• $A \subset \mathbb{R}^n$: $\overline{A} = \{x \in \mathbb{R}^n;\ \forall r > 0 : B(x,r) \cap A \neq \emptyset\}$

• $\operatorname{Int} A \cap H(A) = \emptyset$: $x \in \operatorname{Int} A \Rightarrow \exists r > 0 : B(x,r) \subset A \Rightarrow$

$\Rightarrow B(x,r) \cap (\mathbb{R}^n \setminus A) = \emptyset \Rightarrow x \notin H(A)$

• $\overline{A} = \operatorname{Int} A \cup H(A)$:

$A \cup H(A)$

„$\supset$" jasná

„$\subset$" $x \in \overline{A}$, $x \notin H(A)$

$\Downarrow$

$\exists r > 0$: $B(x,r) \cap A = \emptyset$ není pravda

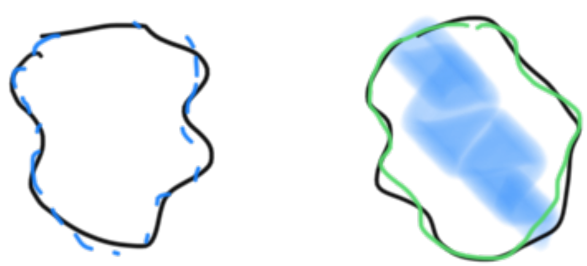
$B(x,r) \cap (\mathbb{R}^n \setminus A) = \emptyset$

$\Updownarrow$

$B(x,r) \subset A \Rightarrow x \in \operatorname{Int} A$

• $H(A) = \overline{A} \setminus \operatorname{Int} A$: „$\subset$"

„$\supset$" z předch.



• $H(A) = H(\mathbb{R}^n \setminus A)$

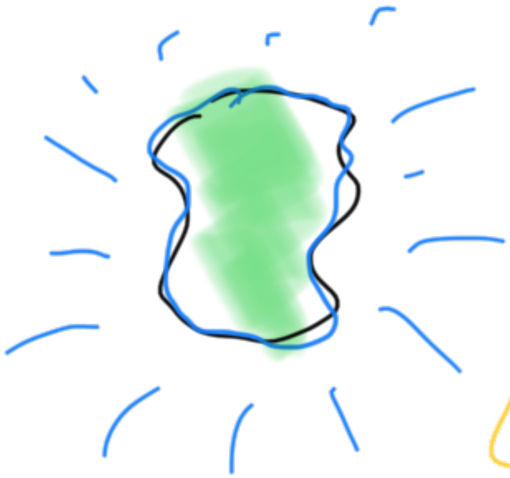
$x \in H(A) \Leftrightarrow \forall r > 0 : B(x,r) \cap \overbrace{A}^{\mathbb{R}^n \setminus (\mathbb{R}^n \setminus A)} \neq \emptyset$

&

$B(x,r) \cap (\mathbb{R}^n \setminus A) \neq \emptyset$

• $\operatorname{Int} A = \mathbb{R}^n \setminus \overline{\mathbb{R}^n \setminus A}$

- $\operatorname{Int} A = \mathbb{R}^n \setminus \underbrace{\overline{\mathbb{R}^n \setminus A}}$



$$\| $$

$$\{x \in \mathbb{R}^n ;\ \forall r>0 : B(x,r) \cap (\mathbb{R}^n \setminus A) \neq \emptyset\}$$

$$\{x \in \mathbb{R}^n ;\ \exists r>0 : \underbrace{B(x,r) \cap (\mathbb{R}^n \setminus A) = \emptyset}_{B(x,r) \subset A}\} = \operatorname{Int} A$$

- $\overline{A} = \mathbb{R}^n \setminus \operatorname{Int}(\mathbb{R}^n \setminus A)$

$r \Uparrow$

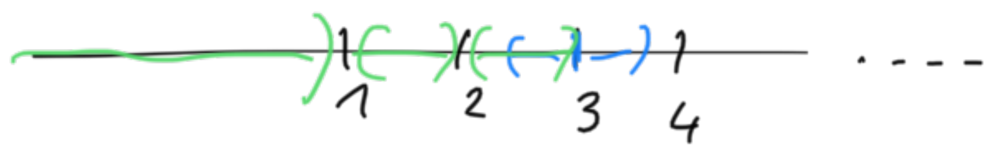
$$B = \mathbb{R}^n \setminus A$$

$$\operatorname{Int} B = \mathbb{R}^n \setminus \overline{\mathbb{R}^n \setminus B} = \mathbb{R}^n \setminus \overline{A}$$

$$\mathbb{R}^n \setminus \operatorname{Int} B = \overline{A}$$

Podmnožiny $\mathbb{R}$:

$\mathbb{N}$



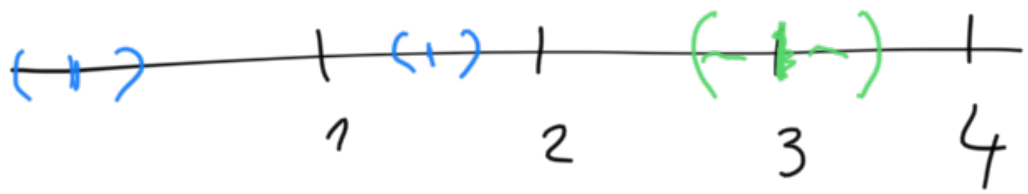
$\mathbb{N}$ není otevřená, $\operatorname{Int} \mathbb{N} = \emptyset$

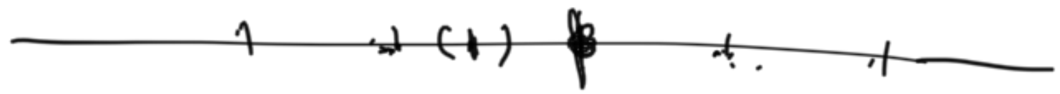
$\mathbb{R} \setminus \mathbb{N} = (-\infty, 1) \cup (1,2) \cup (2,3) \cup \ldots\ldots$ .... otevřená (sjednocení otev. V3) ol. ol. ol.

$\Rightarrow \mathbb{N}$ je uzavřená (doplněk je otevřená, V5)

$\overline{\mathbb{N}} = \mathbb{N}$

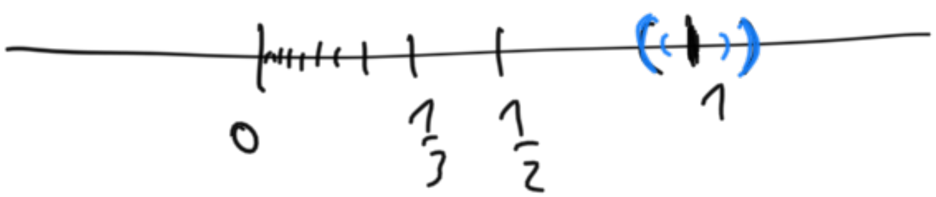
$H(\mathbb{N}) = \overline{\mathbb{N}} \setminus \operatorname{Int} \mathbb{N} = \mathbb{N}$





$A = \{ \frac{1}{n};\ n \in \mathbb{N} \}$

A není otevřená; $\operatorname{Int} A = \emptyset$



A není uzavřená:

$x^n = \frac{1}{n} \in A$

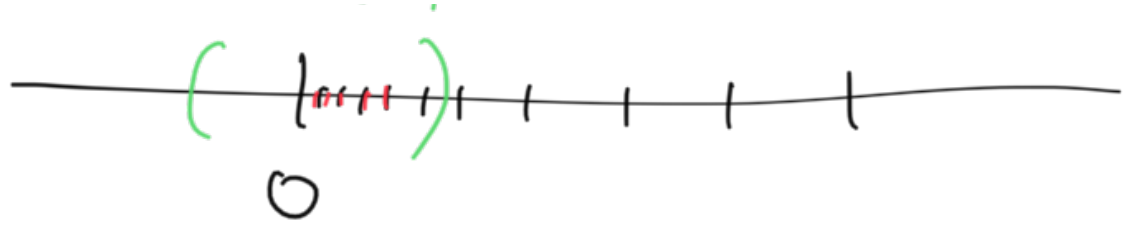
$x^n \to 0 \notin A \Rightarrow$ A není uzav. V5

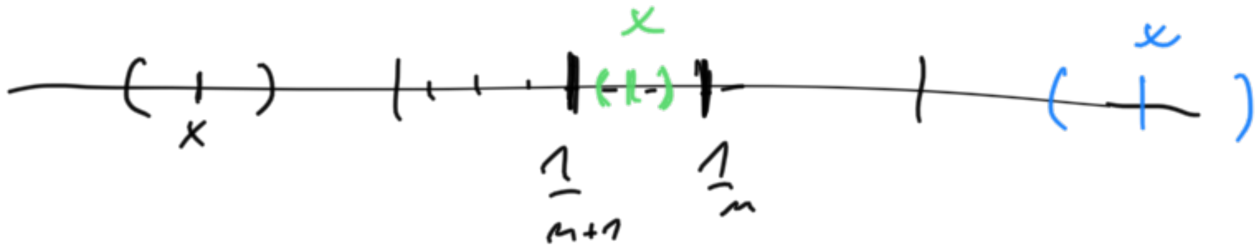
$\overline{A} = A \cup \{0\}$

$H(A) = \overline{A} \setminus \operatorname{Int} A = A \cup \{0\} = \overline{A}$

$B(0,r)$

$n > \frac{1}{r}:\ \frac{1}{n} \in B(0,r)$



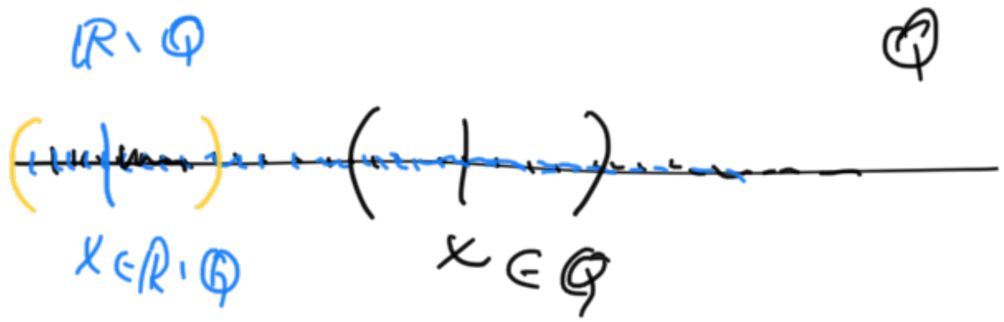


$\mathbb{Q}$ .... ani otev., ani uzav.

Je-li $x \in \mathbb{Q}$ a $r > 0$, pak z věty o hustotě irac. čísel $\Rightarrow B(x,r) \cap (\mathbb{R}^n \setminus \mathbb{Q}) \neq \emptyset$

$\Rightarrow x$ není vnitřní bod $\mathbb{Q}$

$\Rightarrow \operatorname{Int} \mathbb{Q} = \emptyset$



$\overline{\mathbb{Q}} = \mathbb{R}$: Je-li $x \in \mathbb{R}$ a $r > 0$, pak dle věty o hustotě rac. čísel:

$B(x,r) \cap \mathbb{Q} \neq \emptyset \Rightarrow x \in \overline{\mathbb{Q}}$

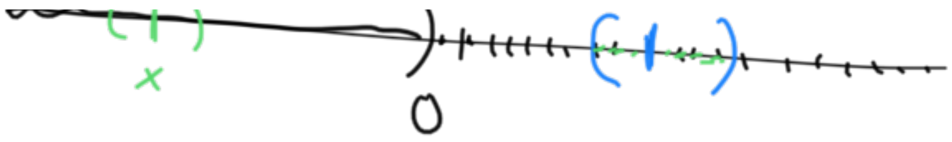
$$H(\mathbb{Q}) = \overline{\mathbb{Q}} \setminus \operatorname{Int} \mathbb{Q} = \mathbb{R} \setminus \emptyset = \mathbb{R}$$

$$A = (-\infty, 0) \cup \{x \in \mathbb{Q};\ x > 0\}$$

$x \in \mathbb{Q}$, $x > 0$, $r > 0$ libovolné $\Rightarrow$

$B(x,r) \cap (\mathbb{R} \setminus \mathbb{Q}) \neq \emptyset$

$\operatorname{Int} A = (-\infty, 0)$



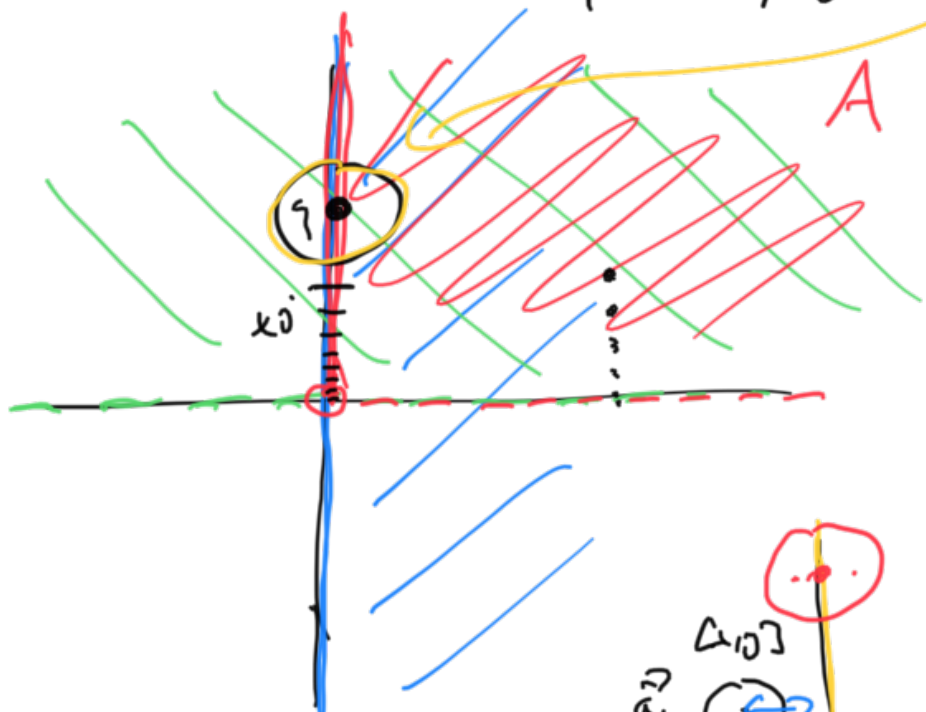
$\Rightarrow x \notin \operatorname{Int} A$

$$\overline{A} = \mathbb{R}$$

$\Rightarrow$ A není otevřená ani uzavřená

$H(A) = \overline{A} \setminus \operatorname{Int} A = \langle 0, +\infty)$

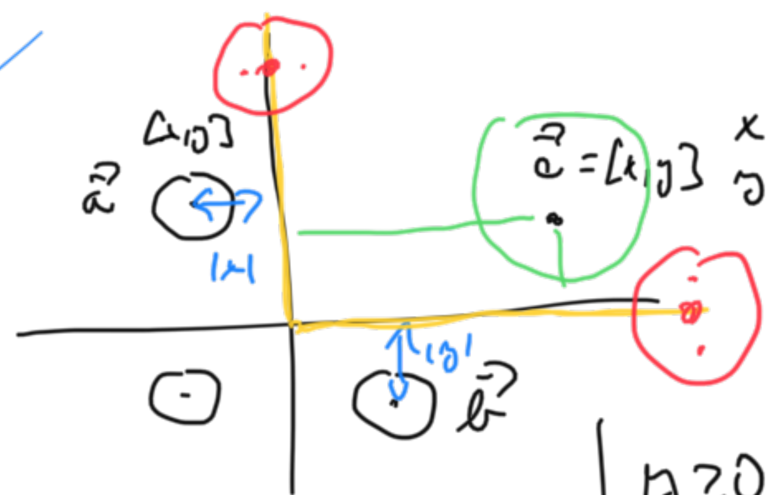
$A = \{(x, y) \in \mathbb{R}^2 ;\ x \geq 0,\ y > 0\}$ ani uzavřená ani otevřená



V4

$\vec{x}^j = [0, \frac{1}{j}] \to o = [0, 0]$

$\vec{x}^j \in A$, $o \notin A$



$x > 0$
$y > 0$

$\vec{c} \in \operatorname{Int} A$

$B(\vec{c}, \min\{x, y\}) \subset A$

$\vec{a} = [x, y],\ x < 0$

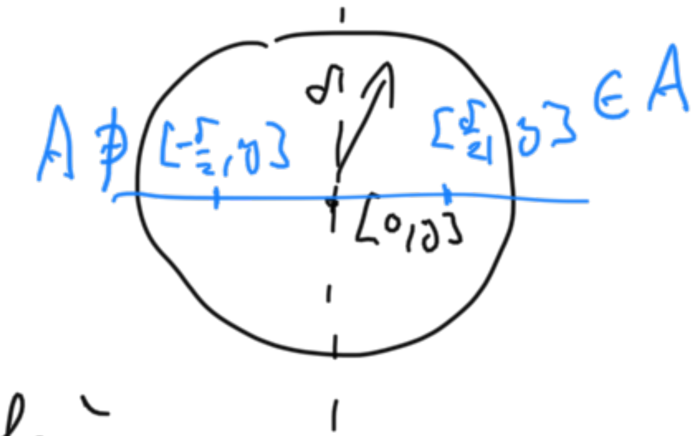
$y \geq 0 :\ [0, y] \in \mathcal{H}(A)$

$B(\vec{a}, |x|) \cap A = \emptyset$

$\vec{b} = [x,y],\ y<0$

$B(\vec{b}, |y|) \cap A = \emptyset$

$B([0,y],\delta)$



podobně $x \geq 0$: $[x,0] \in \partial(A)$

Celkem: $H(A) = \{[x,0];\ x \geq 0\} \cup \{[0,y];\ y \geq 0\}$

$\overline{A} = \{[x,y];\ x \geq 0,\ y \geq 0\}$

$\operatorname{Int} A = \{[x,y];\ x > 0,\ y > 0\}$