Inverzní matice

$$\left(\begin{array}{c|c} \underbrace{\begin{matrix}1&2&1&1\\1&2&2&2\\2&2&2&2\\1&2&2&1\end{matrix}}_{A} & \begin{matrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{matrix}\end{array}\right) \sim \begin{array}{r}-\text{II}.\\ \\ -\text{II}.\\ -\text{II}.\end{array}\left(\begin{array}{cccc|cccc}0&0&-1&-1&1&-1&0&0\\1&2&2&2&0&1&0&0\\1&0&0&0&0&-1&1&0\\0&0&0&-1&0&-1&0&1\end{array}\right) \sim \begin{array}{r}-\text{IV}.\\ -\text{III}.\\ \\ \\ \end{array}\left(\begin{array}{cccc|cccc}0&0&-1&0&1&0&0&-1\\0&2&2&2&0&2&-1&0\\1&0&0&0&0&-1&1&0\\0&0&0&-1&0&-1&0&1\end{array}\right) \sim$$

$$\sim \begin{array}{r}\cdot(-1)\\ \\ \\ \cdot(-1)\end{array}\left(\begin{array}{cccc|cccc}0&0&1&0&-1&0&0&1\\0&2&2&2&0&2&-1&0\\1&0&0&0&0&-1&1&0\\0&0&0&1&0&1&0&-1\end{array}\right) \sim \overset{\text{Záměna ř.}}{\left(\begin{array}{cccc|cccc}1&0&0&0&0&-1&1&0\\0&2&2&2&0&2&-1&0\\0&0&1&0&-1&0&0&1\\0&0&0&1&0&1&0&-1\end{array}\right)} \sim$$

$$\sim \begin{array}{r} \\ -2\cdot\text{IV}.\\ \\ \\ \end{array}\left(\begin{array}{cccc|cccc}1&0&0&0&0&-1&1&0\\0&2&2&0&0&0&-1&2\\0&0&1&0&-1&0&0&1\\0&0&0&1&0&1&0&-1\end{array}\right) \sim \begin{array}{r} \\ -2\cdot\text{III}.\\ \\ \\ \end{array}\left(\begin{array}{cccc|cccc}1&0&0&0&0&-1&1&0\\0&2&0&0&2&0&-1&0\\0&0&1&0&-1&0&0&1\\0&0&0&1&0&1&0&-1\end{array}\right) \sim$$

$$\sim \begin{array}{r} \\ \cdot\frac{1}{2}\\ \\ \\ \end{array}\left(\begin{array}{c|c}\begin{matrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{matrix} & \underbrace{\begin{matrix}0&-1&1&0\\1&0&-\frac{1}{2}&0\\-1&0&0&1\\0&1&0&-1\end{matrix}}_{A^{-1}}\end{array}\right)$$

$$\left(\begin{array}{cccc|cccc}-7 & 2 & -9 & -2 & 1 & 0 & 0 & 0\\ 14 & -4 & 18 & 0 & 0 & 1 & 0 & 0\\ 12 & -4 & 16 & 0 & 0 & 0 & 1 & 0\\ 9 & -2 & 15 & 2 & 0 & 0 & 0 & 1\end{array}\right) \sim \begin{array}{r} \\ +2\cdot\text{I} \\ -\text{II}. \\ +\text{I}. \end{array}\left(\begin{array}{cccc|cccc}-7 & 2 & -9 & -2 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & -4 & 2 & 1 & 0 & 0\\ -2 & 0 & -2 & 0 & 0 & -1 & 1 & 0\\ 2 & 0 & 6 & 0 & 1 & 0 & 0 & 1\end{array}\right) \sim$$

rámène ř.

$$\sim \left(\begin{array}{cccc|cccc}2 & 0 & 6 & 0 & 1 & 0 & 0 & 1\\ -7 & 2 & -9 & -2 & 1 & 0 & 0 & 0\\ -2 & 0 & -2 & 0 & 0 & -1 & 1 & 0\\ 0 & 0 & 0 & -4 & 2 & 1 & 0 & 0\end{array}\right) \sim \begin{array}{r} \\ +\frac{7}{2}\cdot\text{I} \\ +\text{I}. \\ \\ \end{array}\left(\begin{array}{cccc|cccc}2 & 0 & 6 & 0 & 1 & 0 & 0 & 1\\ 0 & 2 & 12 & -2 & \frac{9}{2} & 0 & 0 & \frac{7}{2}\\ 0 & 0 & 4 & 0 & 1 & -1 & 1 & 1\\ 0 & 0 & 0 & -4 & 2 & 1 & 0 & 0\end{array}\right) \sim$$

$$\sim \begin{array}{r} \\ \cdot 2 \\ \\ \\ \end{array}\left(\begin{array}{cccc|cccc}2 & 0 & 6 & 0 & 1 & 0 & 0 & 1\\ 0 & 4 & 24 & -4 & 9 & 0 & 0 & 7\\ 0 & 0 & 4 & 0 & 1 & -1 & 1 & 1\\ 0 & 0 & 0 & -4 & 2 & 1 & 0 & 0\end{array}\right) \sim \begin{array}{r} \\ -\underline{\text{IV}}. \\ \\ \\ \end{array}\left(\begin{array}{cccc|cccc}2 & 0 & 6 & 0 & 1 & 0 & 0 & 1\\ 0 & 4 & 24 & 0 & 7 & -1 & 0 & 7\\ 0 & 0 & 4 & 0 & 1 & -1 & 1 & 1\\ 0 & 0 & 0 & -4 & 2 & 1 & 0 & 0\end{array}\right) \sim$$

$$\sim \begin{array}{r} \cdot 2 \\ -6\cdot\underline{\text{III}} \\ \\ \\ \end{array}\left(\begin{array}{cccc|cccc}4 & 0 & 12 & 0 & 2 & 0 & 0 & 2\\ 0 & 4 & 0 & 0 & 1 & 5 & -6 & 1\\ 0 & 0 & 4 & 0 & 1 & -1 & 1 & 1\\ 0 & 0 & 0 & -4 & 2 & 1 & 0 & 0\end{array}\right) \sim \begin{array}{r} -3\cdot\underline{\text{III}}. \\ \\ \\ \cdot(-1) \end{array}\left(\begin{matrix}4 & 0 & 0 & 0\\ 0 & 4 & 0 & 0\\ 0 & 0 & 4 & 0\\ 0 & 0 & 0 & 4\end{matrix}\,\middle|\,\underbrace{\begin{matrix}-1 & 3 & -3 & -1\\ 1 & 5 & -6 & 1\\ 1 & -1 & 1 & 1\\ -2 & -1 & 0 & 0\end{matrix}}_{}\right)$$

$$A^{} = \frac{1}{4} \cdot \begin{pmatrix} -1 & 3 & -3 & -1 \\ 1 & 5 & -6 & 1 \\ 1 & -1 & 1 & 1 \\ -2 & -1 & 0 & 0 \end{pmatrix}$$

$x \in \mathbb{R}$ parametr

$$\left(\begin{array}{cccc|cccc} 1 & -1 & 0 & 14 & 1 & 0 & 0 & 0 \\ 10 & 2 & 4 & 28 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 7 & 0 & 0 & 1 & 0 \\ 7 & 4 & 4 & 7x & 0 & 0 & 0 & 1 \end{array}\right) \sim \begin{array}{r} -2\cdot\overline{\underline{III}}. \\ -4\cdot\overline{\underline{III}}. \\ \\ -x\cdot\overline{\underline{III}} \end{array} \left(\begin{array}{cccc|cccc} 1 & -1 & 0 & 0 & 1 & 0 & -2 & 0 \\ 10 & 2 & 4 & 0 & 0 & 1 & -4 & 0 \\ 0 & 0 & 0 & 7 & 0 & 0 & 1 & 0 \\ 7 & 4 & 4 & 0 & 0 & 0 & -x & 1 \end{array}\right) \sim$$

$$\sim \begin{array}{r} \\ -10\cdot I \\ \\ -7\cdot I \end{array} \left(\begin{array}{cccc|cccc} 1 & -1 & 0 & 0 & 1 & 0 & -2 & 0 \\ 0 & 12 & 4 & 0 & -10 & 1 & 16 & 0 \\ 0 & 0 & 0 & 7 & 0 & 0 & 1 & 0 \\ 0 & 11 & 4 & 0 & -7 & 0 & 14-x & 1 \end{array}\right) \sim \begin{array}{r} \\ -\overline{\underline{IV}}. \\ \\ \\ \end{array} \left(\begin{array}{cccc|cccc} 1 & -1 & 0 & 0 & 1 & 0 & -2 & 0 \\ 0 & 1 & 0 & 0 & -3 & 1 & 2+x & -1 \\ 0 & 0 & 0 & 7 & 0 & 0 & 1 & 0 \\ 0 & 11 & 4 & 0 & -7 & 0 & 14-x & 1 \end{array}\right) \sim$$

$$\sim \begin{array}{r} +\overline{\underline{II}}. \\ \\ \\ -11\cdot\overline{\underline{II}}. \end{array} \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -2 & 1 & x & -1 \\ 0 & 1 & 0 & 0 & -3 & 1 & 2+x & -1 \\ 0 & 0 & 0 & 7 & 0 & 0 & 1 & 0 \\ 0 & 0 & 4 & 0 & 26 & -11 & -8-12x & 12 \end{array}\right) \sim$$

záměna

$$\begin{array}{r} \\ \\ \cdot\frac{1}{4} \\ \cdot\frac{1}{7} \end{array} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & \\ 0 & 1 & 0 & 0 & \\ 0 & 0 & 1 & 0 & \\ 0 & 0 & 0 & 1 & \end{array} \underbrace{\begin{array}{cccc} -2 & 1 & x & -1 \\ -3 & 1 & 2+x & -1 \\ \frac{13}{2} & -\frac{11}{4} & -2-3x & 3 \\ 0 & 0 & \frac{1}{7} & 0 \end{array}}\right)$$

Determinanty

A^{-1}

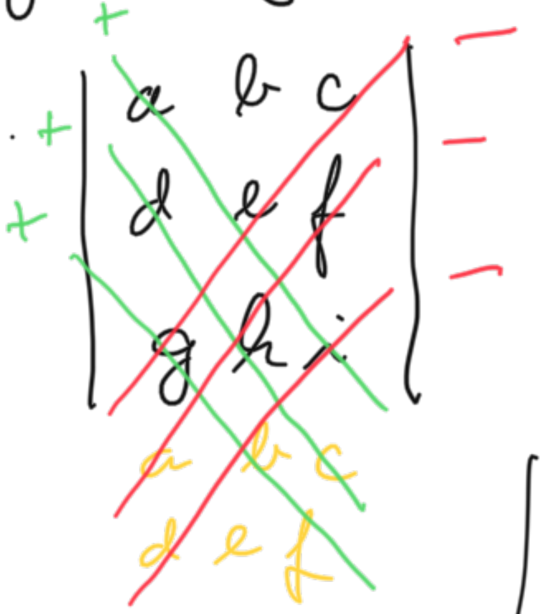
$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a \cdot \begin{vmatrix} e & f \\ h & i \end{vmatrix} - d \begin{vmatrix} b & c \\ h & i \end{vmatrix} + g \begin{vmatrix} b & c \\ e & f \end{vmatrix} =$$

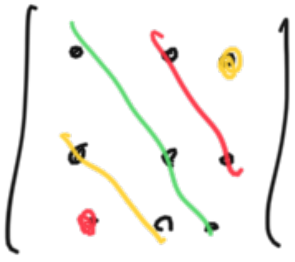
$$= a(ei - fh) - d(bi - ch) + g(bf - ce) =$$

$$= aei - afh - bdi + cdh + bfg - ceg =$$

$$= \underline{aei} + \underline{cdh} + \underline{bfg} - \underline{afh} - \underline{bdi} - \underline{ceg}$$



Sarrusovo pravidlo



$$\begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 2 & 3 \\ 3 & 5 & 4 & 1 & 2 \\ 2 & 2 & 2 & 3 & 7 \end{vmatrix} = \begin{array}{|ccccc|} & -\text{II.} & & & \\ \overset{+}{1} & \overset{-}{0} & 1 & 1 & 1 \\ 2 & \overset{+}{0} & 1 & 2 & 3 \\ 3 & \overset{-}{1} & 4 & 1 & 2 \\ 2 & 0 & 2 & 3 & 7 \end{array} \underset{\text{rozvoj}}{=} -1 \cdot \begin{vmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & 2 & 3 \\ 2 & 2 & 3 & 7 \end{vmatrix} =$$

$$\begin{vmatrix} 8 & 8 & 8 & 0 & 2 \end{vmatrix} \qquad \begin{vmatrix} 8 & 0 & 8 & 0 & 2 \end{vmatrix} \ \text{dle 2.sl.} \qquad \begin{vmatrix} 8 & 8 & 0 & 2 \end{vmatrix}$$

$$= - \begin{vmatrix} \overset{-II.}{{}^{+}0} & 1 & 1 & 1 \\ {}^{-}1 & 1 & 2 & 3 \\ 0 & 2 & 3 & 7 \\ 0 & 8 & 0 & 2 \end{vmatrix} \underset{\substack{\text{rozvoj} \\ \text{dle 1.sl.}}}{=} -(-1)\cdot 1 \cdot \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 7 \\ 8 & 0 & 2 \end{vmatrix} =$$

$$1\cdot 3\cdot 2 + 2\cdot 0\cdot 1 + 1\cdot 7\cdot 8 - 8\cdot 3\cdot 1 - 7\cdot 0\cdot 1 - 2\cdot 2\cdot 1 =$$

$$= 6 + 56 - 24 - 4 =$$

$$= 34$$

$$\begin{array}{r} \\ -3\cdot I \\ \\ \end{array} \begin{vmatrix} 1 & 1 & 1 \\ -1 & 0 & 4 \\ 8 & 0 & 2 \end{vmatrix} =$$

$$= - \begin{vmatrix} -1 & 4 \\ 8 & 2 \end{vmatrix} = -(-2-32) =$$

$$= 34$$

$$\begin{vmatrix} -1 & 3 & 3 & 1 & 7 \\ 1 & 4 & 4 & -6 & 1 \\ 2 & 1 & -1 & 1 & 1 \\ 1 & -2 & -1 & 1 & -6 \\ 9 & 1 & 2 & 2 & 1 \end{vmatrix} = \begin{vmatrix} -1 & \overset{-III.}{0} & \overset{+IV.}{4} & 1 & 7 \\ 1 & 0 & -2 & -6 & 1 \\ 2 & 2 & 0 & 1 & 1 \\ 1 & -1 & 0 & 1 & -6 \\ 9 & -1 & 4 & 2 & 1 \end{vmatrix} = \begin{array}{r} +2\cdot II. \\ \\ \\ \\ -I. \end{array} \begin{vmatrix} 1^{+} & 0^{-} & {}^{+}0 & -11 & 9 \\ 1 & 0 & {}^{-}{-2} & -6 & 1 \\ 2 & 2 & 0 & 1 & 1 \\ 1 & -1 & 0 & 1 & -6 \\ 10 & -1 & 0 & 1 & -6 \end{vmatrix} =$$

rozvoj dle 3. sl.

$$= (-1)\cdot(-2)\cdot \begin{vmatrix} 1 & 0 & -11 & 9 \\ 2 & 2 & 1 & 1 \\ 1 & -1 & 1 & -6 \\ 10 & -1 & 1 & -6 \end{vmatrix} = 2\cdot \begin{array}{r} + \\ - \\ + \\ -\mathrm{III}\cdot \; - \end{array} \begin{vmatrix} 1 & 0 & -11 & 9 \\ 2 & 2 & 1 & 1 \\ 1 & -1 & 1 & -6 \\ 9 & 0 & 0 & 0 \end{vmatrix}$$

rozvoj dle 4. ř.

$$= -2\cdot 9\cdot \begin{vmatrix} 0 & -11 & 9 \\ 2 & 1 & 1 \\ -1 & 1 & -6 \end{vmatrix} =$$

$$= -2\cdot 9\cdot \begin{array}{r} + \\ - \\ + \end{array} \begin{vmatrix} 0 & -11 & 9 \\ 0 & 3 & -11 \\ -1 & 1 & -6 \end{vmatrix} \begin{array}{l} \\ +2\cdot\mathrm{III} \\ \\ \end{array} =$$

rozvoj dle 1. sl.

$$-2\cdot 9\cdot(-1) \begin{vmatrix} -11 & 9 \\ 3 & -11 \end{vmatrix} = 2\cdot 9\cdot\left(11^2 - 3\cdot 9\right) =$$

$$= 2\cdot 9\left(121 - 27\right) =$$

$$= \underline{2\cdot 9\cdot 94} = 1692$$