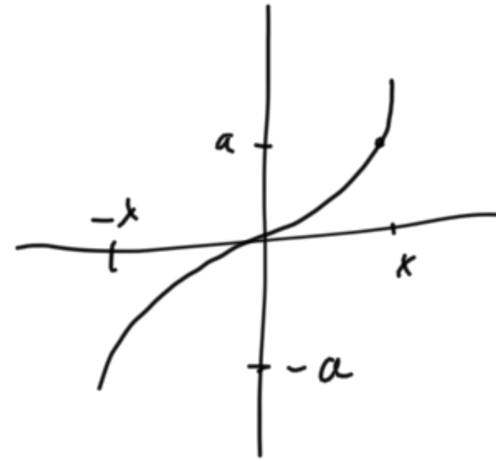
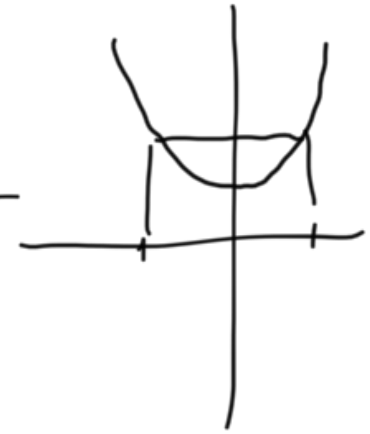
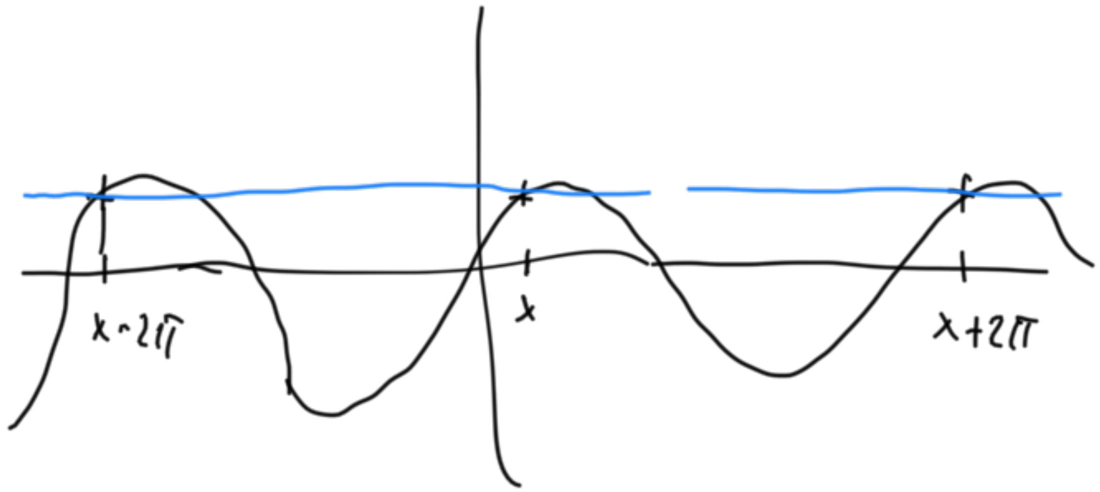
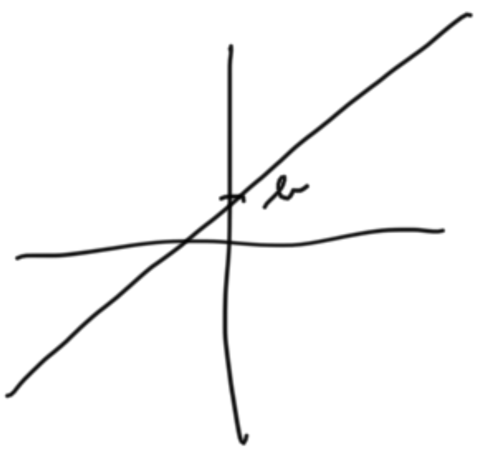


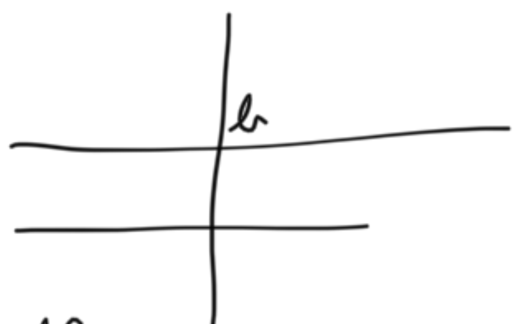




<u>Př.:</u> $a, b \in \mathbb{R}$ $f(x) = ax + b,\ x \in \mathbb{R}$ ..... afinní funkce

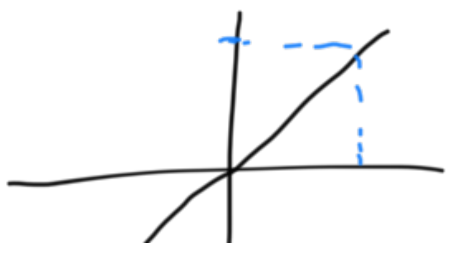


<u>$a=0$</u> .... $f(x)=b$ .... konstantní funkce



$H_f = \{b\}$

<u>$b=0$</u> ... $f(x)=ax$ ..... lineární funkce



$a \neq 0$ $H_f = \mathbb{R}$

Nechť $y \in \mathbb{R}$.
Pak pro $x = \frac{y}{a}$
je $f(x) = a\frac{y}{a} = y$

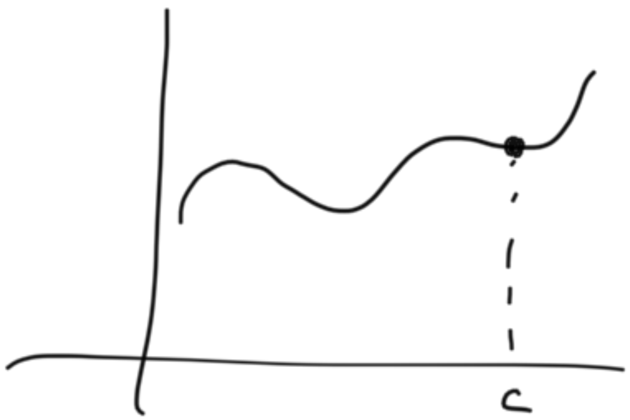
lichá $f(-x) = a(-x) = -ax = -f(x)$

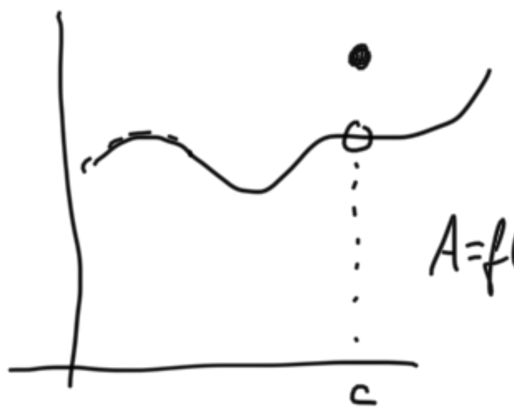
$a > 0$ rostoucí

$a < 0$ klesající

$$B(c,\varepsilon) = \{x \in \mathbb{R};\ |x-c| < \varepsilon\}$$

$$P(c,\varepsilon) = B(c,\varepsilon) \setminus \{c\} = (c-\varepsilon, c) \cup (c, c+\varepsilon) = \{x \in \mathbb{R};\ 0 < |x-c| < \varepsilon\}$$





$A = f(c)$

$$\lim_{x \to c} f(x) = A$$

$$\forall \varepsilon > 0\ \exists \delta > 0\ \underline{\forall x \in P(c,\delta)}: f(x) = B(A,\varepsilon)$$

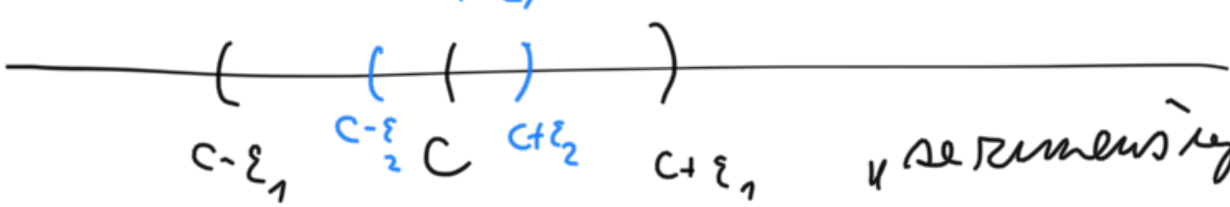
$\forall x \in B(c,\delta)$ $\qquad$ $= B(f(c),\varepsilon)$

$x = c$: $f(c) \in B(f(c),\varepsilon)$

automaticky

$\varepsilon_2 < \varepsilon_1$

$B(c,\varepsilon_2) \subset B(c,\varepsilon_1)$

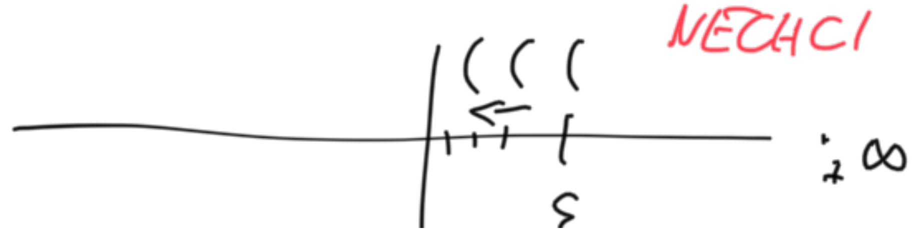


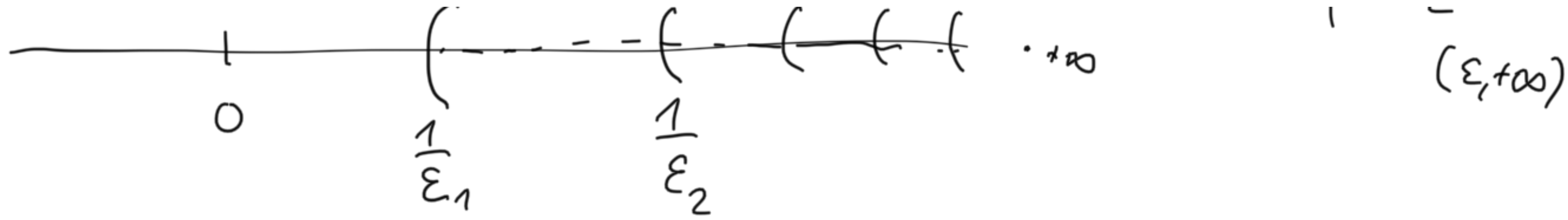
„zmenšující se ε se $B(c,\varepsilon)$ zmenšuje

(přibližuje se k bodu c)"

CHCI $\qquad \varepsilon_2 < \varepsilon_1$

NECHCI





$$\lim a_n = A \in \mathbb{R}^* \iff \forall \varepsilon \in \mathbb{R}, \varepsilon > 0 \ \exists n_0 \in \mathbb{N} \ \forall n \geq n_0 : a_n \in B(A, \varepsilon)$$

Předpokládejme, že $A \in \mathbb{R}$ je limitou f v c & $+\infty$ je limitou f v c.

Zvolme $\varepsilon > 0$ tak, aby $A + \varepsilon < \frac{1}{\varepsilon}$.

$\varepsilon < 1$ & $\frac{1}{\varepsilon} > |A| + 1$

$\varepsilon < \frac{1}{|A|+1}$

$A + \varepsilon \leq |A| + \varepsilon < |A| + 1 < \frac{1}{\varepsilon}$

To znamená, že $B(A, \varepsilon) \cap B(+\infty, \varepsilon) = \emptyset$.

$\exists \delta_1 > 0 \ \forall x \in P(c, \delta_1) : f(x) \in B(A, \varepsilon)$

$\exists \delta_2 > 0 \ \forall x \in P(c, \delta_2) : f(x) \in B(+\infty, \varepsilon)$

Položme $\delta = \min\{\delta_1, \delta_2\}$. Pak $\forall x \in P(c, \delta)$ $f(x) \in B(A, \varepsilon)$ & $f(x) \in B(+\infty, \varepsilon)$

a to je spor □

Pozn.: 1) Jestliže $\lim\limits_{x \to c} f(x) = A$, pak f nemusí být definována na nějakém prstencovém okolí c.

V bodě c může a nemusí být f definována.

2) Limitu můžeme počítat v bodě — vlastním ($c \in \mathbb{R}$) / nevlastním ($c = \pm\infty$)

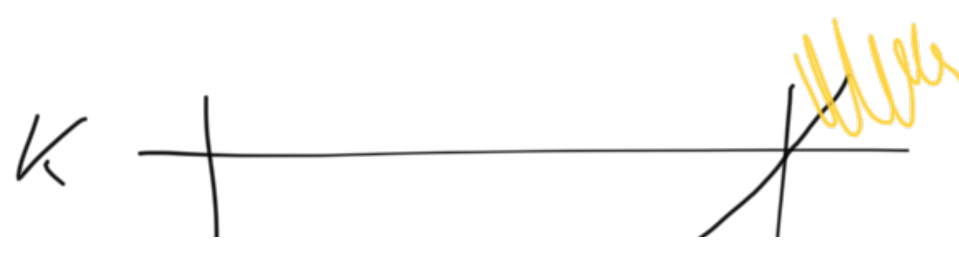
$$\lim_{x \to c} f(x) = \begin{cases} \text{neexistuje} \\ \text{existuje} \begin{cases} \text{vlastní, tj. } \in \mathbb{R} \\ \text{nevlastní, tj. } \begin{cases} +\infty \\ -\infty \end{cases} \end{cases} \end{cases}$$

3) Pro $c \in \mathbb{R}$, $A \in \mathbb{R}$

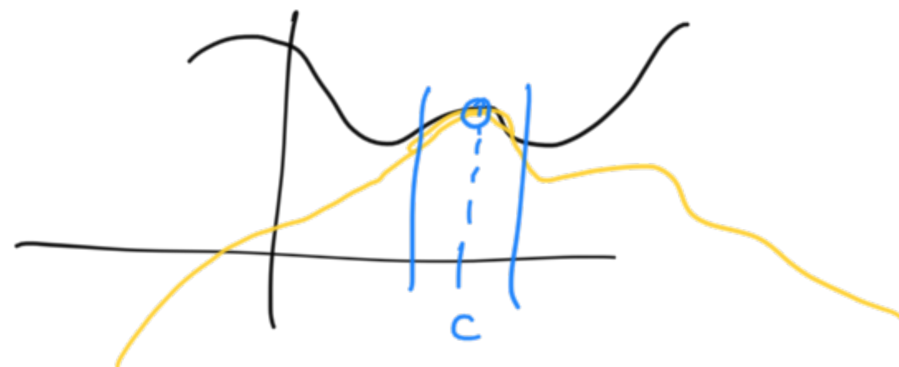
$$\lim_{x \to c} f(x) = A \iff \forall \varepsilon \in \mathbb{R}, \varepsilon > 0 \ \exists \delta \in \mathbb{R}, \delta > 0 : \forall x \in \mathbb{R} : 0 < |x - c| < \delta \Downarrow |f(x) - A| < \varepsilon.$$

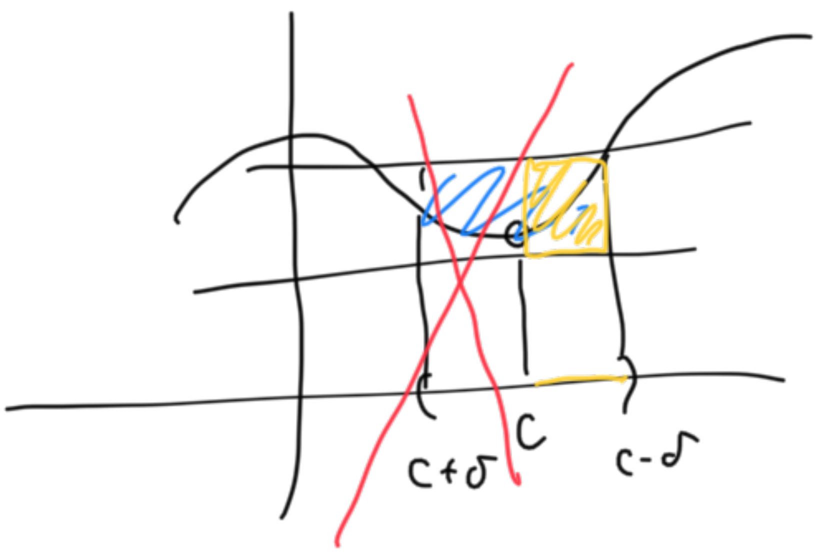
nebo pro $c = +\infty$, $A = +\infty$

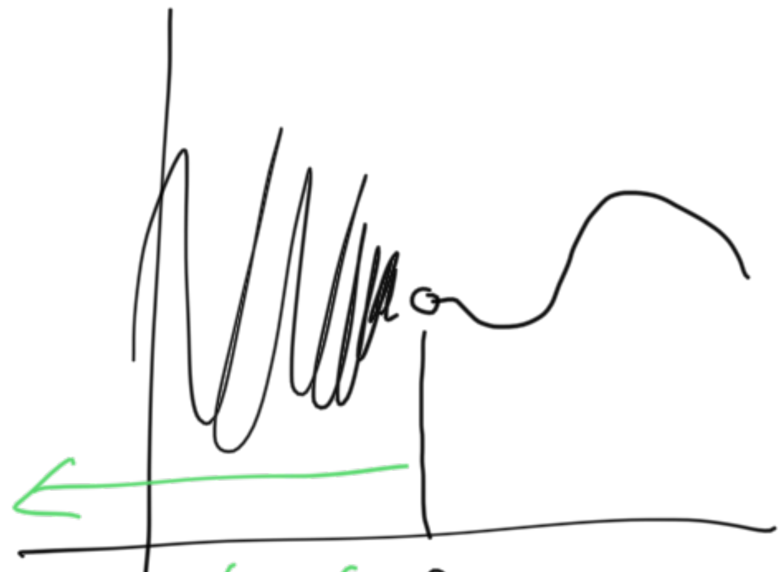
$$\lim_{x \to +\infty} f(x) = +\infty \iff \forall K \in \mathbb{R} \ \exists L \in \mathbb{R} \ \forall x \in \mathbb{R} : x > L \Rightarrow f(x) > K$$



4) Jestliže se fce f a g rovnají na nějakém prstencovém okolí $c \in \mathbb{R}^*$, pak jestliže existuje jedna z limit $\lim_{x \to c} f(x)$, $\lim_{x \to c} g(x)$, pak existuje i druhá a rovnají se.







$\lim\limits_{x\to c} f(x)=A$: $\quad \forall \varepsilon>0\ \exists \delta>0\ \forall x\in P(c,\delta): f(x)\in B(A,\varepsilon)$

$\Downarrow$

$\lim\limits_{x\to c+} f(x)=A \quad \forall \varepsilon>0\ \exists \delta>0\ \forall x\in P^+(c,\delta): f(x)\in B(A,\varepsilon)$

$\lim\limits_{x\to c-} f(x)=A \quad \forall \varepsilon>0\ \exists \delta>0\ \forall x\in P^-(c,\delta): f(x)\in B(A,\varepsilon)$

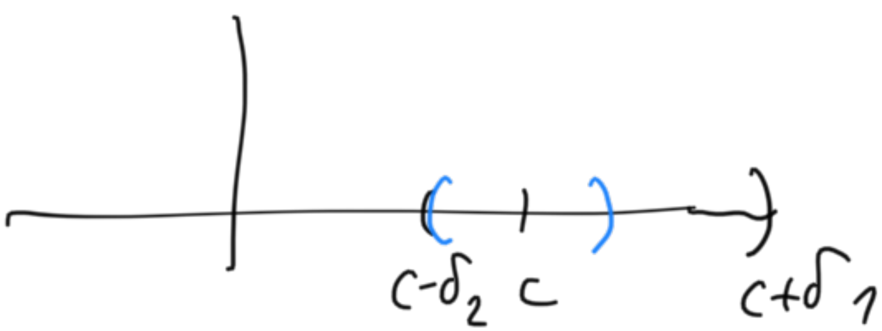
$P^+(c,\delta)\subset P(c,\delta)$

$P^-(c,\delta)\subset P(c,\delta)$

„$\Uparrow$" Nechť $\varepsilon>0$.

$\exists \delta_1>0: \forall x\in P^+(c,\delta_1): f(x)\in B(A,\varepsilon)$

$\exists \delta_2>0 \quad \forall x\in P^-(c,\delta_2): f(x)\in B(A,\varepsilon)$



Položím $\delta=\min\{\delta_1,\delta_2\}$.

Pak $\forall x\in P(c,\delta)$

$x\in(c,c+\delta) \Rightarrow x\in(c,c+\delta_1)\in P^+(c,\delta_1)$

$x \in (c-\delta_1, c) \Rightarrow x \in (c-\delta_2, c) \in P^-(c, \delta_2)$

a tedy $f(x) \in B(A, \varepsilon)$