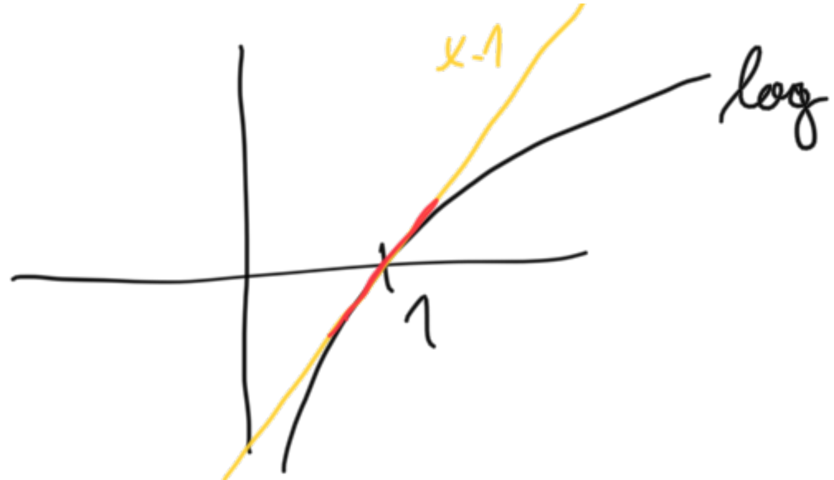
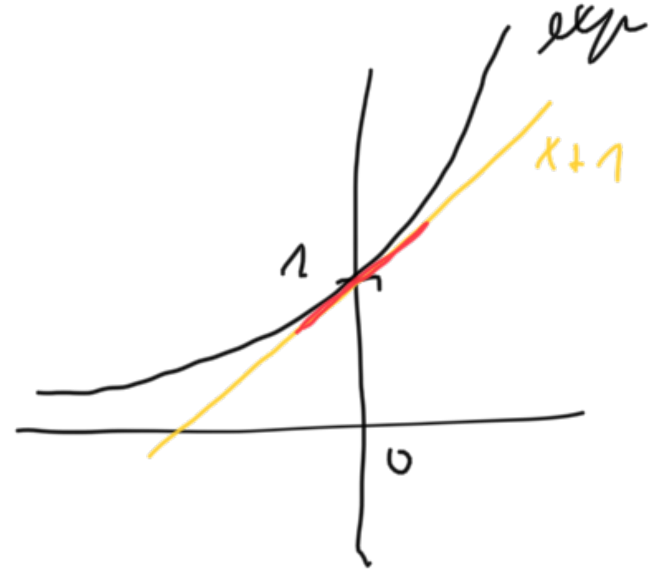
$$\lim_{x\to 1} \frac{\log x}{x-1} = 1$$



„x blízko 1, pak $\frac{\log x}{x-1}$ blízko 1, tj. $\log x$ blízko $x-1$"



$$\lim_{x\to 0} \frac{\exp x - 1}{x} = \lim_{x\to 0} \frac{e^x - 1}{x} = 1$$

„x blízko 0 ... $e^x - 1$ blízko x

e^x blízko $x+1$"

$$\lim_{x\to 1} \frac{\log(x^2)}{x^3-1} = \lim_{x\to 1} \frac{2\log x}{x^3-1} = \lim_{x\to 1} 2 \cdot \overset{\to 1}{\boxed{\frac{\log x}{x-1}}} \cdot \frac{x-1}{x^3-1} \overset{AL}{=} \cancel{?}$$

$$= 2\cdot 1 \cdot \lim_{x\to 1} \frac{x-1}{x^3-1} = 2 \lim_{x\to 1} \frac{\cancel{x-1}}{\cancel{(x-1)}(x^2+x+1)} = 2\cdot\frac{1}{3} = \frac{2}{3}$$

spojitost rac. fce

$$\lim_{x\to+\infty} \underbrace{x^2}_{+\infty} \cdot \underbrace{\log\frac{x^2+1}{x^2+5}}_{0} = \lim_{x\to+\infty} x^2 \cdot \boxed{\frac{\log\frac{x^2+1}{x^2+5}}{\frac{x^2+1}{x^2+5}-1}} \cdot \left(\frac{x^2+1}{x^2+5}-1\right) \overset{AL}{=} \cancel{?}$$

$$= 1 \cdot \lim_{x\to+\infty} x^2 \cdot \left(\frac{x^2+1}{x^2+5}-1\right) =$$

$$= \lim_{x\to+\infty} x^2 \cdot \frac{x^2+1-x^2-5}{x^2+5} =$$

$$= \lim_{x\to+\infty} \frac{-4x^2}{x^2+5} \overset{AL}{=} -4$$

↓ VOLSF

1 $f(y) = \frac{\log y}{y-1}$, $\lim_{y\to 1} f(y) = 1$

$g(x) = \frac{x^2+1}{x^2+5}$, $\lim_{x\to+\infty} g(x) = 1$

(P): $\frac{x^2+1}{x^2+5} = 1$

$x^2+1 = x^2+5$

$1 \neq 5$

mai vero, lo è: $\forall x \in \mathbb{R}$ $g(x) \neq 1$

$$\lim_{x\to+\infty} \frac{\overbrace{\log(x^3+15)}^{+\infty}}{\underbrace{\log(x^{15}+3)}_{+\infty}} = \lim_{x\to+\infty} \frac{\log\left(x^3\left(1+\frac{15}{x^3}\right)\right)}{\log\left(x^{15}\left(1+\frac{3}{x^{15}}\right)\right)} =$$

$\log(x^3)$ ... $\lim$ $\log(x^3) + \log(1+\frac{15}{\ldots})$

$$\frac{\log(\dots)}{\log(x^{15})} = \frac{3\log x}{15\log x} \qquad = \lim_{x\to+\infty} \frac{\log(\dots) + \log\left(1+\frac{\dots}{x^3}\right)}{\log(x^{15}) + \log\left(1+\frac{3}{x^{15}}\right)} =$$

$$= \lim_{x\to+\infty} \frac{3\overbrace{\log x}^{+\infty} + \overbrace{\log\left(\underbrace{1+\frac{15}{x^3}}_{\to 1}\right)}^{0}}{15\log x + \log\left(1+\frac{3}{x^{15}}\right)} = \lim_{x\to+\infty} \frac{3 + \frac{\log\left(1+\frac{15}{x^3}\right)}{\log x}}{15 + \frac{\log\left(1+\frac{3}{x^{15}}\right)}{\underbrace{\log x}_{\downarrow +\infty}}} \overset{AL}{=}$$

$\log\left(1+\frac{15}{x^3}\right) \to 0$, $\log\left(1+\frac{3}{x^{15}}\right) \to 0$

VOLSF $f(y) = \log y$, $\lim_{y\to 1} \log y = 0$

$g(x) = 1 + \frac{15}{x^3}$, $\lim_{x\to+\infty} g(x) \overset{AL}{=} 1$

(S): log je spojitá fce

$$= \frac{3 + \frac{0}{+\infty}}{15 + \frac{0}{+\infty}} = \frac{3}{15} = \frac{1}{5}$$

$$\lim_{x\to 0} \frac{2^x - 1}{x} = \lim_{x\to 0} \frac{\exp(x\cdot\log 2) - 1}{x} = \lim_{x\to 0} \boxed{\frac{\exp(x\cdot\log 2) - 1}{x\cdot\log 2}} \cdot \log 2 \overset{AL}{=}$$

$$\overset{AL}{=} 1\cdot\log 2 = \log 2$$

$\downarrow$ lineární

$\downarrow$ substituce $y = x\cdot\log 2$

$\underline{a>0}$

$$\lim_{x\to 0}\frac{a^x-1}{x}=\lim_{x\to 0}\frac{e^{x\cdot\log a}-1}{x\cdot\log a}\cdot\log a\overset{AL}{=}1\cdot\log a=\log a$$

$$\lim_{x\to 0}\frac{10^x-\overbrace{2^x}^{\to 1}}{x}\;\cancel{=}\;\lim_{x\to 0}\frac{10^x-1}{x}=\log 10$$

částečné dosazení !

$$\lim_{x\to 0}10^x=1$$

$$\hookrightarrow=\lim_{x\to 0}\frac{1-2^x}{x}=\lim_{x\to 0}-\frac{2^x-1}{x}=-\log 2$$

$$=\lim_{x\to 0}\frac{10^x-1+1-2^x}{x}=\lim_{x\to 0}\frac{10^x-1}{x}-\frac{2^x-1}{x}\overset{AL}{=}\log 10-\log 2=$$

$$=\log 5$$

jinak: $$\lim_{x\to 0}\frac{2^x\left(\frac{10^x}{2^x}-1\right)}{x}=\lim_{x\to 0}2^x\cdot\frac{5^x-1}{x}\overset{AL}{=}2^0\cdot\log 5=\log 5$$

spojitost $x\mapsto 2^x$

$$\lim_{x\to+\infty}\underbrace{\left(\frac{3x+1}{x+3}\right)}_{f}^{\overbrace{5x-x^2}^{g}}=\lim_{x\to+\infty}\exp\left(\underbrace{(5x-x^2)\cdot\log\frac{3x+1}{x+3}}\right)$$

$h(x)$

$$f(x)^{g(x)} = \exp\big(g(x)\cdot \log f(x)\big)$$ ... g je spojitá na I, f je spojitá a kladná na I $\Big\}\Rightarrow$

$x \mapsto f(x)^{g(x)}$ je spojitá na I

$$\lim_{x\to+\infty} h(x) = \lim_{x\to+\infty} \underbrace{(5x-x^2)}_{} \cdot \boxed{\log \frac{3x+1}{\underbrace{x+3}_{}}} \overset{AL}{=} -\infty \cdot \underbrace{\log 3}_{>0} = -\infty$$

$= x^2\left(\frac{5}{x}-1\right) \xrightarrow{AL} -\infty$

$\xrightarrow{AL} 3$

$\to \log 3$ VOLSF (S), vnější fce je log

$$\lim_{x\to+\infty} \exp(h(x)) = 0$$

VOLSF

$f(y) = \exp y$, $\lim_{y\to-\infty} f(y) = 0$ || (S) není splněna, protože počítáme lim v $-\infty$

vnitřní $h(x)$, $\lim_{x\to+\infty} h(x) = -\infty$

(P) je splněna automaticky:

$\forall x \in \mathbb{R}: h(x) \neq -\infty$

$$\lim_{x\to 0}\left(\frac{2^x+8^x}{2}\right)^{\frac{1}{x}} = \lim_{x\to 0} \exp\Big(\underbrace{\frac{1}{x}\cdot\log\frac{2^x+8^x}{2}}_{h(x)}\Big)$$

$$\lim_{x\to 0} h(x) = \lim_{x\to 0}\frac{1}{x}\cdot\log\underbrace{\frac{2^x+8^x}{2}}_{\downarrow\, 1} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0}\frac{1}{x}\cdot\boxed{\frac{\log\frac{2^x+8^x}{2}}{\frac{2^x+8^x}{2}-1}}\left(\frac{2^x+8^x}{2}-1\right) =$$

$$\overset{AL}{\underset{\cancel{?}}{=}} 1\cdot\lim_{x\to 0}\frac{1}{x}\cdot\left(\frac{2^x+8^x}{2}-1\right) =$$

$$= \lim_{x\to 0}\frac{1}{2}\cdot\frac{2^x+8^x-2}{x} =$$

$$= \lim_{x\to 0}\frac{1}{2}\cdot\frac{2^x-1+8^x-1}{x} \overset{AL}{=} \frac{1}{2}(\log 2+\log 8) = \frac{1}{2}\log 16 =$$

$$= \log 4$$

$\log 2$ $\quad$ $\log 8$

$\downarrow$ VOLSF

1

$f(y)=\frac{\log y}{y-1}$

$g(x)=\frac{2^x+8^x}{2}\xrightarrow[x\to 0]{} 1$

(P) ... g je rostoucí (součet rostoucích)

$$\lim_{x\to 0}\exp(h(x)) \underset{\text{VOLSF (S)}}{=} \exp(\log 4) = 4$$

vnější fce $y \mapsto \exp y$

vnitřní $h(x)$

$$\left|\begin{aligned} &\lim_{y \to \log 4} \exp y = \exp(\log 4) \\ &\lim_{x \to 0} h(x) = \log 4 \end{aligned}\right.$$