$f(x,y)=\sqrt{x^2+y^2}$

$D_f=\mathbb{R}^2$

$$\frac{\partial f}{\partial x}(x,y)=\frac{1}{2\sqrt{x^2+y^2}}\cdot 2x=\frac{x}{\sqrt{x^2+y^2}}, \quad [x,y]\neq[0,0]$$

$\frac{\partial f}{\partial x}(0,0)$ .... neexistuje

$x\mapsto f(x,0)=\sqrt{x^2}=|x|$

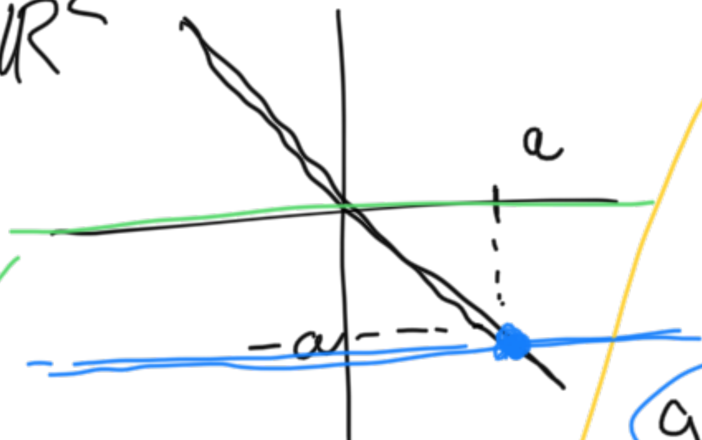
$$\frac{\partial f}{\partial y}(x,y)=\frac{y}{\sqrt{x^2+y^2}}, \quad [x,y]\neq[0,0]$$

$\frac{\partial f}{\partial y}(0,0)$ neexistuje

$f(x,y)=\sqrt[3]{x^3+y^3}$

$D_f=\mathbb{R}^2$

$$\frac{\partial f}{\partial x}(x,y)=\frac{1}{3\sqrt[3]{(x^3+y^3)^2}}\cdot 3x^2=\frac{x^2}{\sqrt[3]{(x^3+y^3)^2}}, \quad \underline{y\neq -x}$$



$$\frac{\partial f}{\partial x}(a,-a)=g'(a)$$

$g(x)=f(x,-a)=\sqrt[3]{x^3-a^3}$ ... spojitá na $\mathbb{R}$

$$g'(a)=\lim_{x\to a} g'(x)=\lim_{x\to a}\frac{\partial f}{\partial x}(x,-a)=$$

$$=\lim_{x\to a}\frac{x^2}{\sqrt[3]{(x^3-a^3)^2}}=\frac{a^2}{0+}=+\infty, \quad \underline{a\neq 0}$$

$$\frac{\partial f}{\partial x}(0,0) = 1$$

$$x \mapsto f(x,0) = \sqrt[3]{x^3} = x$$

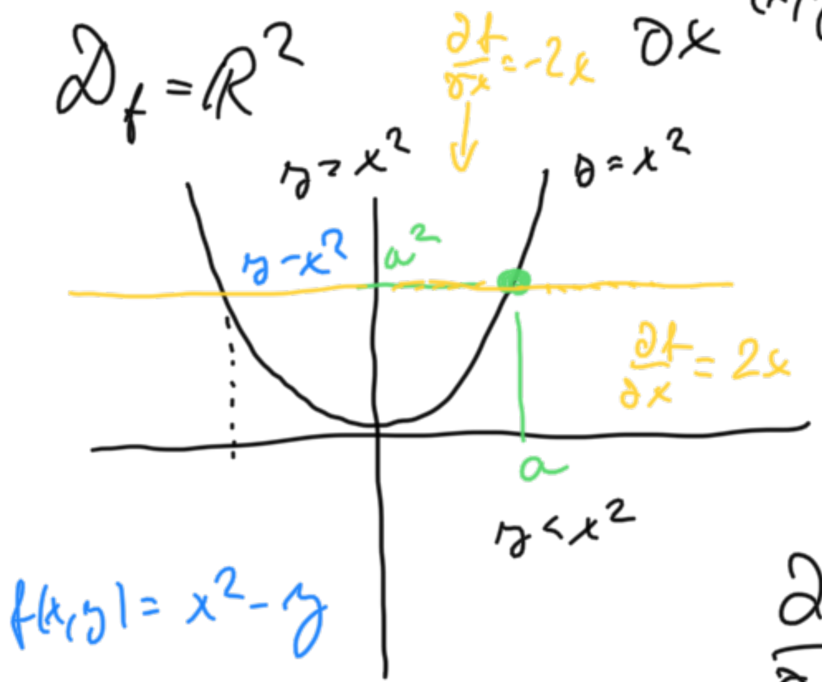
symetrie: $\frac{\partial f}{\partial y} = \frac{y^2}{\sqrt[3]{(x^3+y^3)^2}}$, $y \neq -x$

$\frac{\partial f}{\partial y}(a,-a) = +\infty$ pro $a \neq 0$

$$\frac{\partial f}{\partial y}(0,0) = 1$$

$f(x,y) = |y - x^2|$

$D_f = \mathbb{R}^2$



$$\frac{\partial f}{\partial x}(x,y) = \begin{cases} -2x, & y > x^2 \\ 2x, & y < x^2 \end{cases}$$

$$\frac{\partial f}{\partial y}(x,y) = \begin{cases} 1, & y > x^2 \\ -1, & y < x^2 \end{cases}$$

$$\frac{\partial f}{\partial x}(a,a^2) \underset{\uparrow}{=} \lim_{x \to a} \frac{\partial f}{\partial x}(x,a^2) \quad \begin{cases} \lim_{x \to a+} \\ \lim_{x \to a-} \end{cases}$$

$x \in \mathbb{R}$

f spojitá

a) $\underline{a > 0}$ $\quad \lim_{x \to a+} 2x = 2a$

∄ parc. der. neexistuje

$$\lim_{x \to a-} -2x = -2a$$

b) $\underline{a<0}$

$$\begin{cases} \lim_{x \to a+} -2x = -2a \\ \lim_{x \to a-} 2x = 2a \end{cases} \neq \quad \text{neexistuje}$$

c) $\underline{a=0}$ $\quad \frac{\partial f}{\partial x}(0,0) = \lim_{x \to 0} 2x = \underline{0}$

$$\frac{\partial f}{\partial y}(a,a^2) = \lim_{y \to a^2} \frac{\partial f}{\partial y}(a,y) = \begin{cases} \lim_{y \to a^2+} 1 = 1 \\ \lim_{y \to a^2-} -1 = -1 \end{cases} \neq \quad \text{neexistuje}$$

$f(x,y,k) = x \cdot y \cdot k$

$\varphi_1(s,t) = s + t^2$

$\varphi_2(s,t) = s \cdot t$

$\varphi_3(s,t) = s$

$f: \mathbb{R}^3 \to \mathbb{R}$

$\varphi_j: \mathbb{R}^2 \to \mathbb{R}$

$F: \mathbb{R}^2 \to \mathbb{R}$

$F(s,t) = \underline{f}(\underline{\varphi_1(s,t)}, \underline{\varphi_2(s,t)}, \underline{\varphi_3(s,t)})$

$\frac{\partial F}{\ } \quad \frac{\partial F}{\ }$

$\| F(s,t) = (s+t^2) \cdot s \cdot t \cdot s =$

$$\frac{\partial F}{\partial s}(s,t) = \frac{\partial f}{\partial x}\left(\varphi_1(s,t),\dots,\varphi_3(s,t)\right)\cdot\frac{\partial \varphi_1}{\partial s}(s,t) + \frac{\partial f}{\partial y}(\dots)\cdot\frac{\partial \varphi_2}{\partial s}(s,t) + \frac{\partial f}{\partial z}(\dots)\cdot\frac{\partial \varphi_3}{\partial s}(s,t)$$

$$\frac{\partial f}{\partial x}(x,y,z) = yz \qquad \frac{\partial \varphi_1}{\partial s}(s,t) = 1 \qquad \frac{\partial \varphi_1}{\partial t}(s,t) = 2t$$

$$\frac{\partial f}{\partial y}(x,y,z) = xz \qquad \frac{\partial \varphi_2}{\partial s}(s,t) = t \qquad \frac{\partial \varphi_2}{\partial t}(s,t) = s$$

$$\frac{\partial f}{\partial z}(x,y,z) = xy \qquad \frac{\partial \varphi_3}{\partial s}(s,t) = 1 \qquad \frac{\partial \varphi_3}{\partial t}(s,t) = 0$$

na $\mathbb{R}^3$ na $\mathbb{R}^2$

$$\frac{\partial F}{\partial s}(s,t) = \varphi_2(s,t)\cdot\varphi_3(s,t)\cdot 1 + \varphi_1(s,t)\cdot\varphi_3(s,t)\cdot t + \varphi_1(s,t)\cdot\varphi_2(s,t)\cdot 1 =$$

$$= sts + (s+t^2)s\cdot t + (s+t^2)st = st(3s+2t^2)$$

$$\frac{\partial F}{\partial t}(s,t) = sts\cdot 2t + (s+t^2)\cdot s\cdot s + 0 = s^2(s+3t^2)$$

$$f(x,y) = xy\,\frac{x^2-y^2}{x^2+y^2}\,,\quad [x,y]\neq[0,0]$$

$$f(0,0)=0$$

Spočítme $\frac{\partial^2 f}{\partial x \partial y}(0,0)$ a $\frac{\partial^2 f}{\partial y \partial x}(0,0)$:

$$\frac{\partial f}{\partial x}(x,y) = y\cdot\frac{x^2-y^2}{x^2+y^2} + xy\,\frac{2x(x^2+y^2)-(x^2-y^2)\cdot 2x}{(x^2+y^2)^2} =$$

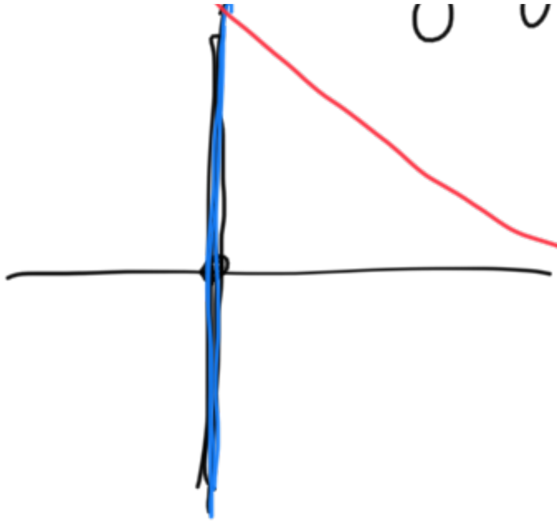
$$= y\left(\frac{x^2-y^2}{x^2+y^2} + \frac{4x^2y^2}{(x^2+y^2)^2}\right),\quad [x,y]\neq[0,0]$$

$$\frac{\partial f}{\partial x}(0,0) = 0$$

$$x \mapsto f(x,0) = 0 \quad \forall x\in\mathbb{R}$$

$$\frac{\partial^2 f}{\partial x \partial y}(0,0) = g'(y) = -1$$

$$g(y) = \frac{\partial f}{\partial x}(0,y) = -y\,,\quad \text{pro } \forall y\in\mathbb{R}\ !$$



$$\frac{\partial f}{\partial y}(x,y) = x\left(\frac{x^2-y^2}{x^2+y^2} - \frac{4x^2y^2}{(x^2+y^2)^2}\right), \quad [x,y]\neq[0,0]$$

$y \mapsto f(0,y)=0$ pro $\forall y\in\mathbb{R}$, tedy $\dfrac{\partial f}{\partial y}(0,0)=0$

$g(x)=\dfrac{\partial f}{\partial y}(x,0)=x$ pro $\forall x\in\mathbb{R}$ $\Rightarrow$ $\dfrac{\partial^2 f}{\partial y\,\partial x}(0,0)=1$