$$\lim \left(\overbrace{\sqrt{\underbrace{n+15}_{\downarrow\, +\infty}}}^{+\infty} - \overbrace{\sqrt{\underbrace{n+1}_{\downarrow\, +\infty}}}^{+\infty} \right) \overset{?}{\underset{AL}{=}} \cancel{\lim \sqrt{n+15} - \lim \sqrt{n+1} = (+\infty) - (+\infty)}$$

není def.

Trik: $a^2 - b^2 = (a-b)(a+b)$

$$\left(\sqrt{n+15} - \sqrt{n+1}\right) \frac{\left(\sqrt{n+15} + \sqrt{n+1}\right)}{\sqrt{n+15} + \sqrt{n+1}} = \frac{n+15-(n+1)}{\sqrt{n+15} + \sqrt{n+1}} = \frac{14}{\sqrt{n+15} + \sqrt{n+1}}$$

$$\lim \left(\sqrt{n+15} - \sqrt{n+1}\right) = \lim \frac{14}{\sqrt{n+15} + \sqrt{n+1}} \overset{\cancel{?}}{\underset{AL}{=}} \frac{14}{\underbrace{\lim \sqrt{n+15}}_{=+\infty} + \underbrace{\lim \sqrt{n+1}}_{=+\infty}} = \frac{14}{+\infty + (+\infty)} =$$

$$= \frac{14}{+\infty} = 0$$

"$\sqrt{a_n} \to \sqrt{a}$"

$$\lim \left(\overbrace{\sqrt{\underbrace{n^2+5n-1}_{+\infty}}}^{+\infty} - \overbrace{\sqrt{\underbrace{n^2+3}_{+\infty}}}^{+\infty} \right) = \lim \left(\sqrt{n^2+5n-1} - \sqrt{n^2+3}\right) \cdot \frac{\sqrt{n^2+5n-1} + \sqrt{n^2+3}}{\sqrt{n^2+5n-1} + \sqrt{n^2+3}} =$$

dominantní n

$$= \lim \frac{(n^2+5n-1) - (n^2+3)}{\sqrt{n^2+5n-1} + \sqrt{n^2+3}} = \lim \frac{\overbrace{5n-4}}{\underbrace{\sqrt{n^2+5n-1}}_{\text{dom. } n^2} + \underbrace{\sqrt{n^2+3}}_{\text{dom. } n^2}} \cancel{\underset{AL}{=} \frac{+\infty}{+\infty + \infty} = \frac{+\infty}{+\infty}} =$$

$$= \lim \frac{n\left(5 - \frac{4}{\ldots}\right)}{\ldots}$$

$\to 5$ AL

$$= \lim \frac{n(5 - \frac{4}{n})}{\sqrt{n^2(1+\frac{5}{n}-\frac{1}{n^2})} + \sqrt{n^2(1+\frac{3}{n^2})}} = \lim \frac{\cancel{n}\overbrace{(5-\frac{4}{n})}^{\text{AL}}}{\cancel{n}\left(\sqrt{\underbrace{1+\frac{5}{n}-\frac{1}{n^2}}_{\downarrow \text{ AL } 1}} + \sqrt{\underbrace{1+\frac{3}{n^2}}_{\downarrow \text{ AL } 1}}\right)} \stackrel{\text{AL}}{=}$$

$$\boxed{1} \xrightarrow{\text{„}\sqrt{a_n} \to \sqrt{a}\text{“}} 1 \qquad \boxed{1} \to 1$$

$$= \frac{5}{1+1} = \frac{5}{2}$$

$$\lim \left(\underbrace{\sqrt{n^4+1}}_{+\infty} - \underbrace{\sqrt{n^2+1}}_{+\infty} \right) = \lim n^2 \left(\overbrace{\sqrt{\underbrace{1+\frac{1}{n^4}}_{\downarrow \text{AL } 1}}}^{1 \;\; \sqrt{a_n} \to \sqrt{a}} - \overbrace{\sqrt{\underbrace{\frac{1}{n^2}+\frac{1}{n^4}}_{\downarrow \text{AL } 0}}}^{0} \right) \stackrel{\text{AL}}{=} +\infty \cdot (1+0) = +\infty$$

„nejsou stejně velké“

$$\sqrt{n^4} - \sqrt{n^2} = n^2 - n = \underbrace{n^2}_{+\infty}\Big(\underbrace{1-\frac{1}{n}}_{1}\Big) \to +\infty$$

$$\lim \frac{\sqrt[3]{n^3+1} - n}{\sqrt[4]{n^4+1} - \sqrt{n^2+1}} = \lim \frac{\sqrt[3]{n^3+1} - \sqrt[3]{n^3}}{\sqrt[4]{n^4+1} - \sqrt[4]{(n^2+1)^2}} =$$

$\sqrt[3]{a}-\sqrt[3]{b}$

$a^3-b^3=(a-b)(a^2+ab+b^2)$

$a^4-b^4=(a-b)(a^3+a^2b+ab^2+b^3)$

$$=\lim \frac{\sqrt[3]{n^3+1}-\sqrt[3]{n^3}}{\sqrt[4]{n^4+1}-\sqrt[4]{(n^2+1)^2}}\cdot\frac{\left(\sqrt[3]{n^3+1}\right)^2+\sqrt[3]{n^3+1}\cdot\sqrt[3]{n^3}+\left(\sqrt[3]{n^3}\right)^2}{\left(\sqrt[3]{n^3+1}\right)^2+\sqrt[3]{n^3+1}\cdot\sqrt[3]{n^3}+\left(\sqrt[3]{n^3}\right)^2}\cdot\frac{\left(\sqrt[4]{n^4+1}\right)^3+\left(\sqrt[4]{n^4+1}\right)^2\sqrt[4]{(n^2+1)^2}+\sqrt[4]{n^4+1}\cdot\left(\sqrt[4]{(n^2+1)^2}\right)^2+\left(\sqrt[4]{(n^2+1)^2}\right)^3}{''}$$

$$=\lim \frac{n^3+1-n^3}{n^4+1-(n^2+1)^2}\cdot\boxed{\frac{\left(\sqrt[4]{n^4+1}\right)^3+\left(\sqrt[4]{n^4+1}\right)^2\sqrt{n^2+1}+\sqrt[4]{n^4+1}\cdot\left(\sqrt{n^2+1}\right)^2+\left(\sqrt{n^2+1}\right)^3}{\left(\sqrt[3]{n^3+1}\right)^2+\sqrt[3]{n^3+1}\cdot n+n^2}}=$$

$$=\lim \frac{1}{\underbrace{-2n^2}_{\downarrow AL}}\cdot\frac{n^3\left(\left(\sqrt[4]{1+\frac{1}{n^4}}\right)^3+\left(\sqrt[4]{1+\frac{1}{n^4}}\right)^2\cdot\sqrt{1+\frac{1}{n^2}}+\sqrt[4]{1+\frac{1}{n^4}}\cdot\left(\sqrt{1+\frac{1}{n^2}}\right)^2+\left(\sqrt{1+\frac{1}{n^2}}\right)^3\right)}{n^2\left(\left(\sqrt[3]{1+\frac{1}{n^3}}\right)^2+\sqrt[3]{1+\frac{1}{n^3}}\cdot 1+1\right)}=$$

$\frac{+\infty}{+\infty}$

$\frac{1}{-\infty}=0$

$\sqrt[4]{a_n}\to\sqrt[4]{a}$

$$=\lim \frac{1}{-2n}\cdot\frac{\left(\sqrt[4]{\underbrace{1+\frac{1}{n^4}}_{AL\to 1}}\right)^3+\left(\sqrt[4]{\underbrace{1+\frac{1}{n^4}}_{AL\to 1}}\right)^2\sqrt{1+\frac{1}{n^2}}+\sqrt[4]{1+\frac{1}{n^4}}\left(1+\frac{1}{n}\right)+\left(\sqrt{1+\frac{1}{n}}\right)^3}{\left(\sqrt[3]{1+\frac{1}{n^3}}\right)^2+\sqrt[3]{1+\frac{1}{n^3}}+1}=$$

$1 \leftarrow AL$ $\to 1$ AL $\to 1$ AL

$\downarrow AL$ 1 $\quad\quad$ $\downarrow AL$ 1

$\downarrow \sqrt[3]{a_n} \to \sqrt[3]{a}$ $\quad\quad$ $\downarrow$

$\sqrt[3]{1} = 1$ $\quad\quad$ 1

$\downarrow$ AL (rovnini)

$$\overset{AL}{=} \frac{1}{-\infty} \cdot \frac{1 + 1\cdot 1 + 1\cdot 1 + 1}{1+1+1} = \frac{1}{-\infty}\cdot\frac{4}{3} = 0$$

$$\boxed{\lim \sqrt[n]{n} = 1}$$

$k \in \mathbb{N}$ daluo pevnĕ, nemĕní se

$$\sqrt[k]{a_n} \to \sqrt[k]{a}$$

a) $\forall n \in \mathbb{N}: \sqrt[n]{n} \geq 1$

$$n \geq 1 \quad / \sqrt[n]{\ }$$
$$\sqrt[n]{n} \geq \sqrt[n]{1} = 1$$

$\sqrt[1000]{1000} \doteq 1{,}007$

TIP: $\lim \sqrt[n]{n} = 1$

b) ukážu, že $\lim \underbrace{\overbrace{\sqrt[n]{n} - 1}^{a_n}}_{\geq 0} = 0$

$\sqrt[n]{\ } \quad 1$

$a_n = \sqrt[n]{n} - 1$

$a_n + 1 = \sqrt[n]{n}$

$$n = (1+a_n)^n = 1 + n\cdot a_n + \frac{n(n-1)}{2}\cdot a_n^2 + \dots + a_n^n \geq n\cdot a_n$$

$n \geq n\cdot a_n$

$a_n \leq 1$

na nic

$$\geq \frac{n(n-1)}{2} a_n^2$$

$$n \geq \frac{n(n-1)}{2} a_n^2$$

$$a_n^2 \leq \frac{2}{n-1}$$

$$0 \leq a_n \leq \sqrt{\frac{2}{n-1}}$$

$\downarrow$ 0 $\quad \downarrow$ 2POL 0 $\quad \downarrow$ 0 $\quad \sqrt{b_n} \to \sqrt{b}$

$a \in \mathbb{R},\ a > 0$

$$\lim \sqrt[n]{a} = 1$$

1) $\underline{a \geq 1}$: $\exists n_0 \in \mathbb{N} : n_0 \geq a$ $\quad n$

$$\forall n \in \mathbb{N},\ n \geq n_0: \quad 1 \leq \sqrt[n]{a} \leq \sqrt[n]{n}$$

$$(a \leq n_0 \leq n) \qquad 1 \xleftarrow{} \qquad \sqrt[n]{a} \xrightarrow{2POL} 1 \qquad \sqrt[n]{n} \to 1$$

2) $\underline{a < 1}$:

$$b_n = \frac{1}{a_n} = \frac{1}{\sqrt[n]{a}} = \sqrt[n]{\frac{1}{a}}, \quad \frac{1}{a} > 1 \quad \Rightarrow \lim b_n = 1 \text{ dle 1)}$$

$$\lim a_n = \lim \frac{1}{b_n} \overset{\cancel{?}}{\underset{AL}{=}} \frac{1}{\lim b_n} = \frac{1}{1} = 1$$