Najděte všechna řešení soust. $Ax=b$ pro tři vektory pravých stran:

$$A=\begin{pmatrix}1&2&3\\7&8&9\\13&14&15\end{pmatrix},\quad b_1=\begin{pmatrix}4\\10\\16\end{pmatrix},\quad b_2=\begin{pmatrix}5\\11\\12\end{pmatrix},\quad b_3=\begin{pmatrix}6\\12\\16\end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc}1&2&3&4&5&6\\7&8&9&10&11&12\\13&14&15&16&12&16\end{array}\right)\sim\begin{array}{r}\\-7\cdot\mathrm{I}\\-2\cdot\mathrm{II}\end{array}\left(\begin{array}{ccc|ccc}1&2&3&4&5&6\\0&-6&-12&-18&-24&-30\\-1&-2&-3&-4&-10&-8\end{array}\right)\sim$$

$$\sim\begin{array}{r}\\ \\ \cdot\left(-\frac{1}{6}\right)\\+\mathrm{I}.\end{array}\begin{array}{c}\begin{array}{ccc}x&y&z\end{array}\\ \left(\begin{array}{ccc|c|c|c}1&2&3&4&5&6\\0&1&2&3&4&5\\0&0&0&0&-5&-2\end{array}\right)\\ \begin{array}{cc}\neq 0&\neq 0\end{array}\end{array}$$

Pro b_2 a b_3 soustava nemá řešení!

Pro b_1:

$z=t\in\mathbb{R}$

$y+2t=3$ | $x+2(3-2t)+3t=4$

$y=3-2t$ | $x=-2+t$

řešení: $\{[-2+t,\ 3-2t,\ t];\ t\in\mathbb{R}\}$

Pro která $b=\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}$ má soustava $Ax=b$ řešení?

$$A = \begin{pmatrix} 1 & 2 & 2 & 2 \\ 2 & 2 & 2 & 0 \\ 1 & 4 & 3 & 4 \\ 5 & 4 & 1 & -8 \end{pmatrix} \quad , \quad \text{řešte pro } b = \begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \end{pmatrix}.$$

$$\left(\begin{array}{cccc|c} 1 & 2 & 2 & 2 & b_1 \\ 2 & 2 & 2 & 0 & b_2 \\ 1 & 4 & 3 & 4 & b_3 \\ 5 & 4 & 1 & -8 & b_4 \end{array}\right) \sim \begin{array}{c} \\ -2\cdot I. \\ -I. \\ -5\cdot I. \end{array} \left(\begin{array}{cccc|c} 1 & 2 & 2 & 2 & b_1 \\ 0 & -2 & -2 & -4 & b_2 - 2b_1 \\ 0 & 2 & 1 & 2 & b_3 - b_1 \\ 0 & -6 & -9 & -18 & b_4 - 5b_1 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{cccc|c} 1 & 2 & 2 & 2 & b_1 \\ 0 & 2 & 1 & 2 & b_3 - b_1 \\ 0 & -2 & -2 & -4 & b_2 - 2b_1 \\ 0 & -6 & -9 & -18 & b_4 - 5b_1 \end{array}\right) \sim \begin{array}{c} \\ \\ +II. \\ +3\cdot II. \end{array} \left(\begin{array}{cccc|c} 1 & 2 & 2 & 2 & b_1 \\ 0 & 2 & 1 & 2 & b_3 - b_1 \\ 0 & 0 & -1 & -2 & b_2 + b_3 - 3b_1 \\ 0 & 0 & -6 & -12 & 3b_3 + b_4 - 8b_1 \end{array}\right) \sim$$

$$\sim \begin{array}{c} \\ \\ \\ -6\cdot III. \end{array} \left(\begin{array}{cccc|c} 1 & 2 & 2 & 2 & b_1 \\ 0 & 2 & 1 & 2 & b_3 - b_1 \\ 0 & 0 & -1 & -2 & b_2 + b_3 - 3b_1 \\ 0 & 0 & 0 & 0 & -6b_2 - 3b_3 + b_4 + 10b_1 \end{array}\right)$$

v Kronecker. tvrzení $\Rightarrow$ soustava má řeš. $\Leftrightarrow$

$$-6b_2-3b_3+b_4+10b_1=0$$

$$\Leftrightarrow \boxed{b_4=-10b_1+6b_2+3b_3}$$

Pro $b=\begin{pmatrix}1\\1\\1\\-1\end{pmatrix}$:

$$\left(\begin{array}{cccc|c} 1 & 2 & 2 & 2 & 1 \\ 0 & 2 & 1 & 2 & 0 \\ 0 & 0 & -1 & -2 & -1 \end{array}\right)$$

$$\left\{[2s, -\tfrac{1}{2}, 1-2s, s],\ s\in\mathbb{R}\right\}$$

$f:\mathbb{R}^3\to\mathbb{R}^2$, $f(x,y,z)=[x-y,\ y+3z]$.

Je f lineární? Pokud ano, najděte reprez. matici.

a) ověříme z def.:

$c\in\mathbb{R}$ libovolné: $f(cx,cy,cz)=[cx-cy,\ cy+3cz]=[c(x-y), c(y+3z)]=$

$$=c[x-y,\ y+3z]=c\,f(x,y,z)$$

$f(x_1+x_2, \ldots$

$$f(x_1+x_2, y_1+y_2, z_1+z_2) = [x_1+x_2-(y_1+y_2),\ y_1+y_2+3(z_1+z_2)] =$$

$$= [(x_1-y_1)+(x_2-y_2),\ (y_1+3z_1)+(y_2+3z_2)] =$$

$$= [x_1-y_1,\ y_1+3z_1] + [x_2-y_2,\ y_2+3z_2] =$$

$$= f(x_1,y_1,z_1) + f(x_2,y_2,z_2)$$

$\Rightarrow$ f je lineární dle def.

$$\left.\begin{aligned} f(e^1) &= f(1,0,0) = [1,0] \\ f(e^2) &= f(0,1,0) = [-1,1] \\ f(e^3) &= f(0,0,1) = [0,3] \end{aligned}\right\} \Rightarrow A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$$

b) ukážeme repr. matici: $\quad f(x,y,z) = [x-y,\ y+3z]$

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x-y \\ y+3z \end{pmatrix}$$

ověřím, že platí vzorec $f(x,y,z) = A\begin{pmatrix}x\\y\\z\end{pmatrix}$