$$\lim_{x\to 0+} \frac{\sqrt{e^x - \cos\sqrt{x}}}{x^2} = \lim_{x\to 0+} \sqrt{\frac{e^x - \cos\sqrt{x}}{x^4}}$$

$$\frac{e^x - 1}{x}\cdot\frac{1}{x^3} + \frac{1-\cos\sqrt{x}}{x}\cdot\frac{1}{x^3}$$

$$e^{g(x)} = \lim_{y\to -\infty} e^y$$

$$\lim_{x\to 0} g(x) = -\infty$$

VOCSF

(P) $\exists \cancel{\delta > 0 : \forall x \in B(0,} \delta) : g(x) \neq -\infty$

$\forall x \in D_g$

$$f(y) = e^y$$

$$\log(2^n + 3^n + 4^n) = n\log 4 + \log\left(1 + \frac{1}{2^n} + \left(\frac{3}{4}\right)^n\right)$$

HEINE

pro spojitost

(log je spojitý v 1), $a_n = 1 + \frac{1}{2^n} + \left(\frac{3}{4}\right)^n \to 1$

$$\lim \log(\cdots) = \log 1 = 0$$

$\underline{\sqrt{a_n} \to \sqrt{a}}$ | $\underline{\text{HEINE}}$

$$\lim_{x\to 0-} \left(\frac{(\cos x)^x + 2^x}{2}\right)^{\frac{1}{x}} = \lim \exp\Big(\underbrace{\tfrac{1}{x} \log(\cdots)}\Big)$$

$f(x)^{g(x)}$

$g(x) = x - \frac{\pi}{2}$ prosté

VOLSF

$$\downarrow \quad \lim \frac{3\sin 2(y+\frac{\pi}{2}) - 2\cos(y+\frac{\pi}{2})}{\cdots} = \frac{\cdots}{\cdots}$$

$$\lim_{x\to\frac{\pi}{2}} \frac{3\sin 2x - 2\cos x}{\sin 4x} = \lim_{y\to 0} \qquad \sin 4\left(y+\frac{\pi}{2}\right) \qquad \lim_{y}$$

$y = x-\frac{\pi}{2}$ $\qquad x\to\frac{\pi}{2} \Rightarrow y\to 0$

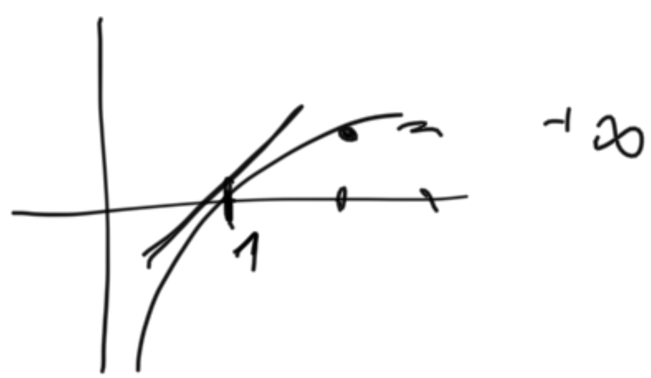
$x = y+\frac{\pi}{2}$

$$\lim_{x\to\frac{\pi}{4}} (\operatorname{tg} x)^{\operatorname{tg} 2x} = \exp \lim \left(\operatorname{tg} 2x \cdot \underbrace{\lg \overbrace{\operatorname{tg} x}^{\to 1}}_{(\operatorname{tg} x - 1)}\right) \text{ VOLSF}$$

$$\frac{\sin 2x}{\cos 2x}\left(\frac{\sin x-\cos x}{\cos x}\right) = \frac{2\sin x\,\cancel{\cos x}}{\underbrace{\cancel{\cos^2 x-\sin^2 x}}_{\cos x+\sin x}} \cdot \frac{\cancel{\sin x-\cos x}}{\cancel{\cos x}}$$

$$\lim_{x\to+\infty} (x+2)\lg(2x+1) - (x+1)\lg(2x+2) - \lg x$$

$$x\left(\lg(\ ) - \lg(\ )\right) + \underbrace{2\lg - 1\lg - \lg}$$



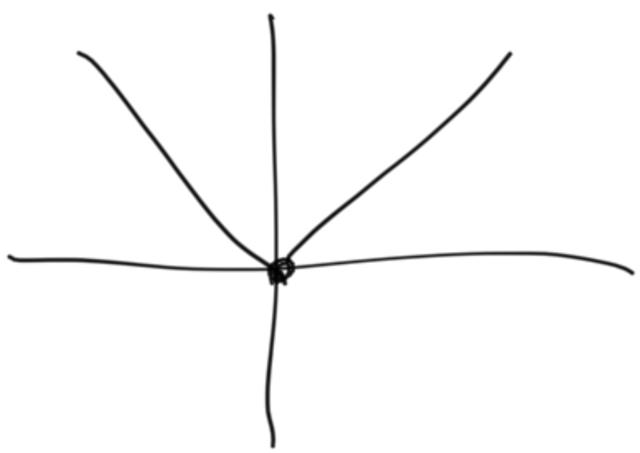
$f(x) = x\operatorname{sgn} x$ $\qquad$ Spojitá na $\mathbb{R}\setminus\{0\}$

(x spoj. na $\mathbb{R}$
sg spoj. na $\mathbb{R}\setminus\{0\}$, aritmetika spoj. fcí)

$$\lim_{x\to 0} f(x) = 0 = f(0) \quad \dots \quad f \text{ je spoj. v } 0$$

nulová x omezená

dokonaly f je spoj. na $\mathbb{R} = D_f$



$x \cdot 1$

$x \cdot (-1)$

$f(x) = |x|$

$(-1)^n \cdot \frac{1}{n}$

$$\underbrace{(-1)^n a_n}_{b_n} \begin{cases} a_n \to 0 \Rightarrow b_n \to 0 \\ a_n \to A \in \mathbb{R}^* \setminus \{0\} \Rightarrow b_n \text{ nemá limitu} \\ a_n \text{ nemá lim. } \dots \text{ nevíme} \end{cases}$$

$$(-1)^n \cdot (-1)^n = 1$$

$$\lim \left(\frac{\log n}{\log(n+1)}\right)^{n \cdot \log n} \overset{=}{\text{HEINE}} \lim_{x \to +\infty} \left(\frac{\lg x}{\lg(x+1)}\right)^{x \lg x} =$$

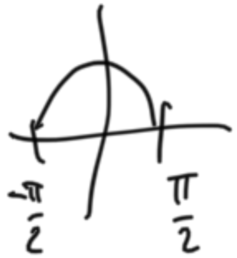
$$= \lim_{x \to +\infty} \exp\left(x \lg x \underbrace{\boxed{\lg\left(\frac{\lg x}{\lg(x+1)}\right)}}\right)$$

$$\lim_{x \to +\infty} \frac{\log x}{\log(x+1)} = \lim_{x \to +\infty} \frac{\lg x}{\lg\left(x\left(1+\frac{1}{x}\right)\right)} = \lim \frac{\lg x}{\lg x + \lg\left(1+\frac{1}{x}\right)} =$$

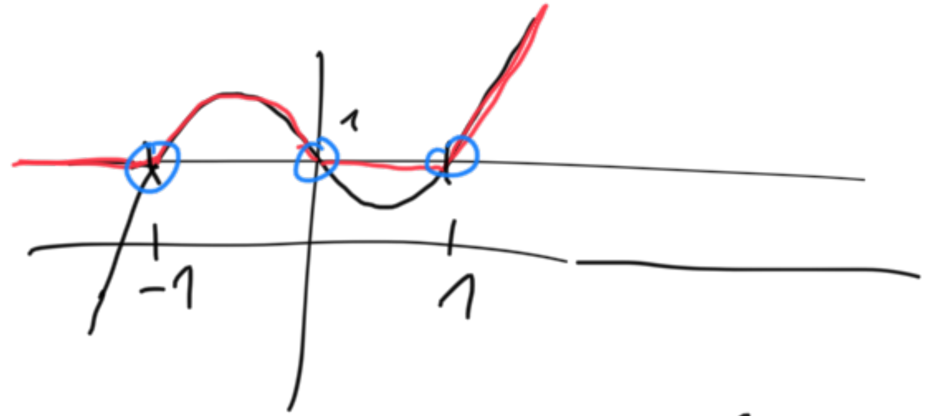
$$= \lim \frac{1}{1 + \frac{\overbrace{\lg\left(1+\frac{1}{x}\right)}^{\to 0}}{\underbrace{\lg x}_{\to +\infty}}} \overset{AL}{=} \frac{1}{1+0} = 1$$

$$f(x) = (\cos x)^{\max\{1, x^3 - x + 1\}} = \exp\left(\max(\dots) \cdot \lg \cos x\right)$$

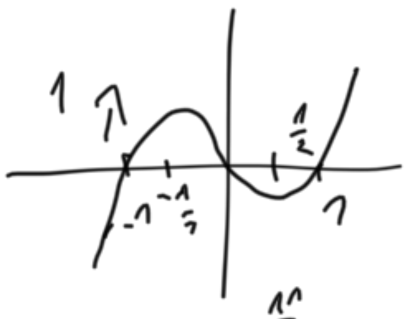
$x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

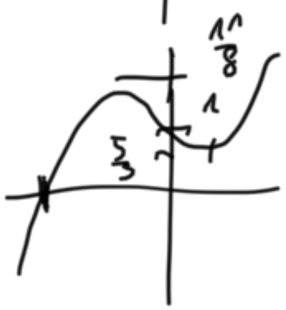


f je spojitá na D_f (aritmetika + skládání spojitých)



$$g(x) = \max\{1, \underbrace{x^3 - x + 1}_{h}\}$$





$$x(x^2-1)+1$$

$$x(x+1)(x-1)+1$$

$$h\left(\frac{1}{2}\right) = \frac{1}{8} - \frac{1}{2} + 1 = \frac{1}{8} + \frac{1}{2} = \frac{5}{8}$$

$$h\left(-\frac{1}{2}\right) = -\frac{1}{8} + \frac{1}{2} + 1 = \frac{11}{8}$$

$$g(x) = \begin{cases} 1, & x \in (-\infty, -1\rangle \\ x^3 - x + 1, & x \in \langle -1, 0\rangle \\ 1, & x \in \langle 0, 1\rangle \\ x^3 - x + 1, & x \in \langle 1, +\infty) \end{cases}$$

$$f(x) = \exp(g(x) \cdot \log \cos x)$$

$$f'(x) = \dots\dots\dots \begin{cases} x \in \left(-\frac{\pi}{2}, -1\right) \cup (0, 1) \quad g(x) = 1 \\ x \in (-1, 0) \cup \left(1, \frac{\pi}{2}\right) \end{cases}$$

$f'(0) = \dots$

$f(0) = \lim_{x \to 0} f'(x)$ $\qquad g(x) = x^3 - x + 1$

spigolose

$$f'_+(0) = \lim_{x \to 0^+} f'(x) = \dots$$
$$f'_-(0) = \lim_{x \to 0^-} f'(x) = \dots$$

$$\lim \frac{\log(n^3 - 12n + 1) - 3\log n}{\log(n^2 + 12) - 2\log n} = \lim \frac{\log\left(1 - \frac{12}{n^2} + \frac{1}{n^3}\right)}{\log\left(1 + \frac{12}{n^2}\right)} =$$

HEINE
VOLSF
2X

$$= \lim_{x \to +\infty} \frac{\frac{12}{x^2}}{-\frac{12}{x^2} + \frac{1}{x^3}} = \lim \frac{12}{-12 + \frac{1}{x}}$$

$$\frac{\log\left(1 - \frac{12}{x^2} + \frac{1}{x^3}\right)}{-\frac{12}{x^2} + \frac{1}{x^3}} \cdot \left(-\frac{12}{x^2} + \frac{1}{x^3}\right)$$