}$$

$$f'(x) = \cancel{\frac{1}{2}} \frac{1}{\sqrt{\operatorname{arctg}(\log^2 x)}} \cdot \frac{1}{1+\log^4 x} \cdot \cancel{2} \log x \cdot \frac{1}{x}, \quad x \in (0,+\infty) \setminus \{1\}$$

$$f'_+(1) \overset{V44}{=} \lim_{x\to 1+} f'(x) = \lim_{x\to 1+} \frac{\log x}{x \cdot \sqrt{\operatorname{arctg}(\log^2 x)} \cdot (1+\log^4 x)} \overset{AL}{=} \cancel{?}$$

$$= \lim_{x\to 1+} \frac{\log x}{\sqrt{\operatorname{arctg}(\log^2 x)}} = \lim_{x\to 1+} \sqrt{\frac{\log^2 x}{\operatorname{arctg}(\log^2 x)}} =$$

$$= 1$$

$$\lim_{y\to 0} \frac{\operatorname{arctg} y}{y} = 1$$

VOLSF

vnitřní $\log^2 x \xrightarrow{x\to 1} 0$

(P)

$$f'_-(1) = \lim_{x\to 1-} = \dots\dots = \lim - \sqrt{\frac{\log^2 x}{\operatorname{arctg}(\log^2 x)}} = -1$$

$f'(1)$ neexistuje

$$f(x) = (x+2)^2 \cdot \sqrt{|x^2-4|}$$

$D_f = \mathbb{R}$, f je spojitá na $\mathbb{R}$ (aritmetika + skládání spoj. fcí)

$$x^2 - 4 = 0 \Leftrightarrow x = \pm 2$$

$$f'(x) = 2(x+2)\cdot 1 \cdot \sqrt{|x^2-4|} + (x+2)^2 \cdot \frac{1}{\cancel{2}\sqrt{|x^2-4|}} \cdot \operatorname{sgn}(x^2-4) \cdot \cancel{2}x,$$

$x \in \mathbb{R} \setminus \{-2, 2\}$

$g(y) = |y|$

$$g'(y) = \left\{\begin{array}{l} 1, \ y > 0 \\ -1, \ y < 0 \end{array}\right\} = \operatorname{sgn} y, \ \boxed{y \neq 0}$$

$$= \frac{2(x+2)\cdot|x^2-4| + (x+2)^2 x \cdot \operatorname{sgn}(x^2-4)}{\sqrt{|x^2-4|}}$$

$x \in (-2,2)$: $\dfrac{2(x+2)(4-x^2)+(x+2)^2 x(-1)}{\sqrt{4-x^2}} = \dfrac{\sqrt{4-x^2}\left(2(x+2)^2(2-x)-(x+2)^2\cdot x\right)}{4-x^2} =$

$$= \frac{\sqrt{4-x^2}\left((x+2)^2(4-2x-x)\right)}{(2-x)(2+x)} = \frac{(x+2)(4-3x)\sqrt{4-x^2}}{2-x} =$$

$$= \frac{(x+2)(3x-4)\sqrt{4-x^2}}{x-2}$$

$x>2$, $x<-2$: $\dfrac{2(x+2)(x^2-4)+(x+2)^2\cdot x\cdot 1}{\sqrt{x^2-4}} = \dfrac{(x+2)(3x-4)\sqrt{x^2-4}}{x-2}$

$$= \frac{(x+2)(3x-4)\sqrt{|x^2-4|}}{x-2}, \quad x \in \mathbb{R} \setminus \{-2,2\}$$

$$f'_+(2) \overset{U44}{=} \lim_{x\to 2+} f'(x) = \lim_{x\to 2+} \frac{(x+2)(3x-4)\sqrt{|x^2-4|}}{x-2} =$$

f je spojitá v 2

$$\overset{AL}{=} 4 \cdot 2 \cdot \lim_{x \to 2+} \frac{\sqrt{|x+2||x-2|}}{x-2} \overset{AL + \sqrt{}}{=}$$

$$= 4 \cdot 2 \cdot \sqrt{4} \lim_{x \to 2+} \sqrt{\frac{x-2}{(x-2)^2}} = 16 \cdot \lim_{x \to 2+} \frac{1}{\sqrt{x-2}} = +\infty$$

$$f'_-(2) = 16 \cdot \lim_{x \to 2-} -\frac{1}{\sqrt{2-x}} = -\infty$$

$$f'(-2) \underset{f \text{ spoj. v } -2}{\overset{V44}{=}} \lim_{x \to -2} f'(x) = \lim_{x \to -2} \frac{(x+2)(3x-4)\sqrt{|x^2-4|}}{x-2} \overset{AL}{=} 0$$

<u>Závěr</u>: $f'(x) = \dfrac{(x+2)(3x-4)\sqrt{|x^2-4|}}{x-2}$, $x \in \mathbb{R} \setminus \{2\}$

$f'_\pm(2) = \pm\infty$, $f'(2)$ neexistuje